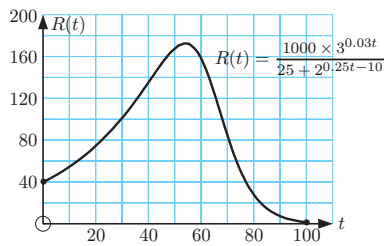
7 **a**



b ≈ 77.2 million tonnes per year

c ≈ 170 million tonnes per year, ≈ 50.2 years after mining begins

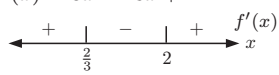
d when $t \approx 28.0$ years, and $t \approx 63.1$ years

REVIEW SET 21C

1 **a** $A = -3$, $B = 7$

b local maximum at $(-1, 9)$, local minimum at $(1, 5)$

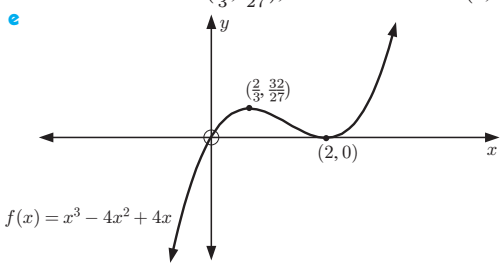
2 **a** y-intercept is 0 **b** $f'(x) = 3x^2 - 8x + 4$



c increasing for $x \leqslant \frac{2}{3}$, $x \geqslant 2$, decreasing for $\frac{2}{3} \leqslant x \leqslant 2$

d local maximum at $(\frac{2}{3}, \frac{32}{27})$, local minimum at $(2, 0)$

e



3 **a** **i** €312 **ii** €1218.75

b **i** 9.1 euros per km h^{-1} **ii** 7.5 euros per km h^{-1}

c 3 km h^{-1}

4 **a** $y = \dfrac{1}{x^2}$ m **c** $C'(x) = 4x - \dfrac{8}{x^2}$

d 1.26 m by 1.26 m by 0.630 m **e** \$9.52

5 **b** 42 days

6 maximum value is $25\frac{1}{3}$, minimum value is $-1\frac{2}{3}$

7 **a** Domain $= \{t \mid 0 \leqslant t \leqslant 5.9\}$ **b** 49.8 m **c** 36.0 m

EXERCISE 22A

1 **a** 0.31 **b** 0.305 **c** $3.050\,85 \times 10^{-1}$

2 **a** **i** If Farouk studies for the test, Farouk scores a good mark.

ii If Farouk does not study for the test, Farouk does not score a good mark.

b **i** $q \Rightarrow p$ **ii** $\neg q \Rightarrow \neg p$ **c**

p	q	$p \Rightarrow q$	$\neg p$	$\neg q$	$\neg q \Rightarrow \neg p$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

same truth table column
$\therefore$ logically equivalent

3 **a** ≈ 0.355 **b** $k \approx 6.01$

4 **a** A$(0, 9)$, B$(6, 0)$ **b** M$(3, 4.5)$ **c** $-\frac{3}{2}$

d $4x - 6y = -15$

5 **a** ≈ 26 cm **b**

Length (cm)	*Frequency*
$0 < x \leqslant 10$	15
$10 < x \leqslant 20$	40
$20 < x \leqslant 30$	75
$30 < x \leqslant 40$	51
$40 < x \leqslant 50$	19

c 26 cm

6 **a** $f'(x) = 10x^4 - 10x$ **b** $y = -2$

7 **a** *Option A*: \$15 124.98, *Option B*: \$15 167.93

b *Option B*, by \$42.95

8 **a** 9.46×10^{12} km **b** 2.08×10^{18} km

9 **a** **i** $x = 3, 4, 5, 10$

ii $x = 3, 5$

iii $x = 1, 3, 5, 7, 8, 9$

b

p	q	$\neg p$	$\neg p \wedge q$	$p \veebar q$
T	T	F	F	F
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

10 **a** $\mathrm{P}(G) = \frac{1}{3}$ **b** 75 'reds'

c **i** \$0 **ii** The game is fair.

11 **a** $x = 5$ **b** C$(-5, 2)$

c $5x + 2y + 21 = 0$ **d** D$(5, -23)$

12 **a** **i** H_0: *time spent on co-curricular activities* and *grade average* are independent.

ii df $= (3 - 1) \times (3 - 1) = 4$ **iii** $\chi^2_{calc} \approx 5.31$

b H_0 is not rejected

c At a 1% level, the test does not support the principal's belief.

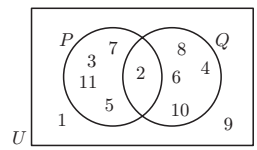
13 **a** $y = f(c)$ **b** $x = b, c, d$ **c** $x \leqslant b, x \geqslant d$

d negative **e** The tangent lines are parallel.

14 **a** €11 737 **b** £214 462.09

15 **a** 6260.9 m^2 **b** 6300 m^2 **c** 0.625%

16 **a**

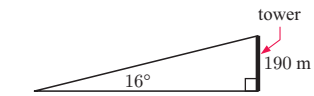


b **i** $\{2\}$

ii $\{1, 2, 3, 5, 7, 9, 11\}$

17 **a** **i** A$(-3, 0)$ **ii** B$(\frac{3}{2}, -\frac{81}{4})$ **b** $(-2, -8)$ and $(5, -8)$

18 **a**



b $x \approx 663$

c ≈ 689 m

19 **a**

Time (t min)	*Frequency*
$0 < t \leqslant 10$	5
$10 < t \leqslant 20$	4
$20 < t \leqslant 30$	10
$30 < t \leqslant 40$	17
$40 < t \leqslant 50$	14
$50 < t \leqslant 60$	3
$60 < t \leqslant 70$	5

b $\overline{x} \approx 35.3$, $s \approx 15.6$

c 37.9%

20 **a** $2x^2 + 8x - 10$ **b** $f'(x) = 4x + 8$

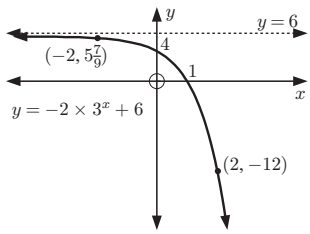
c **i** $a = -\frac{13}{4}$ **ii** $(-\frac{13}{4}, -\frac{119}{8})$

21 **a** **i** 2004.38 yuan **ii** 1974.31 yuan **b** €604.38

22 **a** **i** 3.2 m **ii** 2.56 m **b** $4 \times (0.8)^n$ m **c** 4.6 cm

23 **a** $\mathrm{P}(A') = 1 - a$ **b** **i** $2a(1 - a)$ **ii** a^2 **c** $a \approx 0.755$

24 **a** 4
b When $x = 2$, $y = -12$. When $x = -2$, $y = 5\frac{7}{9}$.
c $y = 6$ **d**



