Chapter 22

Miscellaneous problems

Contents:

A SHORT QUESTIONS

EXERCISE 22A

1 Calculate the value of $\dfrac{(1.2 \times 10^{-1})^2}{4.72 \times 10^{-2}}$, giving your answer:

a correct to two decimal places **b** correct to 3 significant figures

c in scientific notation correct to 6 significant figures.

2 Two propositions p and q are defined as follows:

p: Farouk studies for the test. q: Farouk scores a good mark.

a Write in words: **i** $p \Rightarrow q$ **ii** the inverse of $p \Rightarrow q$.

b Write in symbols: **i** the converse of $p \Rightarrow q$ **ii** the contrapositive of $p \Rightarrow q$.

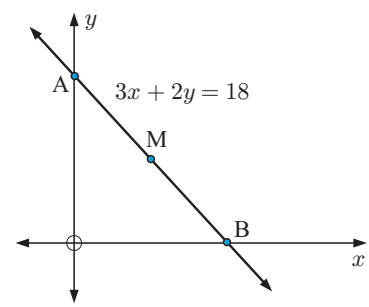
c Use a truth table to show that the contrapositive of $p \Rightarrow q$ is equivalent to the original statement $p \Rightarrow q$.

3 A random variable X is normally distributed with mean 2.5 and standard deviation 6.7.

a Find the probability that a randomly chosen item from this population has a negative value.

b Given that $\mathrm{P}(X \geqslant k) = 0.3$, find k.

4 The line $3x + 2y = 18$ cuts the axes at A and B. M is the midpoint of the line segment AB as shown.



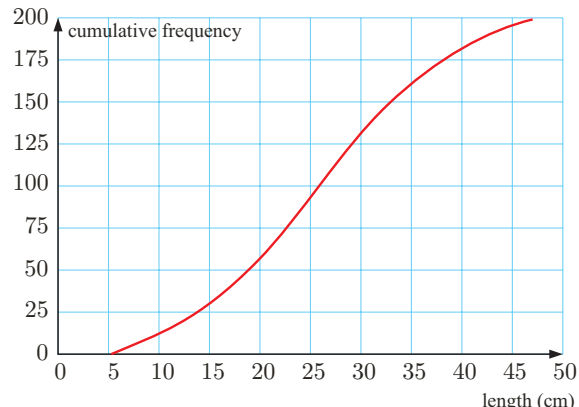
a Find the coordinates of A and B.

b Find the coordinates of M.

c Find the gradient of the line AB.

d Find the equation of the perpendicular bisector of AB in the form $ax+by=c$, where $a, b \in \mathbb{Z}$ and $a > 0$.

5 The lengths of a random sample of 200 fish caught one day from the local jetty are displayed in the cumulative frequency curve.



a Write down the median length of fish caught.

b Complete the following table of values for the lengths of fish. Give your answers to the nearest 5 fish.

Length (cm)	$0 < x \leqslant 10$	$10 < x \leqslant 20$	$20 < x \leqslant 30$	$30 < x \leqslant 40$	$40 < x \leqslant 50$
Frequency	15	$55 - 15 = 40$	$130 - 55 = 75$		

c Use your table in **b** to estimate the mean length of fish caught from the jetty.

6 Consider the function $f(x) = 2x^5 - 5x^2 + 1$.

a Find $f'(x)$. **b** Find the equation of the tangent to $y = f(x)$ at the point $(1, -2)$.

7 Romeo has \$12 000 which he would like to invest for four years until he finishes his University degree. The following options are available:

Option A: 5.8% p.a. interest compounding monthly.
Option B: 5.9% p.a. interest compounding quarterly.

a For each option, calculate the value of the investment after four years.

b Which option is better, and by how much?

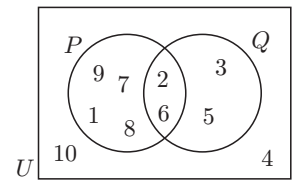
8 A *light-year* (ly) is the distance light travels in a vacuum in one year.

a Light travels at 2.998×10^5 km s^{-1}. Assuming there are 365.25 days in a year, calculate the distance light travels in 1 year. Write your answer correct to 3 significant figures, in the form $a \times 10^k$ km where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

b The Andromeda Galaxy is approximately 2.2×10^5 ly wide. Write this distance in kilometres, in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

9 Let U be the universal set $\{x \mid 1 \leqslant x \leqslant 10,\ x \in \mathbb{Z}\}$.
P and Q are subsets of U, as shown in the Venn diagram.
Let p and q be the propositions:

p: x is an element of P.
q: x is an element of Q.



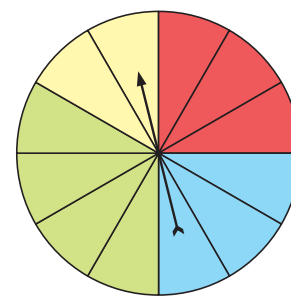
a For what values of x are the following statements true:

i $\neg p$ **ii** $\neg p \wedge q$ **iii** $p \veebar q$?

b Complete the truth table:

p	q	$\neg p$	$\neg p \wedge q$	$p \veebar q$
T	T			
T	F			
F	T			
F	F			

10 Consider the spinner alongside.



a Find the probability that the next spin will finish on green.

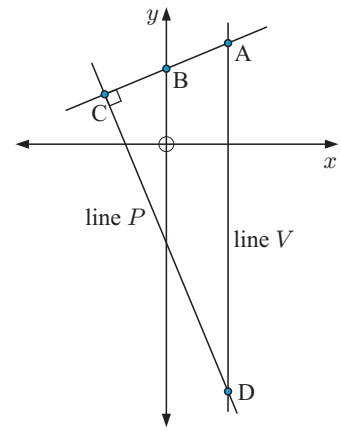
b How many 'reds' would you expect if the spinner was spun 300 times?

c A game is played with this spinner where green pays \$1.50, blue pays \$2, red pays \$6, and yellow pays \$15. It costs \$5 to play the game.

i Calculate the expected gain from this game.

ii Determine whether the game is fair.

11 Line V is vertical and passes through $A(5, 6)$ as shown.



a Write down the equation of line V.

b $B(0, 4)$ is the midpoint of line segment AC. Write down the coordinates of point C.

c Line P passes through C and is perpendicular to AC. Write down the equation of line P, giving your answer in the form $ax + by + d = 0$.

d Write down the coordinates of D, the point of intersection of lines P and V.

12 A high school principal believes that academic success is related to the students' involvement with co-curricular activities. To investigate this further, the principal compiled the following information.

