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International Baccalaureate®
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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

| 2 | 2 | 1 | 4 | – | 7 | 4 | 0 | 3 |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical Studies SL** *formula booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Let $p = \dfrac{2\cos x - \tan x}{\sqrt{y} - z}$.

    (a) Calculate the value of p when $x = 45°$, $y = 8192$, and $z = 64$. Write down your full calculator display. *[2]*

    (b) Write down your answer to part (a)

        (i) correct to two decimal places;

        (ii) correct to four significant figures;

        (iii) in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. *[4]*

Working:

Answers:

(a) ....

(b) (i) ....

    (ii) ....

    (iii) ....

Turn over

2. A class of 13 Mathematics students received the following grades in their final IB examination.

$$3 \quad 5 \quad 3 \quad 4 \quad 7 \quad 3 \quad 2 \quad 7 \quad 5 \quad 6 \quad 5 \quad 3 \quad 4$$

For these grades, find

(a) the mode; *[1]*

(b) the median; *[2]*

(c) the upper quartile; *[1]*

(d) the interquartile range. *[2]*

Working:

Answers:

(a) ....

(b) ....

(c) ....

(d) ....

3. Consider the three propositions *p*, *q* and *r*.

> *p*: *The food is well cooked*
> *q*: *The drinks are chilled*
> *r*: *Dinner is spoilt*

(a)   Write the following compound proposition in words.

$$(p \wedge q) \Rightarrow \neg r$$ [3]

(b)   Complete the following truth table.

p	*q*	*r*	$p \wedge q$	$\neg r$	$(p \wedge q) \Rightarrow \neg r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

[3]

Working:

Answers:

(a)   . . . .
. . . .
. . . .
. . . .

4.   A study was carried out to determine whether the country chosen by students for their university studies was influenced by a person's gender. A random sample was taken. The results are shown in the following table.

	Country Chosen		
	USA	**Australia**	**UK**
Male	55	26	40
Female	25	31	41

A χ^2 test was performed at the 1% significance level.
The critical value for this test is 9.210.

(a)   State the null hypothesis.   *[1]*

(b)   Write down the number of degrees of freedom.   *[1]*

(c)   Write down

   (i)   the χ^2 statistic;

   (ii)   the associated *p*-value.   *[2]*

(d)   State, giving a reason, whether the null hypothesis should be accepted.   *[2]*

Working:

Answers:

(a)   . . . .
. . . .
. . . .

(b)   . . . .

(c)   (i)   . . . .

   (ii)   . . . .

(d)   . . . .
. . . .
. . . .

5. *In this question give all answers correct to two decimal places.*

Dumisani has received a scholarship of 5000 US dollars (USD) to study in Singapore.
He has to travel from South Africa and must change USD for his air fare of 6600 South African rand (ZAR).

The exchange rate is $1\,\text{USD} = 8.2421\,\text{ZAR}$.

(a)   Calculate the number of USD that Dumisani must change to pay for his air fare.   *[2]*

On arrival in Singapore, Dumisani changes 3000 USD to Singapore dollars (SGD) to pay for his school fees. There is a 2.8 % commission charged on the exchange.

(b)   Calculate the value, **in USD**, of the commission that Dumisani has to pay.   *[2]*

The exchange rate is $1\,\text{USD} = 1.29903\,\text{SGD}$.