25 **a** 56.5 cm^3 **b** 3 cm **c** 96.5 cm^2

26 **a** $y = x + 1$ **b** $\approx (-1.73, -0.732)$ and $(1.73, 2.73)$

27 **a** $C(1500) = 2$, when 1500 bars are produced each day, the cost per bar is \$2.
b $C'(x) = 0.000\,008x - 0.008$ **c** $x = 1000$ **d** \$1

28 **a** $\approx 10.5\%$ **b** $\approx$ €6887.21

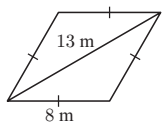
29 **a** 1 200 000 m **b** 1.2348×10^6 m

30 **a** **i** Antonio plays football, and is not good at kicking a ball.
ii If Antonio is not good at kicking a ball, then he plays football.
b **i** $p \veebar q$
ii $\neg p \Rightarrow \neg q$
c

p	q	$\neg q$	$\neg q \Rightarrow p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	(F)

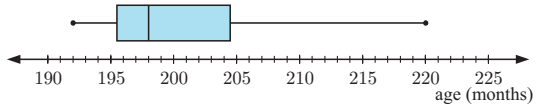
31 **a** $x^2 + (x+3)^2 = (x+6)^2$ **b** $x = 9$ {since $x > 0$}
c 54 cm^2

32 **a**



b 9.33 m
c 71.3°

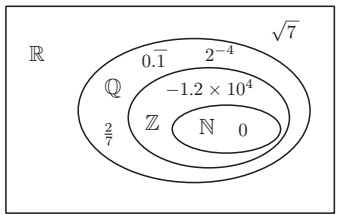
33 **a** **i** 198 months **ii** 18 months **iii** 9 months
b



34 **a** $f'(x) = 6x^2 + 6x^{-3} - 24$ **b** $f'(2) = \frac{3}{4}$
c $3x - 4y = 134$

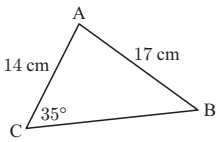
35 **a** $a \approx 1.4588$ **b** 2918 yen **c** 10 282.14 rupees

36



37 **a** **i** 0.05 **ii** 0.55 **iii** 0.45 **b** no, $P(A \cap B) \neq 0$

38 **a**



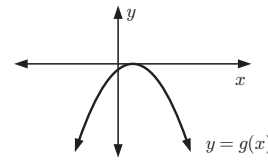
b $\widehat{ABC} \approx 28.19°$
c 106 cm^2

39 **a** **i** 48 **ii** 100.25 **iii** 15.0 **b** 8.33%

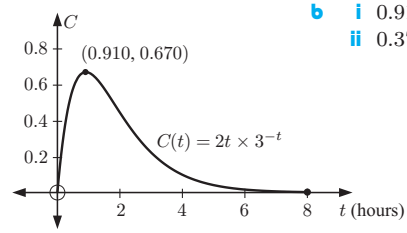
40 **a**

Constant	*a*	*b*	*c*
Value	positive	negative	positive

b none **c**



41 **a**



b **i** $0.910 \leqslant t \leqslant 8$
ii $0.379 < t < 1.79$

42 **a** **i** £24 310.13 **ii** £24 417.91 **b** 0.443%

43 **a** **i** $\{d, s\}$ **ii** $\{h, k\}$ **iii** $\{a, b, d, h, k, r, s\}$
iv $\{f, g\}$
b **i** 7 **ii** 3

44 **a** $\frac{1}{6}$ **b** $\frac{1}{9}$ **c** $\frac{5}{18}$

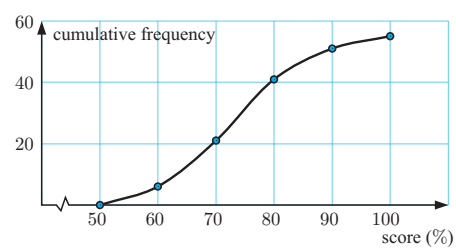
45 **a** $y = 3x^2 - 3x - 18$ **b** -18 **c** $(\frac{1}{2}, -\frac{75}{4})$

46 **a** 89.7 m **b** 1220 m^2 **c** 609 m^3

47 **a**

Score (%)	*Frequency*	*Cum. Freq.*
$50 \leqslant S < 60$	6	6
$60 \leqslant S < 70$	15	21
$70 \leqslant S < 80$	20	41
$80 \leqslant S < 90$	10	51
$90 \leqslant S < 100$	4	55

b



c ≈ 73

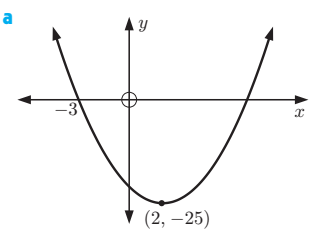
48 **a** 0 **b** $f'(x) = 6x^2 - 10x - 4$ **c** $-\frac{1}{3}$ and 2

49 **a** \$830.76 **b** $\approx$ 35 months

50 **a** 191 m **b** 6.04 m s^{-1}

51 **a** $p = 20$, $q = 30$, $r = 45$
b **i** 0.36 **ii** 0.34 **iii** 0.654

52 **a**



b 7
c **i** $p = -3$, $q = 7$
ii $a = 1$

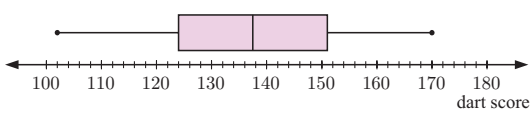
53 **a** **i** 27.6 cm **ii** 23.3 cm **b** 6010 cm^3

54 **a** $\dfrac{dy}{dx} = 3x^2 - 8x + 3$ **b** $y = 6x - 17$

55 **a** **i** €16 **ii** \$30 **b** 1 AUD = 0.8 EUR **c** €58 800

56 **a** 137.5 **b** 27

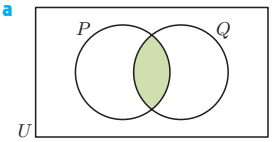
c



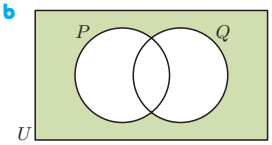
d ≈ 17.7

57 **a** $-5,\ 2,\ 9$ **b** arithmetic sequence **c** 688 **d** 34 150

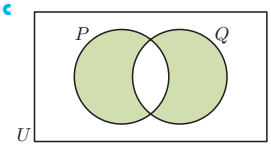
58 **a**



b

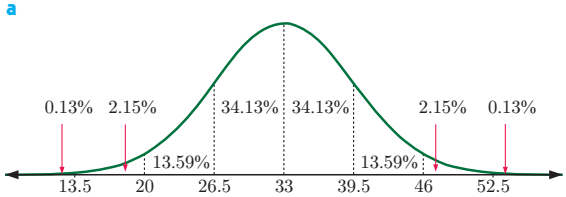


c



59 **a** $a = 4,\ b = 3$ **b** $(11,\ 0)$

60 **a**



b **i** ≈ 0.238 **ii** ≈ 0.109

c $k \approx 36.4$ **d** about 11 customers

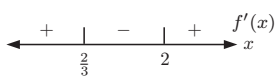
61 **a** H_0: *travel time* and *quality of work* are independent.
H_1: *travel time* and *quality of work* are not independent.

b 1

c Since $p > 0.05$, we do not reject H_0. At a 5% level of significance, *travel time* and *quality of work* are independent.