	Total time spent on co-curricular activities		
Grade average	*Less than 2 hours*	*From 2 to 5 hours*	*More than 5 hours*
1 or 2	15	14	17
3, 4, or 5	31	26	18
6 or 7	22	24	31

The principal performs a χ^2 test on the data at a 1% significance level.

a For this χ^2 test:

i write down the null hypothesis H_0

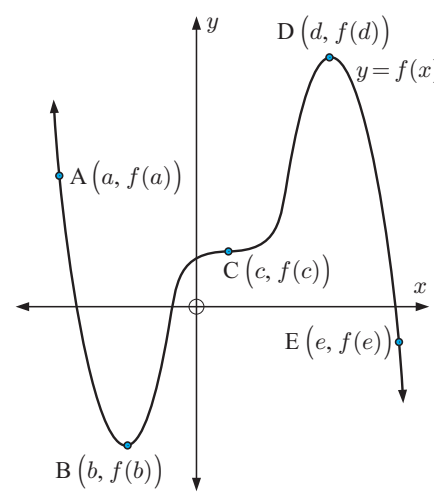
ii explain why there are 4 degrees of freedom

iii calculate the value of the χ^2 test statistic.

b Given the critical value is ≈ 13.3 at a 1% significance level, comment on whether H_0 is rejected.

c Discuss whether this χ^2 test at a 1% significance level supports the principal's belief.

13 Consider the graph of $y = f(x)$ shown opposite.



B is a local minimum, D is a local maximum, and the tangent at C is horizontal.

a Write down the equation of the tangent at C.

b Write down all solutions to $f'(x) = 0$.

c For what values of x is $f(x)$ decreasing?

d Comment on whether $\dfrac{f(b) - f(a)}{b - a}$ is positive, negative, or zero.

e What can be said about the tangent lines at A and E if $f'(a) = f'(e)$?

14 Mr Bond receives £200 000 in retirement money. He decides to spend £10 000 on a holiday to France, leaving the remainder of his retirement fund in the bank.

a When Mr Bond converts his holiday money from British pounds (GBP) to euros (EUR), the exchange rate is 1 GBP = 1.21 EUR. A 3% commission is paid on the transaction. How many euros does Mr Bond receive?

b The remainder of Mr Bond's retirement money is left in an account offering 5.6% p.a. interest, compounding monthly. Calculate the value of his retirement fund after 2 years.

15 A rectangular field is 91.4 m long and 68.5 m wide.

a Calculate the exact area of the field in m^2.

b Round your answer in **a** to two significant figures.

c Calculate the percentage error of your answer in **b**.

16 Suppose $U = \{x \mid 1 \leqslant x \leqslant 11,\ x \in \mathbb{Z}\}$, p: x is a prime number, and q: x is an even number.

a Represent the information on a Venn diagram.

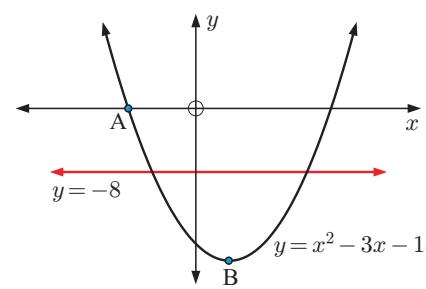
b Write down the truth set for: **i** $p \wedge q$ **ii** $p \vee \neg q$.

17 The graph alongside shows $y = x^2 - 3x - 18$. It cuts the x-axis at A, and its vertex is at B.

a Write down the coordinates of:

i A **ii** B.

b Find the coordinates of the points where the curve $y = x^2 - 3x - 18$ meets the line $y = -8$.



18 A house is x metres from a 190 m high radio tower. The angle of elevation from the house to the top of the tower is 16°.

a Draw a diagram to show this information. **b** Find x.

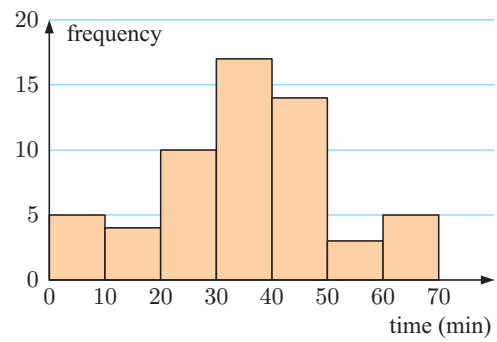
c Find the straight line distance from the top of the tower to the house.

19 The histogram shows the times a group of students spent travelling to school in the morning.

a Construct a frequency table for this data.

b Estimate the mean and standard deviation for the travel time.

c Find the percentage of students who spend more than 40 minutes travelling to school in the morning.



20 **a** Expand the expression $2(x - 1)(x + 5)$.

b Differentiate $f(x) = 2(x - 1)(x + 5)$ with respect to x.

c The tangent to $y = f(x)$ at the point where $x = a$ has gradient -5.

i Find the value of a.

ii Hence, find the coordinates of the point on the curve where the gradient is -5.

21 The table alongside shows exchange rates for the Malaysian ringgit (MYR).

Currency	1 MYR
Euro (EUR)	0.249 715
Chinese yuan (CNY)	2.004 38

a **i** Calculate how many Chinese yuan can be exchanged for 1000 MYR.

ii If this transaction is subject to a 1.5% commission, calculate the amount of yuan received by the purchaser. Give your answer correct to the nearest yuan.

b A tourist exchanges 5000 CNY into Malaysian ringgit. The money is left over after the holiday, however, so it is exchanged into euros ready for the next trip. A 1.5% commission applies to each transaction. Determine the amount of euros received, giving your answer correct to the nearest cent.

22 A ball is dropped from a height of 4 metres. After each bounce, the ball reaches 80% of its previous height.

a Find the height reached by the ball after the:

i first bounce **ii** second bounce.

b Assuming this trend continues, write an expression for the height reached by the ball after the nth bounce.

c Determine the height reached by the ball after the 20th bounce. Write your answer correct to the nearest millimetre.

23 The probability of event A occurring in any given trial of an experiment is $\mathrm{P}(A) = a$.

a Write down $\mathrm{P}(A')$.

b Suppose two trials of the experiment are performed independently. Write in terms of a, the probability of:

i A occurring exactly once **ii** A occurring twice.

c The probability of A occurring at least once in two independent trials is 0.94. Determine the value of a.

24 Consider the function $f(x) = -2 \times 3^x + 6$, where $x \in \mathbb{R}$.

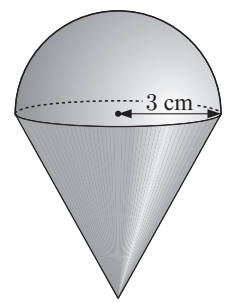
a Find the y-intercept.

b Determine the value of y when $x = 2$ and when $x = -2$.

c Determine the equation of the horizontal asymptote.

d Sketch $y = -2 \times 3^x + 6$, showing the features you have found.

25 A solid metal spinning top is constructed by joining a hemispherical top to a cone-shaped base.
The radius of both the hemisphere and the base of the cone is 3 cm.



a Calculate the volume of the hemispherical top.

b Calculate the height of the cone-shaped base if its volume is half that of the hemisphere.

c Hence, calculate the total outer surface area of the spinning top.

26 **a** Find the equation of the normal to $y = x^2 + \dfrac{3}{x} - 2$ at the point where $x = 1$.

b Write down the coordinates of the point where this normal meets the curve again.

27 A small confectionery company produces x candy bars per day, where $x \leqslant 1500$.
For a given day, the cost per candy bar C depends on x, such that
$C(x) = 0.000\,004x^2 - 0.008x + 5$ dollars.

a Calculate $C(1500)$, and interpret this value. **b** Find $C'(x)$.

c Solve $C'(x) = 0$. **d** Determine the minimum daily cost per candy bar.