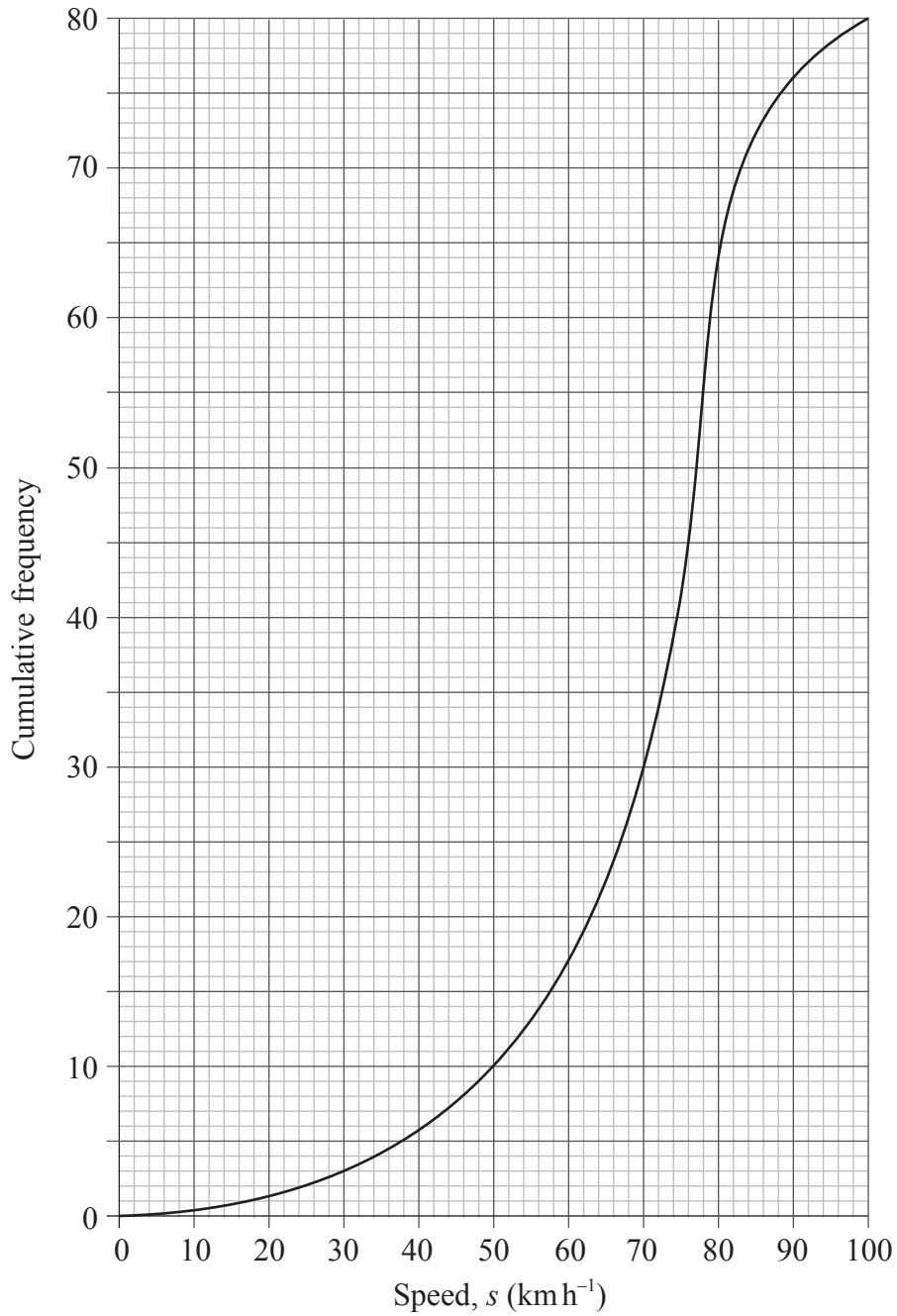(c)   Calculate the number of SGD Dumisani receives.   *[2]*

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

6. The cumulative frequency graph represents the speed, s, in $\text{km}\,\text{h}^{-1}$, of 80 cars passing a speed camera.

(a) Write down the number of cars passing the camera with speed of less than or equal to $50\,\text{km}\,\text{h}^{-1}$. *[1]*

(This question continues on the following page)

(Question 6 continued)

(b) Complete the following grouped frequency table for s, the speed of the cars passing the camera.

$s\left(\mathrm{km\,h^{-1}}\right)$	$0 < s \le 50$	$50 < s \le 70$	$70 < s \le 80$	$80 < s \le 90$	$90 < s \le 100$
Frequency			34		4

[1]

(c) Write down the mid-interval value of the $50 < s \le 70$ interval.   *[1]*

(d) Use your graphic display calculator to find an estimate of

(i)   the mean speed of the cars passing the camera;