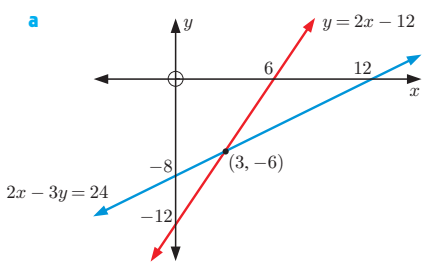
62 **a** $f'(x) = 3x^2 - 8x + 4$ **b** $x = \frac{2}{3}$ or 2

c



d increasing for $x \leqslant \frac{2}{3}$ or $x \geqslant 2$
decreasing for $\frac{2}{3} \leqslant x \leqslant 2$

63 **a**



b A is $(3,\ -6)$ **c** $3x + 2y = -3$

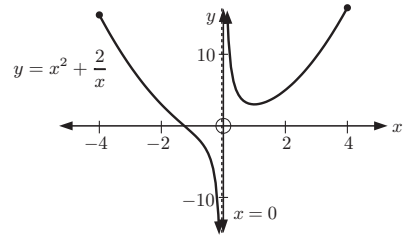
64 **a** $-\frac{1}{2}$ **c** $x \approx 19.3$ **d** 43.3 cm

65 **a** 9.772 cm **b** 61 cm

66 **a** $\frac{1}{5}$ **b** $\mathrm{P}(B \mid A) \neq \mathrm{P}(B)$ **c** $\frac{2}{3}$

67 **a** **i** £50 **ii** £200 **b** £3000 **c** $r = 0.075,\ t = 50$

68 **a**



b $(1,\ 3)$

c $-4 \leqslant x < 0,\ 0 < x \leqslant 1$

69 **a** 70%

b **i** $m \approx 28$ **ii** $n \approx 35$ **iii** $p \approx 42$ **iv** $q = 100$

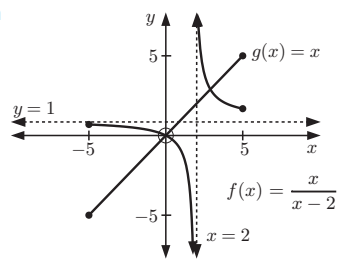
70 **a** 6, 7.2 **b** $u_{10} \approx 31.0$

c 40.2 (the 11th term, and $u_{11} \approx 37.2$)

71 **a** 50 times **b** $3\frac{1}{2}$ (\$3.50) **c** No

72 **a** $a \wedge b$ **b** $\neg(a \vee b)$ **c** $b \wedge \neg a$ **d** $(a \vee b) \wedge c$

73 **a**



b horiz. asymptote $y = 1$, vert. asymptote $x = 2$

c $x = 0$ or 3

74 **a** $\sqrt{45}$ m **b** 7 m **c** 16.6°

75 **a** $y \approx -0.0151x + 25.9$ **b** 25.5 min **c** $r \approx -0.0550$

d Very unreliable, as there is almost no linear relationship between the variables.

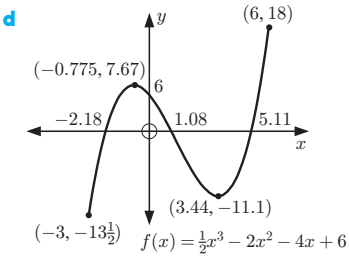
76 **a** $P'(m) = 60 - 2m,\ 0 \leqslant m \leqslant 40$ **c** \$100 000

77 **a** y-intercept is 6, x-intercepts are $\approx -2.18,\ 1.08,\ 5.11$

b local maximum at $\approx (-0.775,\ 7.67)$
local minimum at $\approx (3.44,\ -11.1)$

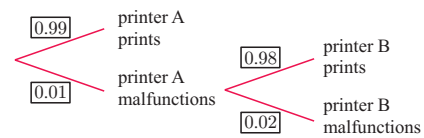
c $f(-3) = -13\frac{1}{2}$, $f(6) = 18$

d



e greatest value of $f(x)$ is 18 (when $x = 6$)
least value of $f(x)$ is $-13\frac{1}{2}$ (when $x = -3$)

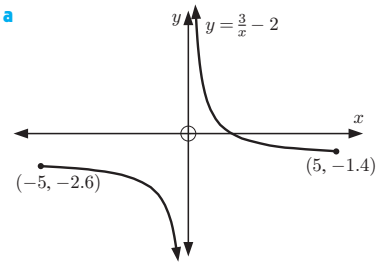
78 **a**



b **i** 0.0002 **ii** 0.0098 **iii** 0.990

79 **a** 343.42 m s^{-1} **b** 2.06×10^5 m **c** 4.41%

80 **a**



b $x = 0$

c $y = 3x - 10$

81 **a** **i** P(0, 5) **ii** Q(8, 0)

b $-\frac{5}{8}$ **c** 32.0° **d** $(-4, -3)$

82 **a** -2 **b** $y = -2x + 6$ **c** $b = -4,\ c = 7$

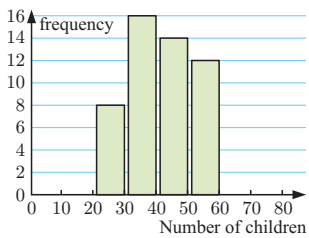
83 **a** positive for each case **b**

Strength of correlation	*Scatter diagram*
Weak	II
Moderate	I
Strong	III

84 **a** 26 days

b 31 - 40 children

c



d **i** $\mu = 41.5$ children, **ii** $\sigma \approx 10.2$ children

85 **a** 10.32 m s^{-1} **b** 37.2 km h^{-1}

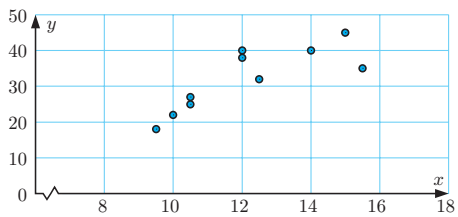
86 **a** 44 students **b** **i** $\frac{9}{44}$ **ii** $\frac{5}{44}$ **iii** $\frac{5}{14}$

87 **a** \$24 000 **b** $r = 0.85$ **c** 7 years

88 **a** 12 cm **b** **i** 67.4° **ii** 113° **c** 20.8 cm

89 **a** $f'(x) = -x$ **b** $(-1, 2\frac{1}{2})$

90 **a**



b $r \approx 0.819$ **c** The statement is true in general.