28 In July 2007, Fari purchased a car for €12 000. In July 2010, the car was valued at €8600.

a Calculate the average annual rate of depreciation on the car.

b If the car continued to depreciate at this same rate, find its value in July 2012.

29 The speed of sound in dry air at 20°C is 343 m s^{-1}. Calculate how many metres sound travels in one hour, giving your answer:

a correct to two significant figures **b** in scientific notation.

30 Propositions p and q are defined as follows: p: Antonio plays football.
q: Antonio is good at kicking a ball.

a Write the following in words:

i $p \wedge \neg q$ **ii** $\neg q \Rightarrow p$

b Write the following in symbolic language:

i Antonio plays football or is good at kicking a ball, but not both.

ii If Antonio does not play football, then he is not good at kicking a ball.

c Use a truth table to show that the implication in **a ii** is not a tautology.

31 The sides of a right angled triangle are x cm, $(x+3)$ cm, and $(x+6)$ cm long.

a Write a quadratic equation in x which links the three sides.

b Solve the equation.

c Hence find the area of the triangle.

32 A sail in the shape of a rhombus has sides of length 8 metres, and the longer diagonal has length 13 metres.

a Draw a diagram and label the given information.

b Find the length of the shorter diagonal of the rhombus.

c Find the measure of the smaller angle in the rhombus.

33 The ages in months of 20 students are:

198, 192, 195, 194, 205, 208, 210, 200, 206, 203,
196, 198, 196, 201, 194, 198, 197, 195, 209, 204.

a Find the:

i median **ii** range **iii** interquartile range of the data.

b Draw a box and whisker plot for the ages of the students.

34 Consider the function $f(x) = 2x^3 - 3x^{-2} - 24x + \frac{3}{4}$.

a Find $f'(x)$. **b** Write down the value of $f'(2)$.

c Given $f(2) = -32$, find the equation of the tangent to $y = f(x)$ at the point where $x = 2$. Write your answer in the form $ax + by = c$ where $a > 0$.

35 Last Tuesday, 48.47 Indian rupee (INR) was equivalent to 70.71 Japanese yen (JPY).

a If the exchange rate of INR to JPY is written in the form $1 : a$, find the value of a. Give your answer correct to 5 significant figures.

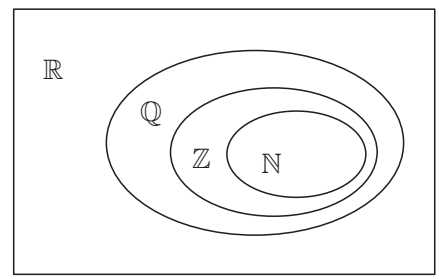
b Calculate the value of 2000 rupees in yen, correct to the nearest yen.

c Calculate the value of 15 000 yen in rupees, correct to 2 decimal places.

36 The set $X = \{\frac{2}{7}, \sqrt{7}, 0, 2^{-4}, 0.\overline{1}, -1.2 \times 10^4\}$ is a subset of $\mathbb{R}$.

The Venn diagram alongside shows subsets of the real numbers.

Place each element of X on the Venn diagram.



37 Two events A and B are independent. $\text{P}(A) = 0.1$ and $\text{P}(B) = 0.5$.

a Calculate: **i** $\text{P}(A \cap B)$ **ii** $\text{P}(A \cup B)$ **iii** $\text{P}((A \cup B)')$

b State, giving a reason, whether events A and B are mutually exclusive.

38 Triangle ABC has $\text{A}\widehat{\text{C}}\text{B} = 35°$, AC = 14 cm, and AB = 17 cm.

a Sketch triangle ABC, showing all of the information provided. Your diagram does not need to be drawn to scale.

b Calculate $\text{A}\widehat{\text{B}}\text{C}$ correct to 2 decimal places.

c Determine the area of triangle ABC, correct to the nearest cm^2.

39 A group of students took an IQ test to measure their intelligence. The results were:

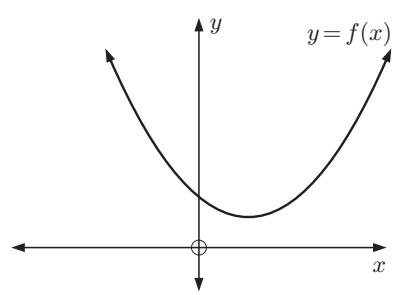
119, 102, 89, 84, 85, 120, 90, 104, 95, 94, 89, 132

a For this data, calculate the:

i range **ii** mean **iii** standard deviation.

b A result of two standard deviations above the mean is classified as "superior intelligence". What proportion of the students within this group have superior intelligence?

40 The graph of $y = f(x)$ is shown opposite, where $f(x) = ax^2 + bx + c$.



a Copy and complete the table below using either *positive*, *negative*, or *zero*.

Constant	a	b	c
Value			

b How many real zeros does $f(x)$ possess?

c Another function $g(x)$ has the form $g(x) = px^2 + qx + r$, where p, q, and r are real constants with signs as follows:

Constant	p	q	r
Value	negative	positive	negative

Given that $g(x)$ has exactly one real zero, sketch a possible graph of $y = g(x)$.

41 A drug in the bloodstream of a patient t hours after being administered, has concentration $C(t) = 2t \times 3^{-t}$, where $0 \leqslant t \leqslant 8$.

a Using your calculator, sketch the graph of $C(t)$ for the domain specified. Clearly show the coordinates of the local maximum.

b For what time interval is the concentration of the drug:

i decreasing **ii** greater than 0.5?

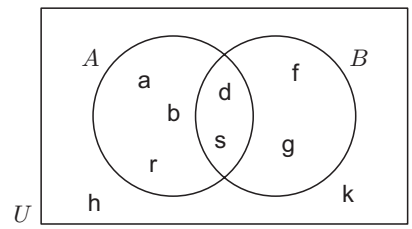
42 £20 000 is to be invested for 4 years.

a Calculate the final value of this investment if interest is offered at:

i 5% p.a. compounded annually **ii** 5% p.a. compounded monthly.

b What is the percentage increase in the final value of the investment when the interest is compounded on a monthly basis, rather than annually?

43 Consider the Venn diagram shown.



a List the letters in:

i $A \cap B$ **ii** $(A \cup B)'$

iii $A \cup B'$ **iv** $A' \cap B$

b Determine:

i $n(A \cup B)$ **ii** $n(A \cap B')$

44 A bag contains 5 red, 3 yellow, and 2 green balloons. Without looking, Mary takes one balloon out of the bag and blows it up. She then takes out a second balloon. Find the probability that:

a Mary selects a red balloon and then a yellow balloon

b the second balloon is green, given that the first balloon is green

c the first balloon is not red and the second balloon is red.

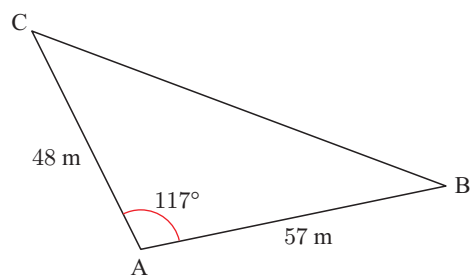
45 A quadratic function has x-intercepts -2 and 3, and the graph of the function passes through $(-3, 18)$.

a Find the equation of the function. Write your answer in the form $y = ax^2 + bx + c$.

b Write down the y-intercept.

c Find the coordinates of the vertex of the graph.