(ii) the standard deviation of the speed of the cars passing the camera.   *[3]*
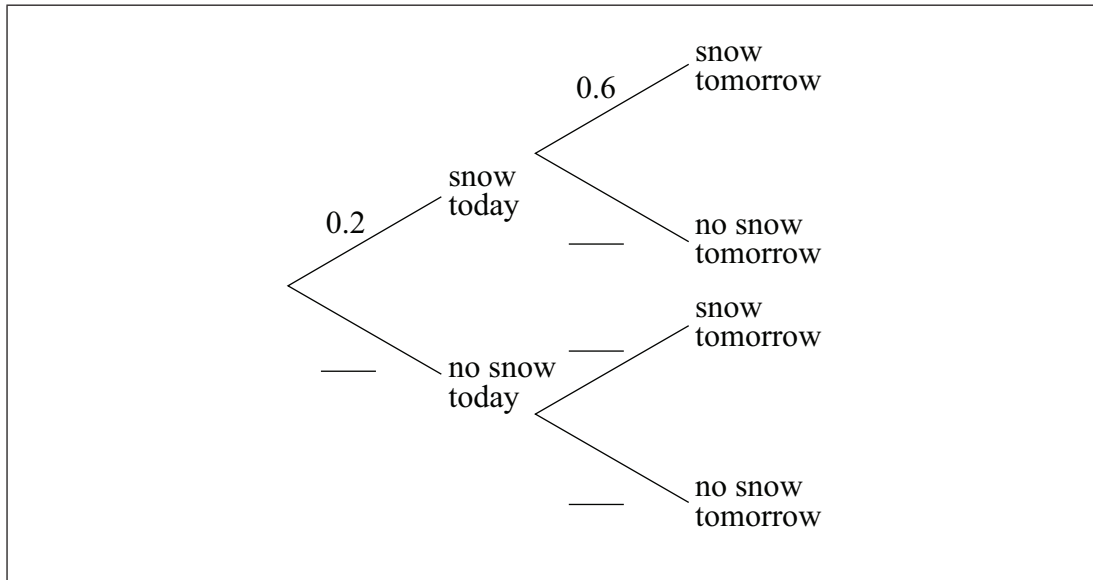
Working:

Answers:

(a) . . . .

(c) . . . .

(d) (i)   . . . .

(ii) . . . .

7. The probability that it snows today is 0.2. If it does snow today, the probability that it will snow tomorrow is 0.6. If it does not snow today, the probability that it will not snow tomorrow is 0.9.

(a) Using the information given, complete the following tree diagram.

[3]

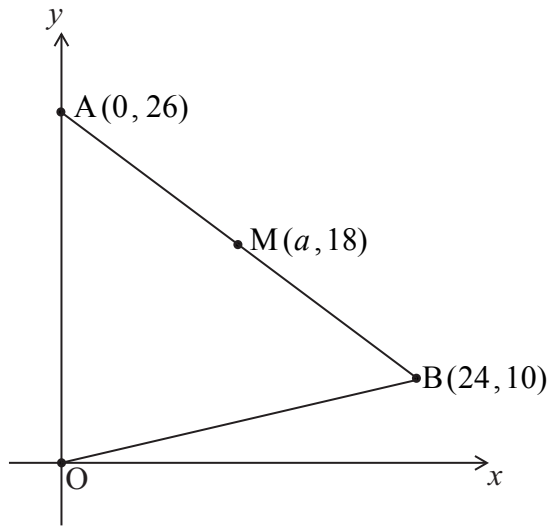(b) Calculate the probability that it will snow tomorrow. *[3]*

Working:

Answers:

(b) . . . .

9. The diagram shows the points $M(a,18)$ and $B(24,10)$. The straight line BM intersects the y-axis at $A(0,26)$. M is the midpoint of the line segment AB.

diagram not to scale

(a) Write down the value of a. [1]

(b) Find the gradient of the line AB. [2]

(c) Decide whether triangle OAM is a right-angled triangle. Justify your answer. [3]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

. . . .

12. Ludmila takes a loan of 320 000 Brazilian Real (BRL) from a bank for two years at a nominal annual interest rate of 10 %, **compounded half yearly**.

(a) Write down the number of times interest is added to the loan in the two years. *[1]*

(b) Calculate the **exact** amount of money that Ludmila must repay at the end of the two years. *[3]*

Ludmila estimates that she will have to repay 360 000 BRL at the end of the two years.

(c) Calculate the percentage error in her estimate. *[2]*

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

Turn over

14. Two propositions are defined as follows:

p : *Quadrilateral ABCD has two diagonals that are equal in length.*
q : *Quadrilateral ABCD is a rectangle.*

(a) Express the following in symbolic form.

"A rectangle always has two diagonals that are equal in length." *[2]*

(b) Write down in symbolic form the converse of the statement in (a). *[1]*

(c) Determine, **without** using a truth table, whether the statements in (a) and (b) are logically equivalent. *[2]*

(d) Write down the name of the statement that is logically equivalent to the converse. *[1]*

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

. . . .

. . . .

(d) . . . .

Turn over