91 **a** $f'(x) = -\dfrac{16}{x^3} + 2$

b $f'(1) = -14$, the gradient of the tangent to $f(x)$ at the point where $x = 1$.

c (2, 3)

92 **a**

a	b	c	$\neg b$	$b \wedge a$	$c \Rightarrow (b \wedge a)$	$\neg b \wedge (c \Rightarrow (b \wedge a))$	$\neg c$	$\neg b \wedge (c \Rightarrow (b \wedge a)) \Rightarrow \neg c$
T	T	T	F	T	T	F	F	T
T	T	F	F	T	T	F	T	T
T	F	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T	T
F	T	T	F	F	F	F	F	T
F	T	F	F	F	T	F	T	T
F	F	T	T	F	F	F	F	T
F	F	F	T	F	T	T	T	T

b tautology

93 **a** **i** €5306.82 **ii** €5632.46

b $V_n = 5000 \times (1.015)^{4n}$ **c** ≈ 11.6 years

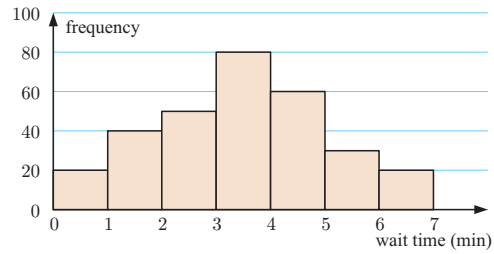
94 **a** 826.563 **b** 830 **c** $8.265\,625 \times 10^2$

95 **a** M(1, 4) **b** $\frac{2}{5}$ **c** $5x + 2y - 13 = 0$ **d** $\frac{13}{5}$

96 **a** 7080 SEK **b** 995.78 AUD **c** 0.422%

97 **a** 3.5 minutes **b** $p = 20,\ q = 30$

c



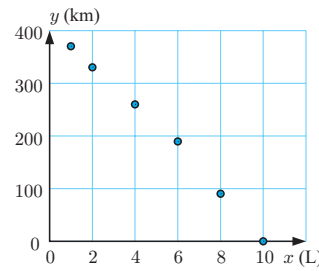
98 **a** **i** $p = 4$ **ii** $q = 4$ **b** -1 **c** C(2, 2)

99 **a** \$450 **b** \$4125

100 **a** **i** 0.41 **ii** 0.59 **b** ≈ 0.814

101 **a** (0, 2.5) **b** $k = 1.5$ **c** $y = 3.5$

102 **a**



b $y \approx -40.7x + 417$

c 4.84 litres

d 42.6 km per litre

103 **a** 70 m **b** 53.9° **c** 3440 m^2

104 **a** \$15 273.01 **b** \$5273.01 **c** \$777.09

105 **a** $a = 4$ **b** $\dfrac{dy}{dx} = 2x - 4$ **c** $(2, -4)$ **d** 2

106 **a** true **b** true **c** true **d** false **e** false **f** true

107 **a** $\mathrm{P}(B) = \frac{1}{3}$ **b** **i** $\frac{16}{21}$ **ii** $\frac{13}{21}$

108 **a** 9 seeds **b** 7 seeds **c** 15 seeds

109 **a** H_0: *intelligence* and *income level* are independent.
H_1: *intelligence* and *income level* are not independent.

b 4

c At a 5% significance level, *intelligence* and *income level* are independent.

110 **a** $f'(x) = x^3 - 4x$

b $f'(-3) = -15,\quad f'(-2) = 0,\quad f'(-1) = 3$

c local minimum **d** $-2 \leqslant x \leqslant 0,\ x \geqslant 2$

111 **a** **i** 11 units **ii** 5 units **b** 53.13° **c** 22 units^2

112 **a** **i** 38 500 MXN **ii** 2172.50 EUR

b **i** 1 EUR $\approx$ 1.2658 USD **ii** 1 EUR $\approx$ 17.7215 MXN

c 90 379.65 MXN

113 **a** $a = 5,\ b = -10$ **b** $y = 310$

114 **a** $u_1 + 3d = 22,\quad u_1 + 9d = 70$

b $u_1 = -2,\ d = 8$ **c** 340

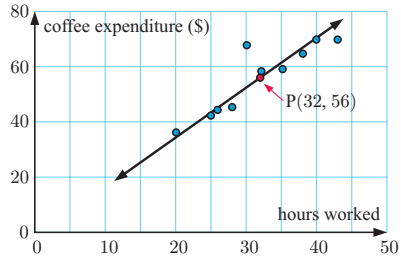
115 **a**

	Left handed	*Right handed*	*Total*
Male	4	26	30
Female	3	17	20
Total	7	43	50

b **i** 0.14 **ii** 0.52 **iii** 0.85

116 **a** $k = -\frac{5}{2}$ **b** $k = \frac{8}{5}$

117 **a, b**



c \$61

d strong positive relationship, the prediction in **c** is reliable

118 **a** 27 **b** $\frac{dy}{dx} = 3x^2$ **c** P(3, 15)

119 **a** $f'(x) = 3x^2 - \frac{5}{x^2}$

b $f'(1) = -2$

The gradient of the tangent to $f(x)$ when $x = 1$, is -2.

c $y = -2x + 8$ **d** $x - 2y = -11$

e meets the graph again at $\approx (1.35, 6.18)$

120 **a** $x = 20$ **b** **i** $\frac{37}{50}$ **ii** $\frac{2}{5}$ **iii** $\frac{17}{50}$ **iv** $\frac{24}{37}$

121 **c** $a = 4,\ b = 1$ **122** **a** £2x **c** $x = 174$

123 **a** P $\approx$ 578 m **b** **i** $\approx 15\,000$ m^2 **ii** $\approx$ 1.50 ha

124 **b** $y = 2x + 3$ **c** $(-2.8, -2.6)$ **d** 6.26 units

125 **a** 13

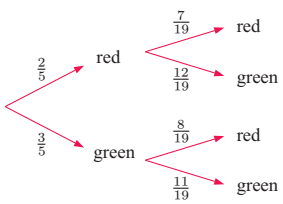
b **i** $w = 3$ **ii** $x = 36$ **iii** $y = 16$ **iv** $z = 211$

c $\frac{z}{y} \approx 13.2$, the mean of the data set

126 **a** ¥300 438.21 **b** $\approx 6.78\%$

127 **a** $u_1 = 59,\ u_2 = 55$ **b** 19th term **c** $k = 11$

128 **a**



b **i** $\frac{14}{95}$ **ii** $\frac{81}{95}$

129 **a** 4 people **b** 393 people **c** 19.9 $\approx$ 20 days

130 **a** 2.83 cm **b** 66°

131 **a** $y = 30 - x$ **b** $A(x) = x(30 - x)$ cm^2

c $A'(x) = 30 - 2x$ **d** $x = 15$, 15 cm $\times$ 15 cm

132 **a** H_0: *drink size* and *time of day* are independent.

b $\chi^2 \approx 7.11$ **c** 4

d $\chi^2 < 9.488$, so at a 5% significance level *drink size* and *time of day* are independent.