46 The diagram shows the plan of a triangular garden bed. The garden bed will be enclosed by a 50 cm high wall and then filled with soil.



a Calculate the length BC.

b Calculate the area of the garden bed.

c Find the volume of soil needed to fill the garden bed.

47 The following results were recorded in a recent Mathematical Studies test.

Score (%)	$50 \leqslant S < 60$	$60 \leqslant S < 70$	$70 \leqslant S < 80$	$80 \leqslant S < 90$	$90 \leqslant S < 100$
Frequency	6	15	20	10	4

a Draw a table of cumulative frequencies.

b Draw a cumulative frequency curve for this information.

c Estimate the median score.

48 A and B are points on the curve $f(x) = 2x^3 - 5x^2 - 4x + 3$ at which the tangents to the curve are parallel to the x-axis.

a Write down the gradient of the tangent at A.

b Find the gradient function of the curve.

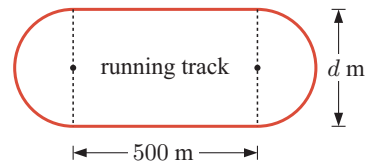
c Find exactly the x-coordinates of points A and B.

49 \$7000 is invested at 7.5% p.a. interest, compounding monthly.

a Calculate the interest earned on this investment after 18 months.

b How long will it take for the investment to earn \$1700 in interest?

50 A running track consists of two straight segments joined by semi-circular ends, as shown.



a If the total perimeter of the track is 1600 metres, determine the diameter of the semi-circular ends.

b Jason takes 4 minutes and 25 seconds to complete a single lap of the track. Calculate Jason's average speed in m s^{-1}.

51 A group of 250 students of ages 13, 14, and 15 were asked to choose which of Art and Music they preferred. The results are shown in the table alongside.

	13	14	15	*Total*
Music	35	p	65	120
Art	55	q	r	130
Total	90	50	110	250

a Calculate the values of p, q, and r.

b A student from the group is selected at random. Calculate the probability that this student:

i is 13 years old **ii** is not 13 years old, and Music is their preferred subject

iii is not 15 given that their preferred subject is Art.

52 The vertex of a quadratic function is $(2, -25)$, and one of the x-intercepts is -3.

a Sketch the function, showing the information provided.

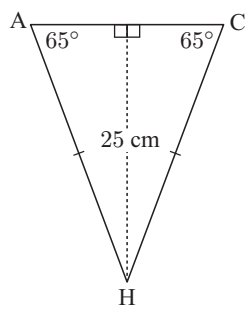
b Write down the other x-intercept.

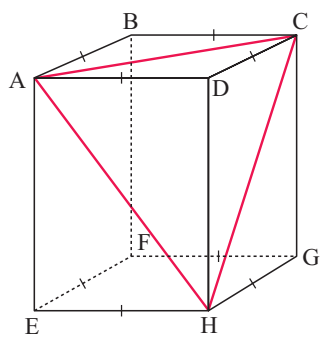
c The quadratic function can be written in the form $y = a(x - p)(x - q)$.
Determine the values of: **i** p and q **ii** a.

53 Triangle ACH is isosceles with altitude 25 cm and base angles $\text{H}\widehat{\text{A}}\text{C} = \text{H}\widehat{\text{C}}\text{A} = 65°$.

a Calculate the length of:

i AH **ii** AC





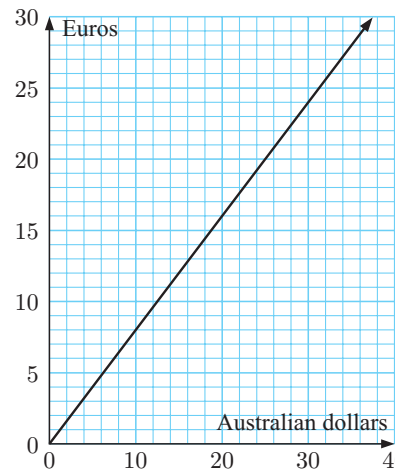
b Triangle ACH lies within the square-based rectangular prism shown. Determine the volume of this square-based prism.

54 The curve with equation $y = x^3 - 4x^2 + 3x + 1$ passes through A$(3, 1)$.

a Find $\frac{dy}{dx}$.

b Determine the equation of the tangent to this curve at A.

55 The straight line graph shows the relationship between Australian dollars and euros.



a Estimate the value of:

i 20 Australian dollars in euros

ii 24 euros in Australian dollars.

b Write down the exchange rate from Australian dollars to euros.

c The sum of 75 000 AUD is converted into euros. If the transaction is charged 2% commission, how many euros are actually received?

56 During a darts competition, players record their best 3-dart score out of 180. The results are shown below:

132	146	154	113	126	140	137	148	156	133	121	102
117	142	168	135	170	138	161	146	159	115	122	126

a Determine the median score. **b** Calculate the interquartile range.

c Draw a box and whisker plot to display the data.

d Use technology to find the standard deviation of the data.

57 A sequence is specified by the formula $t_n = 7n - 12$.

a List the first three terms of the sequence.

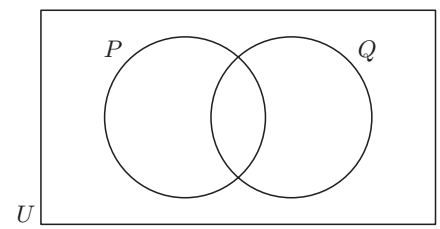
b Determine whether these terms form an arithmetic or geometric sequence.

c Find the 100th term of the sequence.

d Hence, or otherwise, find the sum of the first 100 terms.

58 Use Venn diagrams like the one alongside to illustrate the truth set for the compound propositions:

a $p \wedge q$ **b** $\neg(p \vee q)$ **c** $p \veebar q$

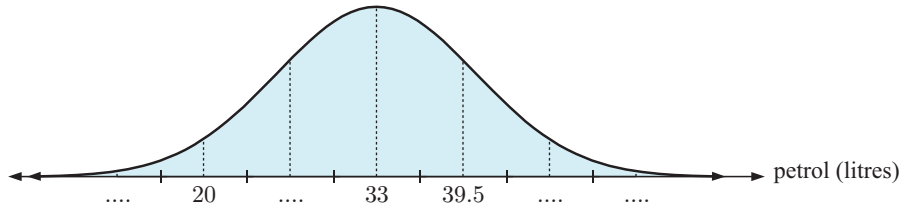


59 **a** The normal to $y = 3x^2 - ax + 2b$ at the point $(1, 5)$ has equation $y = -\frac{1}{2}x + \frac{11}{2}$. Find a and b.

b Find the coordinates of the point where the normal meets the x-axis.

60 The amount of petrol bought by customers at a service station is normally distributed with a mean of 33 litres and a standard deviation of 6.5 litres.

a Copy and complete the values on the normal distribution curve below.



b Find the probability that a randomly chosen customer bought:

i between 35 and 40 litres **ii** at most 25 litres.

c 30% of customers bought k or more litres of petrol. Find k.

d In one day the service station had 500 customers. How many of these customers would be expected to buy less than 20 litres of petrol?

61 Nunu performs a chi-squared test to see if there is any association between the *time taken to travel to work* in the morning (short time or long time) and the *quality of work* she accomplishes in the day (good or poor). She performs this test at the 5% level of significance.

a Write down the null and alternate hypotheses.

b Determine the number of degrees of freedom for this test.

c The p-value for Nunu's test is 0.082. What conclusion can be drawn? Justify your answer.

62 Consider $f(x) = x^3 - 4x^2 + 4x - 2$.

a Find $f'(x)$.

b Solve $f'(x) = 0$.

c Draw a sign diagram for $f'(x)$.

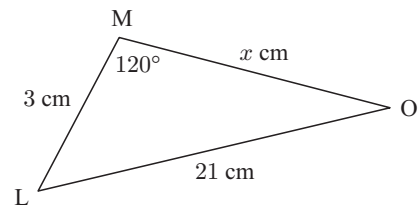
d Find intervals where $f(x)$ is increasing or decreasing.

63 **a** On the same set of axes, draw the graphs of $y = 2x - 12$ and $2x - 3y = 24$.

b Find the coordinates of A, the intersection point of the two lines.

c Find the equation of the line perpendicular to $2x - 3y = 24$ and passing through A.

64 Triangle LMO has $\widehat{\text{LMO}} = 120°$, LM = 3 cm, LO = 21 cm, and MO = x cm.



a Evaluate $\cos 120°$.

b Using the cosine rule, show that $x^2 + 3x - 432 = 0$.

c Hence, find x correct to 3 significant figures.

d Find the perimeter of triangle LMO.

65 A circle has area 300 cm^2.

a Find the radius of the circle correct to 3 decimal places.

b Find the circumference of the circle correct to 2 significant figures.

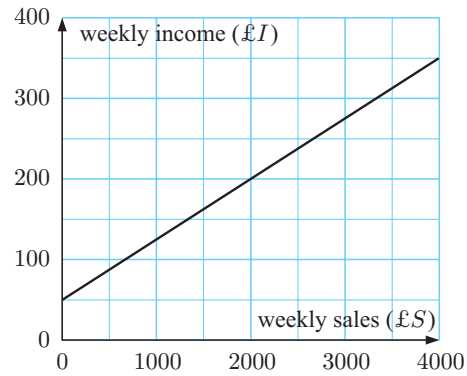
66 For two events A and B, it is known that $\text{P}(A) = \frac{2}{5}$, $\text{P}(B) = \frac{3}{10}$, and $\text{P}(B \mid A) = \frac{1}{2}$.

a Calculate $\text{P}(A \cap B)$.

b Show that A and B are not independent.

c Calculate $\text{P}(A \mid B)$.

67 The weekly income £I of an employee varies depending on their total weekly sales £S. The chart alongside shows the relationship between I and S.



a Use the graph to determine the employee's weekly income if:

i no sales are made

ii £2000 in sales are made.

b In a given week an employee earns £275. Use the graph to estimate their total sales for that week.

c The formula connecting I and S has the form $I = rS + t$, where r and t are both constants. Calculate the values of r and t.

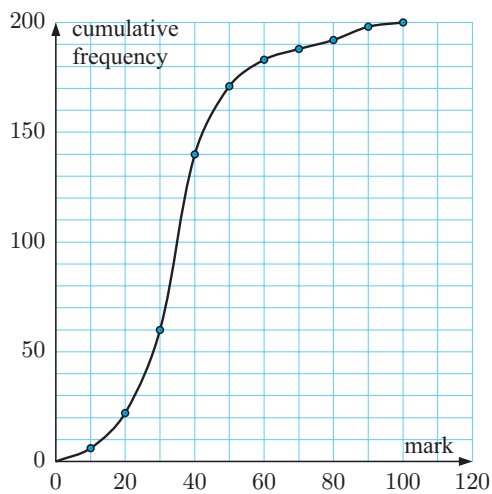
68 A function f is defined by $f(x) = x^2 + \dfrac{2}{x}$ for $-4 \leqslant x \leqslant 4$.

a Sketch $y = f(x)$ for the region $-4 \leqslant x \leqslant 4$, $-15 \leqslant y \leqslant 15$.

b Using technology, write down the coordinates of the local minimum.

c Hence, find the intervals in the given domain where $f(x)$ is decreasing.

69



The examination marks for 200 students are displayed on the cumulative frequency graph shown. The pass mark for the examination was 30.

a What percentage of the students passed?

b A box and whisker plot for the examination data is:

Use the graph to estimate:

i m **ii** n **iii** p **iv** q

70 A geometric sequence has its general term given by $u_n = 6(1.2)^{n-1}$.

a Write down the first two terms of the sequence.

b Find the first term of the sequence which is greater than 30.

c Another geometric sequence has first term 2 and common ratio 1.35. Find the first term of this sequence which is greater than the equivalent term of the first sequence.

71 Consider a game in which the player rolls a single die once. The player wins \$2 for rolling a 1, \$3 for rolling a 2, 3, or 4, and \$5 for rolling a 5 or 6. The game costs \$4 to play.

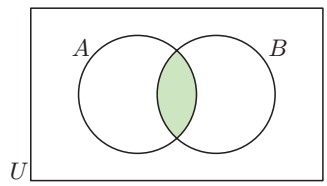
a If the game is played 150 times, how many times would you expect the player to roll a 5 or 6?

b Calculate the expected result from playing this game once.

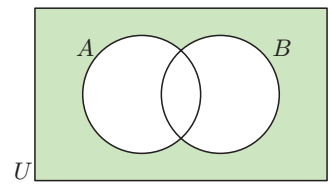
c Determine whether the game is fair.

72 Use logic symbols to describe the shaded area on the following Venn diagrams:

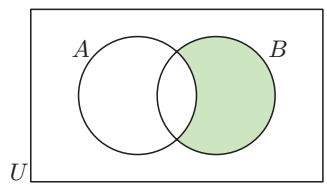
a



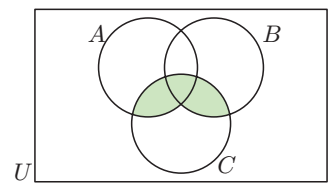
b



c



d



73 The functions f and g are defined for $-5 \leqslant x \leqslant 5$ by $f(x) = \dfrac{x}{x-2}$, $x \neq 2$ and $g(x) = x$.

a Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same set of axes.

b Write down the equations of the horizontal and vertical asymptotes of $y = f(x)$.

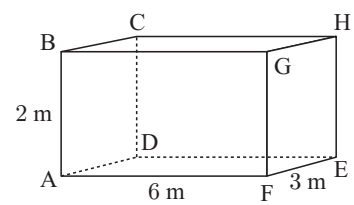
c Find the solutions of $\dfrac{x}{x-2} = x$.

74 For the solid shown, find:

a the length of AE

b the length of BE

c the angle BE makes with the base ADEF.