133 **a** \$14 056.88 **b** 12.524% **c** \$10 756.44

134 **a** $a \veebar b$

$\neg b$

$\therefore\ a$

b

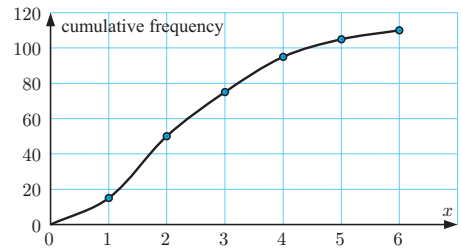
a	b	$a \veebar b$	$\neg b$	$(a \veebar b) \wedge \neg b$	$(a \veebar b) \wedge \neg b \Rightarrow a$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

$\therefore$ the argument is valid (we have a tautology).

135 **a**

Length (x cm)	*Frequency*	*Cumulative frequency*
$0 \leqslant x < 1$	15	15
$1 \leqslant x < 2$	35	50
$2 \leqslant x < 3$	25	75
$3 \leqslant x < 4$	20	95
$4 \leqslant x < 5$	10	105
$5 \leqslant x < 6$	5	110

b



c $\approx$ 3.4 cm

136 **a** **i** 0.3 **ii** 0.6 **iii** $\frac{3}{7} \approx 0.429$ **b** No

137 **a** p: I watch a movie, q: I will relax.

b p true but q false (I watch a movie, but do not relax.)

c If I have not relaxed, then I have not watched a movie.

d

p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	Contrapositive $\neg q \Rightarrow \neg p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

138 **a** OM = 5 cm **b** 400 cm^3 **c** 67.4°

139 **a** **i** $u_n = 13 + 8n$ **ii** $S_n = 4n^2 + 17n$

b **i** $u_{50} = 413$ **ii** $S_{50} = 10\,850$

140 **a** 50 m **b** **i** 100 m **ii** $0 \leqslant x \leqslant 100$ **c** L(50, 30)

EXERCISE 22B

1 **a** \$192 000

b **i** \$1000, \$1600, \$2200 **ii** \$189 600

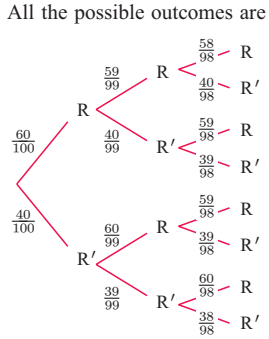
c **i** \$500, \$600, \$720 **ii** \$196 242.12

d *Option 3* **e** \$636.97

2 **a** **i** 0.36 **ii** 0.48 **iii** 0.16

b All the possible outcomes are covered in **a**. **c** 0.216

d

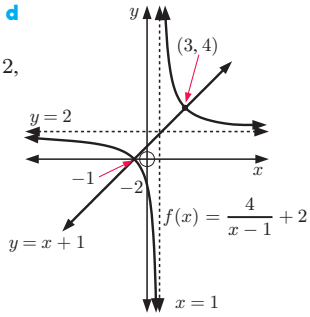


i 0.289

ii 0.939

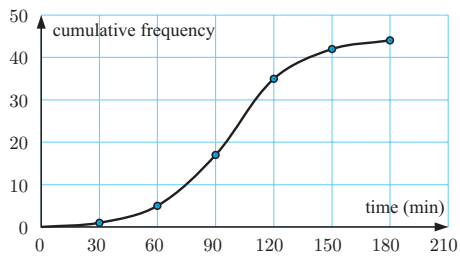
3 **a** x-intercept is -1
y-intercept is -2
b horizontal asymptote $y = 2$,
vertical asymptote $x = 1$
c Domain $\{x \mid x \neq 1\}$,
Range $\{y \mid y \neq 2\}$
e $x = -1$ or 3

d



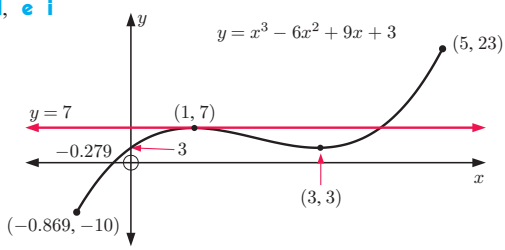
4 **a** **i** A(8, 0) **ii** B(0, 6) **b** **i** $-\frac{3}{4}$ **ii** 10 units
c $y = \frac{4}{3}x + 6$ **d** 37.5 units^2 **e** D(3, 0)

5 **a** 44 players **b** $90 \leqslant t < 120$
c



d **i** $\approx$ 98 min **ii** 96.8 min **iii** no
e "…. between 75 and 117 minutes."

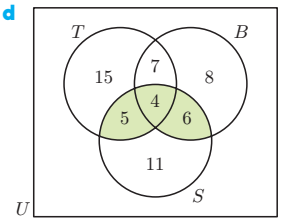
6 **a** $f'(x) = 3x^2 - 12x + p$ **b** $p = 9$, $q = 3$ **c** (3, 3)
d, e **i**



e **ii** $y = 7$ **iii** (4, 7)

7 **c** $\frac{dS}{dr} = 4\pi r - \frac{500\pi}{r^2}$ **d** $r = 5$ cm **e** 5 cm
f ≈ 471 cm^2

8 **a** 56 members **b** **i** 8 **ii** 25 **iii** 5 **c** yes
d **e** 11

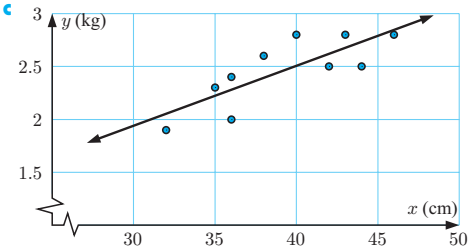


9 **a** **i** $\frac{3}{8}$ **ii** 1 **b** **i** $\frac{25}{64}$ **ii** $\frac{55}{64}$
c **i** $\frac{3}{28}$ **ii** $\frac{15}{28}$ **d** $\frac{15}{28}$

10 **a** PQ $= \sqrt{50}$ units **b** $a = 10$ **c** $\frac{1}{7}$
d $x - 7y + 32 = 0$ **e** S$(-32, 0)$ **f** 5.44°

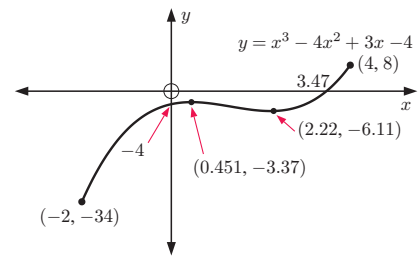
11 **a** ≈ 0.0115 **b** $\approx 17.8\%$ **c** $\approx$ 123 ferrets
d $k \approx 52.8$ cm

12 **a, c**



b $r \approx 0.797$, there is a moderate positive relationship between the variables.
c $y \approx 0.0565x + 0.244$ **d** **i** 2.2 kg **ii** 40 cm

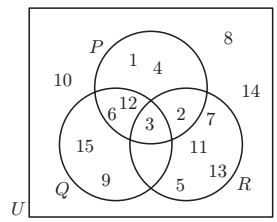
13 **a** $f(0) = -4$, $f(1) = -4$, $f(2) = -6$
b $f'(x) = 3x^2 - 8x + 3$
c (0.451, −3.37) and (2.22, −6.11)
d (0.451, −3.37) is a local maximum,
(2.22, −6.11) is a local minimum
e



14 **a** $a = 0.28$ **b** 0.29 **c** 9 times **d** 6.05 people

15 **a** **i** $u_1 + 4d = 50$, $u_1 + 7d = 80$
ii $u_1 = 10$, $d = 10$ **iii** 10, 20, 30, 40, 50
b **i** 100, 50, 25, 12.5, 6.25 **ii** 200 **iii** 200
iv The terms get successively smaller, and adding the extra terms does not alter the overall sum when rounded to 3 significant figures.
c 1400

16 **a**



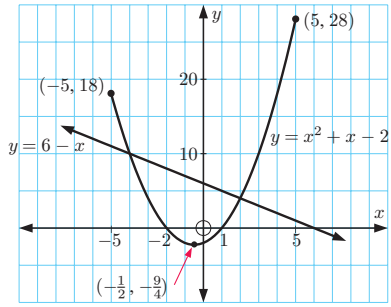
b **i** $x = 3, 6, 12$ **ii** $x = 2, 5, 6, 7, 9, 11, 12, 13, 15$
iii $x = 6, 9, 12, 15$ **iv** $x = 8, 10, 14$
c **i** x is a factor of 12, and a multiple of 3, but is not prime.
ii

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \wedge \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	F	F	F
F	F	F	F	T	F

p is true, q is true, r is false.
iii $x = 6, 12$

17 **a** 1 and -2 **b** $(-\frac{1}{2}, -\frac{9}{4})$

c, d



e $x = -4$ or 2

18 **a** **i** 0.19 **ii** 0.06

b **i** H_0: *attendance* and *performance* are independent.
H_1: *attendance* and *performance* are not independent.
ii 2 **iii** 4.61

c % attendance at lectures

Exam result	0 - 39	40 - 79	80 - 100	*sum*
Pass	21	49	70	140
Fail	9	21	30	60
sum	30	70	100	200

d $\chi^2_{calc} \approx 16.0$

e $\chi^2_{calc} > \chi^2_{crit}$, so at a 10% significance level *attendance* and *performance* are not independent.

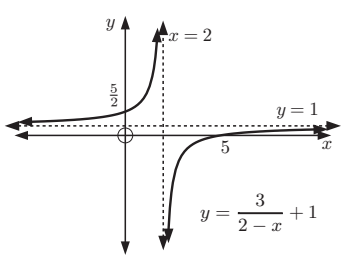
19 **a** **i** DB ≈ 4.09 m **ii** BC ≈ 9.86 m

b **i** $\widehat{ABE} \approx 68.2°$ **ii** $\widehat{DBC} \approx 57.5°$

c 17.0 m^2 **d** AE ≈ 10.9 m

20 **a** x-intercept is 5, y-intercept is $\frac{5}{2}$

b



c **i** $y = 1$ **ii** $x = 2$

d **i** The graph is increasing for $x \geqslant 3$.
iii $a = 1.5$, $b \approx 2.9970$ **iv** $\dfrac{dy}{dx} = 3$

21 **a** **i** 16 000 CHF **ii** 200 000 JPY

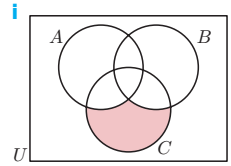
b **i** $a = 1$ **ii** $b = 0.625$

c **i** 1 CHF $= 80$ JPY **ii** 1 CHF $= 0.625$ GBP

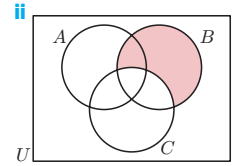
d $c = 128$ **e** 1 134 720 JPY **f** $\approx 7.813 \times 10^{-3}$

22 **a** **i** $P = \{3, 4, 5, 6, 7, 8, 9\}$ **ii** $n(P) = 7$ **iii** finite
iv **(1)** 2 and 15 are in Q, but not P
(2) $R = \{3, 6, 9\}$, all these elements are in P
v **(1)** $\{9\}$ **(2)** $\{9\}$ **(3)** $\{2, 3, 6, 9, 15\}$

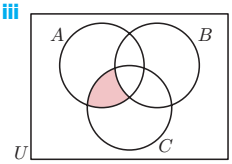
b **i**



ii



iii



23 **a** **i** $\neg q \Rightarrow \neg p$ **ii** $q \Rightarrow p \veebar r$

b **i** If Pepin does not play the guitar, then he does not both ride a motorbike *and* live in Jakarta.

ii

p	q	r	$\neg r$	$q \wedge p$	$\neg(q \wedge p)$	$\neg r \Rightarrow \neg(q \wedge p)$
T	T	T	F	T	F	T
T	T	F	T	T	F	F
T	F	T	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	T	F	T	F	T	T
F	F	T	F	F	T	T
F	F	F	T	F	T	T

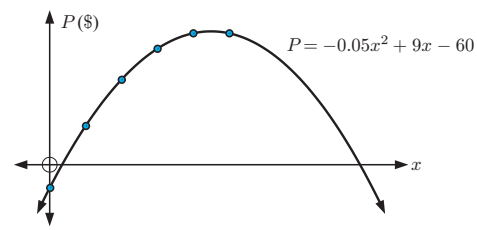
iii not a tautology

iv Pepin does not play the guitar, but he does ride a motorbike and he does live in Jakarta.

24 **a**

x	0	20	40	60	80	100
P	−60	100	220	300	340	340

b



c **i** 90 pies **ii** \$345 **iii** 37 or 143 **iv** \$60

25 **a** AN $= \sqrt{244} \approx 15.6$ cm **b** AM $= \sqrt{269} \approx 16.4$ cm

c 17.7° **d** 39.1 cm^2 **e** 660 cm^2 **f** 600 cm^3

26 **a** 18.7°C **b** 1.38°C

c $r \approx -0.744$, there is a moderate negative correlation between n and T.

d $n = -6.82T + 252$ **e** 118 cups of coffee

f Using the regression line, 47 should be sold, so the owner may be underestimating. However, this is an extrapolation, so the regression line estimate may be unreliable.