75 Consider the following data regarding the time to complete a task compared with a person's age.

Age x (years)	12	18	26	31	36	42
Time y (minutes)	22	30	24	28	26	23

a Use your graphics calculator to find the equation of the regression line for y in terms of x.

b Use your equation to estimate the time for a person aged 28 years to complete the task.

c Use your graphics calculator to find the correlation coefficient r.

d Comment on the reliability of your answer to **b**.

76 The profit when m machines are sold by a firm each month can be determined by the function $P(m) = 60m - 800 - m^2$ thousand dollars, where $0 \leqslant m \leqslant 40$.

a Write down a function for the rate of change in the profit for a given change in the number of machines sold by the firm.

b Show that the monthly profit is maximised when 30 machines are sold.

c Calculate the maximum monthly profit.

77 Consider the function $f(x) = \frac{1}{2}x^3 - 2x^2 - 4x + 6$.

a Find the axes intercepts.

b Find the position and nature of any turning points.

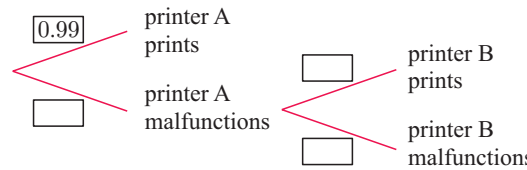
c Find $f(-3)$ and $f(6)$.

d Sketch the function on the domain $-3 \leqslant x \leqslant 6$.

e Find the greatest and least value of $f(x)$ on the domain $-3 \leqslant x \leqslant 6$.

78 A student has access to two printers, A and B. The probability that printer A malfunctions is 1%, and the probability that printer B malfunctions is 2%. When attempting to print, the student always tries printer A first. Printer B is only used if printer A malfunctions.

a Complete the tree diagram by filling in the missing values.



b When the student tries to print a one page document, determine the probability that:

i both printers malfunction

ii printer B prints the document

iii printer A prints the document, given that the document is printed.

79 The speed of sound can be found using the formula $S = 331.3 + 0.606T$ m s^{-1}, where T is the temperature of the air in degrees Celsius.

a Determine the speed of sound at 20°C. Do not round your answer.

b Determine the distance sound travels in 10 minutes at 20°C. Write your answer in the form $a \times 10^k$, where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

c On a hot day, the temperature rises from 20°C to 45°C. Calculate the percentage increase in the speed of sound which results from this change in air temperature.

80 **a** Sketch the graph of $y = \dfrac{3}{x} - 2$ on the domain $-5 \leqslant x \leqslant 5$.

b Write down the equation of the vertical asymptote.

c Find the equation of the normal at the point where $x = 3$.

81 Line A has equation $y = 2x + 5$. Line B has equation $x - 4y = 8$. Point P is the y-intercept of line A and point Q is the x-intercept of line B.

a Write down the coordinates of:

i P **ii** Q.

b Calculate the gradient of PQ.

c Hence, or otherwise, calculate the *acute* angle PQ makes with the x-axis.

d Line A and line B intersect at R. Find the coordinates of R.

82 Line S has equation $2x + y = -2$.

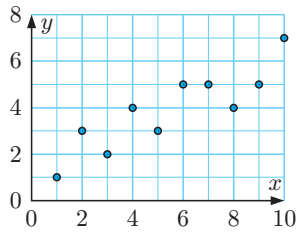
a Write down the gradient of S.

b Line T is parallel to line S and passes through A$(1, 4)$.
Find the equation of line T.

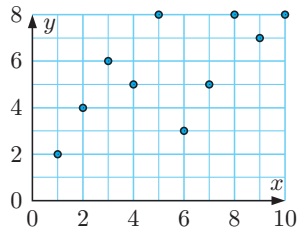
c Line T is a tangent to the quadratic $y = x^2 + bx + c$ at A.
Find b and c.

83 Consider the scatter diagrams shown below.

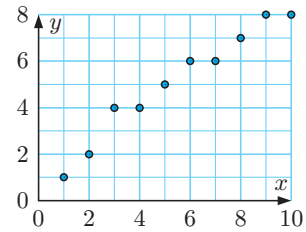
Scatter diagram I



Scatter diagram II



Scatter diagram III



a For each of the cases shown, is the association between x and y positive, negative, or zero?

b Complete the table by matching each description with scatter diagram I, II, or III.

Strength of correlation	*Scatter diagram*
Weak	
Moderate	
Strong	

84 A teacher recorded the number of children who used the school's playground each day for 50 days. The results are displayed alongside.

Number of children	*Frequency*
21 - 30	8
31 - 40	16
41 - 50	14
51 - 60	12
Total	50

a On how many days was the playground used by more than 40 children?

b Find the modal class.

c Draw a column graph to display the data.

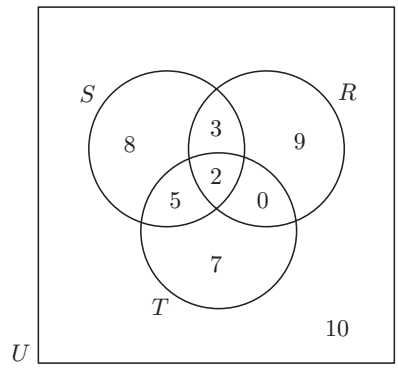
d Estimate the **i** mean **ii** standard deviation of the data.

85 Usain Bolt set a world record at the Beijing Olympics by running the 100 m sprint in 9.69 seconds.

a Calculate his average speed for this race in metres per second, giving your answer correct to 2 decimal places.

b Convert this speed to kilometres per hour.

86 The Venn diagram shows the number of students in a group who play soccer (S), rugby (R), or do track (T).



a Find the total number of students in the group.

b Find the probability that a randomly chosen student:

i plays only rugby

ii takes soccer and track only

iii plays soccer, given that he or she plays rugby.

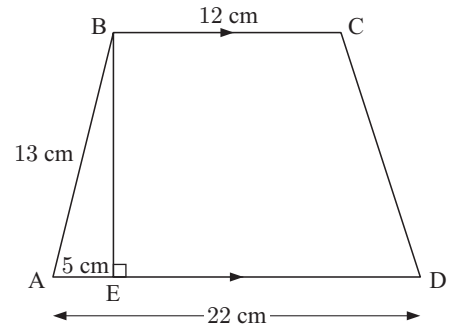
87 The value of a car over time is calculated using the function $v = 24\,000r^t$ dollars, where t is the number of years after it was first purchased, $t \geqslant 0$, and r is a constant, $0 < r < 1$.

a Write down the value of the car when it was first purchased.

b The value of the car after one year was \$20 400. Find the value of r.

c How long will it take for the value of the car to reduce to \$8000? Give your answer to the nearest year.

88 ABCD is a trapezium with BC parallel to AD. AD = 22 cm, BC = 12 cm, AB = 13 cm, and AE = 5 cm.



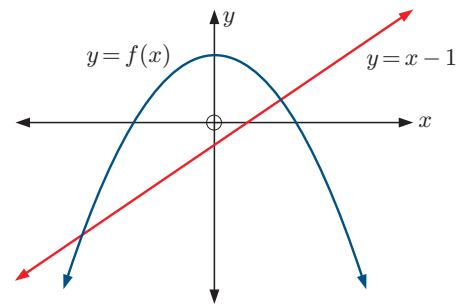
a Calculate the height BE of the trapezium.

b Calculate:

i $\widehat{BAE}$ **ii** $\widehat{ABC}$

c Calculate the length of the diagonal AC.

89 The diagram alongside shows the graphs of $f(x) = -\frac{1}{2}x^2 + 3$ and $y = x - 1$.



a Find $f'(x)$.

b Find the coordinates of the point on $f(x)$ where the tangent is parallel to $y = x - 1$.

90 Ten students were given aptitude tests on language skills and mathematics. The table below shows the results:

Language (x)	12.5	15.0	10.5	12.0	9.5	10.5	15.5	10.0	14.0	12.0
Mathematics (y)	32	45	27	38	18	25	35	22	40	40

a Plot the data on a scatter diagram.

b Find the correlation coefficient r.

c Use your results to comment on the statement:

"Those who do well in language also do well in mathematics."

91 Consider the function $f(x) = \frac{8}{x^2} + 2x - 3$.

a Differentiate $f(x)$ with respect to x.

b Find $f'(1)$ and explain what it represents.

c Find the coordinates of the point where the gradient of the curve is zero.

92 **a** Complete the truth table for the compound proposition $\neg b \wedge (c \Rightarrow (b \wedge a)) \Rightarrow \neg c$

a	b	c	$\neg b$	$b \wedge a$	$c \Rightarrow (b \wedge a)$	$\neg b \wedge (c \Rightarrow (b \wedge a))$	$\neg c$	$\neg b \wedge (c \Rightarrow (b \wedge a)) \Rightarrow \neg c$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

b Is the proposition $\neg b \wedge (c \Rightarrow (b \wedge a)) \Rightarrow \neg c$ a tautology, a contradiction, or neither?

93 Melissa deposits €5000 into a bank account which pays 6% p.a. interest compounding quarterly. No extra money is deposited or withdrawn.

a Determine the total value of the investment after:

i 1 year **ii** 2 years.

b Write down a formula for the total value of the investment after n years.

c Calculate the number of years required for Melissa to double her original deposit.

94 Let $a = 3.5$, $b = 1.2$, $c = 0.4$, and $d = -8$. Find the value of $\dfrac{5(a-d)^2}{b-c}$, giving your answer:

a to 3 decimal places **b** to 2 significant figures

c in the form $a \times 10^k$, where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

95 Consider A$(-4,\ 2)$ and B$(6,\ 6)$.

a Write down the coordinates of M, the midpoint of the line segment AB.

b Find the gradient of AB.

c Line L is perpendicular to AB and passes through M. Determine the equation of line L, giving your answer in the form $ax + by + d = 0$.

d Calculate the x-intercept of line L.

96 Greg needs to travel from Australia to Sweden for work. Prior to leaving, he converts Australian dollars (AUD) into Swedish kronor (SEK). The bank offers the rates in the table shown.

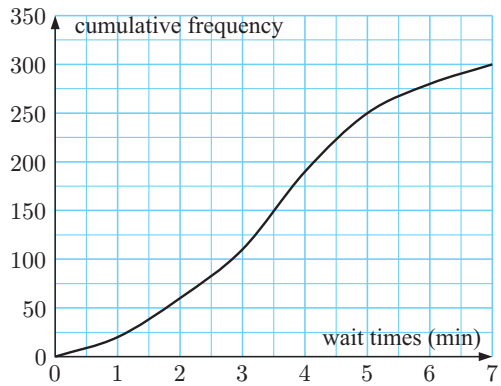
1 AUD	*Buy*	*Sell*
SEK	7.08	7.11

a If Greg exchanges 1000 Australian dollars into Swedish kronor, how much will he receive?

b If Greg immediately exchanged his Swedish kronor back into Australian dollars, how much would he receive?

c Calculate the percentage loss if Greg converted his money from AUD into SEK and immediately back into AUD.

97 The manager of a bank decides to investigate the time customers wait to be served. Most of the results are shown in the table below, and they are illustrated in the cumulative frequency graph alongside it.



Waiting time (t) *in minutes*	*Frequency*
$0 \leqslant t < 1$	p
$1 \leqslant t < 2$	40
$2 \leqslant t < 3$	50
$3 \leqslant t < 4$	80
$4 \leqslant t < 5$	60
$5 \leqslant t < 6$	q
$6 \leqslant t < 7$	20

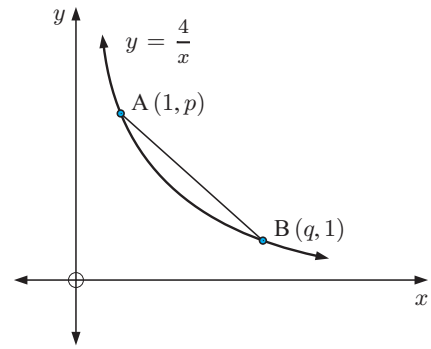
a Use the graph to estimate the median waiting time.

b Determine the values of p and q.

c Hence, draw a frequency histogram for the data.

98 The graph shows $y = \frac{4}{x}$ for $x > 0$.

Points $A(1, p)$ and $B(q, 1)$ lie on the curve.



a Determine the value of:

i p **ii** q

b Evaluate the gradient of line segment AB.

c Point C also lies on $y = \frac{4}{x}$. The tangent at C is parallel to AB. Determine the coordinates of C.

99 A woman deposits \$100 into her son's savings account on his first birthday. She deposits \$125 on his second birthday, \$150 on his third birthday, and so on.

a Calculate the amount of money she will deposit into her son's account on his 15th birthday.

b Find the total amount she will have deposited over the 15 years.

100 The table shows the frequencies of the ages of students at a school.

Age (years)	*Frequency*
13	48
14	75
15	84
16	60
17	33
Total	300

a If a student is randomly selected from this school, find the probability that the student is:

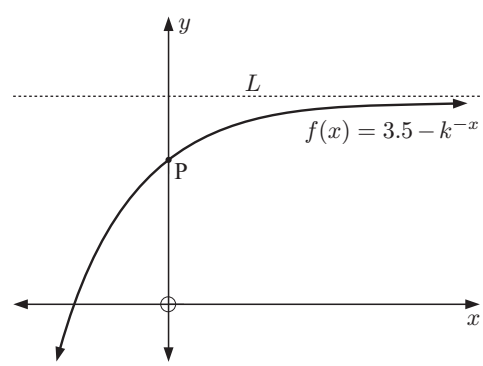
i 13 or 14 years old

ii 15 years of age or older.

b Given that a randomly selected student is older than 14, find the probability that the student is not 17.

101 The graph shows the function $f(x) = 3.5 - k^{-x}$, where k is a positive constant.

The point $(-1, 2)$ lies on the graph.



a Write down the coordinates of the y-intercept, P.

b Find the value of k.

c Find the equation of the horizontal asymptote, L.

102 The table shows the amount of petrol remaining in a motorbike's fuel tank and the number of kilometres travelled. The capacity of the tank is 10 litres.