27 **a** $a = -12$, $b = 6$

b **i** $p = 5$, $q = -4$ **ii** $\frac{2}{5}$ **iii** $c = -6$

28 **a** $p \Rightarrow r$

b If Ryan is going to the zoo, then Peter is going to the zoo.

c **i** $p \Leftrightarrow r$

$$\frac{p \veebar q}{\neg r}$$

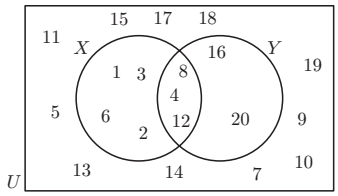
ii

p	q	r	$p \Leftrightarrow r$	$p \veebar q$	$(p \Leftrightarrow r) \wedge (p \veebar q)$	$\neg r$	$(p \Leftrightarrow r) \wedge (p \veebar q) \Rightarrow \neg r$
T	T	T	T	F	F	F	T
T	T	F	F	F	F	T	T
T	F	T	T	T	T	F	F
T	F	F	F	T	F	T	T
F	T	T	F	T	F	F	T
F	T	F	T	T	T	T	T
F	F	T	F	F	F	F	T
F	F	F	T	F	F	T	T

$\therefore$ the argument is not valid.

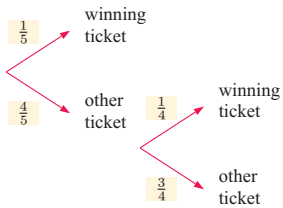
29 **a** **i** $X = \{1, 2, 3, 4, 6, 8, 12\}$
ii $Y = \{4, 8, 12, 16, 20\}$

b



c **i** $X \cap Y = \{4, 8, 12\}$
ii $(X \cup Y)' = \{5, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$

30 **a**



b $\frac{2}{5}$ **c** 100 times **d** **i** $\frac{4}{5}$ **ii** 1
e **i** $\frac{1}{5}$ **ii** $\frac{4}{25}$ **iii** $\frac{16}{125}$ **f** $\frac{61}{125}$ **g** **i** $\frac{4}{5}$
h 96.5%

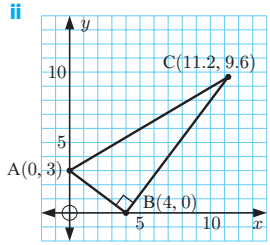
31 **a** $x = 0$ **b** $\dfrac{dy}{dx} = 2 - \dfrac{1}{x^2}$ **c** gradient $= 1$
d $(-0.707, 0.172)$ and $(0.707, 5.83)$
e $\{y \mid y \leqslant 0.172 \text{ or } y \geqslant 5.83\}$

32 **b**

	Male	*Female*	*Total*
Participated	78	72	150
Did not participate	52	48	100
Total	130	120	250

c 1 **d** ≈ 5.41 **e** H_0 is rejected
f Participation in the survey was not independent of gender.
g continuous **h** **i** 12.5 km **ii** 6.19 km **i** 0.367

33 **a, e** **ii**



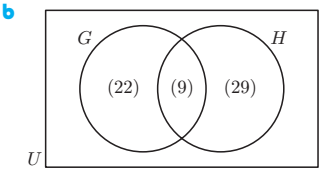
b 5 units
c $-\frac{3}{4}$
d **i** $\frac{4}{3}$
ii $y = \frac{4}{3}x - \frac{16}{3}$
e **i** $b = 9.6$
f 30 units^2
g 22.6°

34 **c** **i** $a = 11\,750$ **ii** $b = 29\,250$
d $\dfrac{dV}{dx} = 1200 - \frac{3}{4}x^2$ **e** $x = 40,\ y = 20$ **f** 32 000 cm^3

35 **a** **i** €511.99 **ii** €1602.45
b $V = 12\,000(1.0105)^{4x}$ euros **c** 22 years
d 4.27% (if $n = 21.9$; 4.25% if $n = 22$)

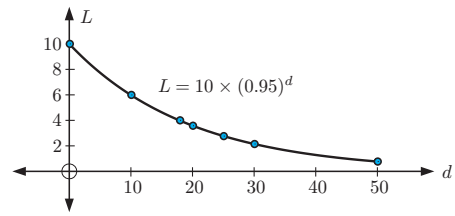
36 **a** 92.4 m^2
b maximum length 16.55 m, maximum width 5.65 m
c 93.5 m^2 **d** 1.87 m^3 **e** \$474.13 **f** \$3.99

37 **a** $n = 9$
b



c $\frac{17}{20}$
d **i** $\frac{2}{5}$ **ii** $\frac{1}{12}$
e **i** $\frac{17}{38}$ **ii** $\frac{9}{38}$

38 **a** $L_0 = 10$ **b** 2.77 units **c** 17.9 m
d



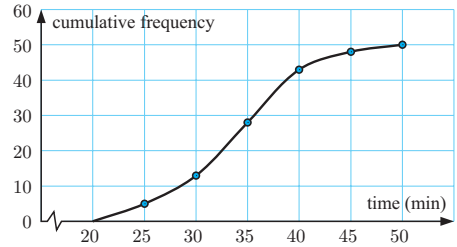
e Between 23.5 m and 44.9 m.

39 **a** 37.5 cm **b** 1500 cm^2 **c** 13 500 cm^3
d 54.8 cm **e** 46.8° **f** 375 cm^2

40 **a**

Time (min)	*Frequency*	*Cumulative frequency*
$20 \leqslant t < 25$	5	5
$25 \leqslant t < 30$	8	13
$30 \leqslant t < 35$	15	28
$35 \leqslant t < 40$	15	43
$40 \leqslant t < 45$	5	48
$45 \leqslant t < 50$	2	50

b



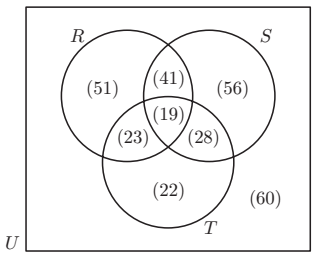
c **i** 34 min **ii** 8 min **iii** 37 runners **iv** 32 min
d 23 min

41 **a** $\$(x - 15)$ **c** \$51 000 **d** $\dfrac{dP}{dx} = 1635 - 18x$
e \$91 per pair, \$51 756 profit

42 **a** **i** 4 **ii** 1 **iii** 4
b **i** $y = x + 7$ **ii** $y = 4x - 8$ **c** (5, 12)
d $y = -x + 3$

43 **a** **i** 55 **iii** 500 500 **iv** $n = 150$
b **i** $u_n = 7n$ **ii** $u_{142} = 994$ **iii** $S_n = \dfrac{7n(n+1)}{2}$
c 429 429

44 **a**



b 60

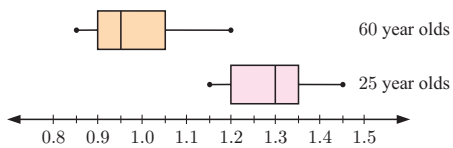
c **i** 6.33%
ii 30.7%

d 60%

45 **a** 25 year old group:
min = 1.15 $Q_1 = 1.2$ median = 1.3 $Q_3 = 1.35$
max = 1.45

60 year old group:
min = 0.85 $Q_1 = 0.9$ median = 0.95 $Q_3 = 1.05$
max = 1.2

b



c The 25 year old group has a higher mean, median and mode than the 60 year old group.

d 25 year old group: IQR = 0.15, range = 0.3
60 year old group: IQR = 0.15, range = 0.35
The IQR is the same for both groups. The range is slightly greater for the 60 year old group.

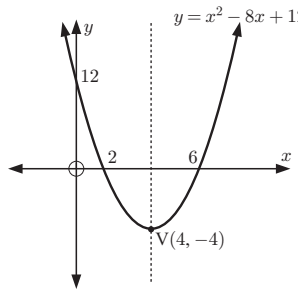
e 60 year old adults will, in general, have a lower bone density than 25 year old adults.

46 **a** x-intercepts are 2 and 6, y-intercept is 12

b $x = 4$

c $(4, -4)$

d



e increasing for $x \geqslant 4$, decreasing for $x \leqslant 4$

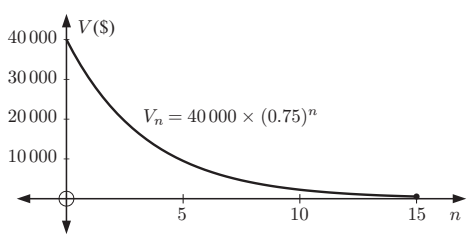
f $y = -2x + 3$

g $y = 4x - 24$

47 **b** QR ≈ 4.624 cm **c** **i** 18.6 cm **ii** 16.5 cm^2

48 **a** **i** $V_1 = \$30\,000$ **iii** $V_n = 40\,000(0.75)^n$

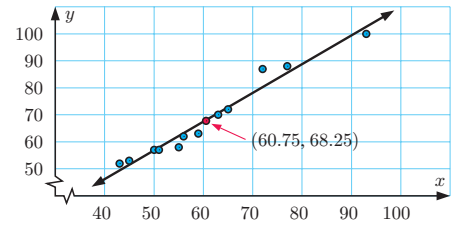
iv



b **i** 5% **ii** $P_2 = \$1323$, $P_3 = \$1389.15$
iii $P_n = 1200(1.05)^n$

c 11 years

49 **a, c**



b (60.75, 68.25)

d **i** $r \approx 0.981$ **ii** $y \approx 1.06x + 3.61$ **e** 94%

50 **a** **i** 100 **ii** 33 **b** **i** 0.76 **ii** 0.74

c **i** 0.258 **ii** 0.546 **iii** 0.379 **iv** 0.439

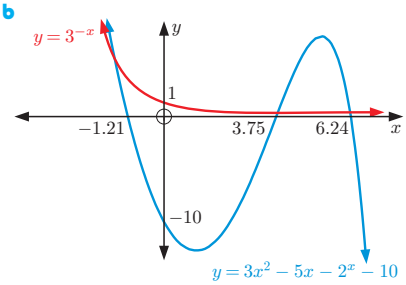
d **i** 0.001 02 **ii** 0.0552

51 **a** **i** x-intercepts are -1.21, 3.75, and 6.24, y-intercept is -10

ii local minimum at $(1.08, -14.0)$, local maximum at $(5.26, 8.38)$

iii As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to \infty$

iv, b



c $x \approx -3.38$, -1.65, 3.75, or 6.24

52 **a** 13.1 km **b** 41.4° **c** 4.83 km **d** 38.3 km^2

e 0.567%

53 **a** In 3 years she will earn \$183 000 under *Option B*, compared with \$126 100 under *Option A*.

b **i** $A_n = 40\,000 \times (1.05)^{n-1}$ **ii** $B_n = 59\,000 + 1000n$

c ≈ 13.1 years

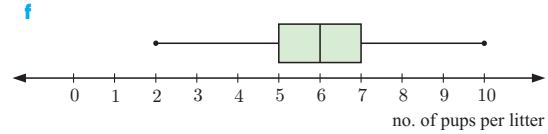
e **i** graph 1 represents T_A, graph 2 represents T_B
ii P(22.3, 1 580 000)

f $0 \leqslant n \leqslant 22$

54 **a** discrete **b** 79 **c** **i** $s = 75$ **ii** $t = 119$

d 487 **e** 6.16

f



g **i** 8 **ii** 2 **iii** 6

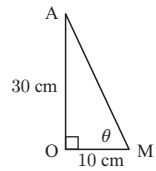
55 **a** -100, for every €1 increase in the selling price, the number of stools sold decreases by 100.

b $N = -100x + 8000$ **c** €$(x - 50)$

e $\dfrac{dP}{dx} = -200x + 13\,000$ **f** $x = 65$, €22 500 profit

56 **a** 20 cm **b** OR = $10\sqrt{2}$ cm

c **i**



ii $\theta \approx 71.6°$

d 1660 cm^2 **e** \$25.52

57 **a** \$41 269.54

b

n (years)	0	1	2	3	4
V_n (\$)	50 000	53 000	56 180	59 550.80	63 123.85

c $V_n = 50\,000 \times (1.06)^n$ dollars **d** $S_n = 3000n$ dollars

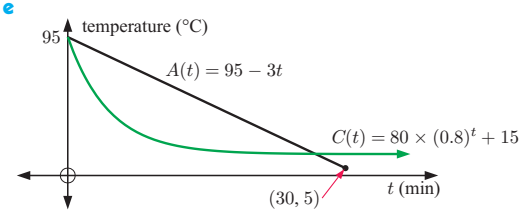
e

n (years)	0	1	2	3	4
T_n (\$)	50 000	56 000	62 180	68 550.80	75 123.85

f 19 years

58 **a** **i** 95°C **ii** 5°C

b, e



c No, we would not expect the temperature to drop at a constant rate in the long term. The coffee's temperature should approach that of its surroundings.

d **i** 95°C **ii** 15.1°C **f** 12.4 min **g** 15°C

h **i** $t = 0$ or 26.6, at these times the approximate temperature $A(t)$ is equal to the actual temperature $C(t)$.

ii 42.6°C

iii This is the maximum amount by which $A(t)$ overestimates the actual temperature.

59 **a** H_0: *movie type* and *gender* are independent.
H_1: *movie type* and *gender* are not independent.

b 28 **c** $\chi^2 \approx 22.7$ **d** 3 **e** 7.81

f $\chi^2 > 7.81$, so we reject H_0, and conclude that *movie type* and *gender* are not independent.

g $\chi^2 \approx 16.9$, which is still > 7.81, so the conclusion is still valid.