Remaining fuel x (litres)	10	8	6	4	2	1
Distance y (km)	0	90	190	260	330	370

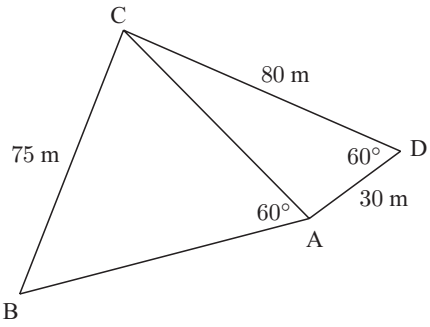
a Plot this data on a scatter diagram.

b Write down the equation of the straight line of regression for y against x.

c The motorbike has travelled 220 km since its tank was refilled. Use your regression equation to estimate the amount of fuel left in the tank.

d Find the average distance travelled per litre over the 220 km.

103



The figure shows two adjacent triangular fields ABC and ACD. AD = 30 m, CD = 80 m, BC = 75 m, $\widehat{ADC} = 60°$, and $\widehat{BAC} = 60°$.

a Calculate the length of AC.

b Calculate the size of $\widehat{ABC}$.

c Find the total area of the fields.

104 $10\,000$ is invested at 8.5% p.a. interest compounded monthly over 5 years. Find:

a the value of the investment after 5 years

b the amount of interest earned

c how much more interest would have been earned if the interest rate had been 9.5% p.a.

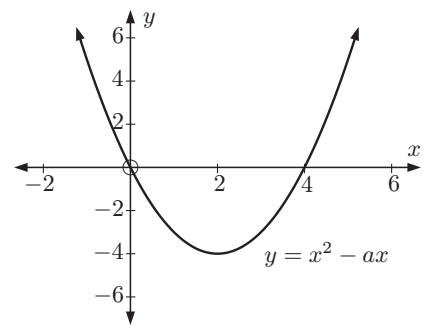
105 The diagram shows the graph of $y = x^2 - ax$.

a Find the value of a.

b Find $\frac{dy}{dx}$.

c Find the coordinates of the minimum point.

d Find the gradient of the tangent to the curve at $x = 3$.



106 Suppose $a = \frac{3}{7}$, $b = 8$, $c = \sqrt{2}$, and $d = 1 \times 10^{-2}$. Classify each of the following statements as true or false:

a $7a \in \mathbb{N}$ **b** $c^2 - b \notin \mathbb{N}$ **c** $\frac{1}{d} \geqslant c^b$

d $\frac{c}{b} \in \mathbb{Q}$ **e** $\sqrt{a - b} \in \mathbb{R}$ **f** $\frac{b}{d} \in \mathbb{Z}$

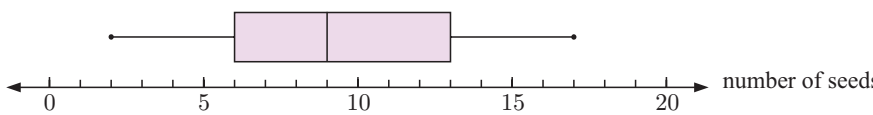
107 For the two events A and B, $\mathrm{P}(A) = \frac{3}{7}$ and $\mathrm{P}(B') = \frac{2}{3}$.

a Determine $\mathrm{P}(B)$.

b Calculate $\mathrm{P}(A \cup B)$ if A and B are:

i mutually exclusive **ii** independent.

108 Margaret picked some mandarins from a tree, and counted the number of seeds in each. Her results are shown in the boxplot below.



Find the:

a median **b** interquartile range **c** range of the data.

109 A χ^2 test at a 5% significance level is used to determine whether *intelligence* is independent of *income level*. For this test, intelligence and income were each split into three classes. The resulting χ^2 test statistic was 8.23.

a State the null and alternative hypotheses.

b Determine the number of degrees of freedom.

c Given that the critical value for this test is ≈ 9.49, what conclusion can be drawn regarding *intelligence* and *income level*?

110 Let $f(x) = \frac{1}{4}x^4 - 2x^2$.

a Find $f'(x)$.

b Evaluate $f'(-3)$, $f'(-2)$, and $f'(-1)$.

c Hence, describe what happens to $f(x)$ at $x = -2$.

d The graph of $y = f(x)$ has exactly 3 turning points. Given $(0,\ 0)$ is a local maximum and $(2,\ -4)$ is a local minimum, write down the intervals for which $f(x)$ is increasing.

111 P$(-3,\ 0)$, Q$(0,\ 4)$, and R$(8,\ 0)$ are the vertices of a triangle. The length of QR is $\sqrt{80}$ units.

a Calculate the length of: **i** PR **ii** PQ.

b Determine the size of angle QPR, giving your answer correct to 2 decimal places.

c Calculate the area of triangle PQR.

112 Consider the following currency conversions between US dollars (USD), Mexican pesos (MXN), and euros (EUR).

$$1 \text{ USD} = 14 \text{ MXN}$$
$$1 \text{ USD} = 0.79 \text{ EUR}$$

a Convert 2750 US dollars into:

i Mexican pesos **ii** euros.

b Write down the exchange rate from euros to:

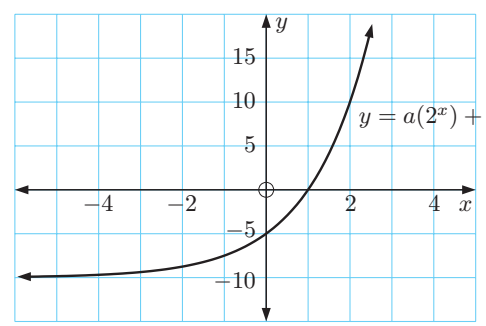
i US dollars **ii** Mexican pesos.

c Hence, convert 5100 euros into Mexican pesos.

113 The graph alongside shows the curve $y = a(2^x) + b$, where a and b are constants.

a Find the values of a and b.

b Find y when $x = 6$.



114 The fourth term of an arithmetic sequence is 22 and the tenth term is 70. Suppose the first term is u_1 and the common difference is d.

a Write down two equations in u_1 and d that satisfy this information.

b Solve the equations to find the values of u_1 and d.

c Find the sum of the first 10 terms of the sequence.

115 The table shows the number of left and right handed writers in a sample of 50 students.

	Left handed	*Right handed*	*Total*
Male	4	26	30
Female			20
Total	7		50

a Complete the table.

b If a student is selected at random from the group, find the probability that the student is:

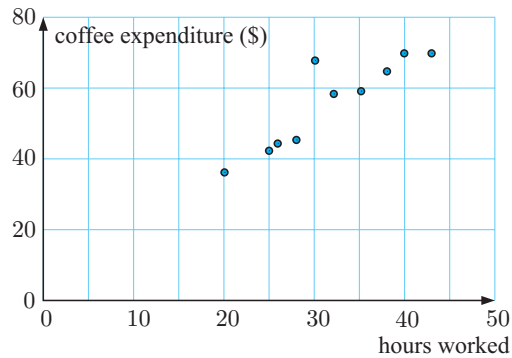
i left handed **ii** male and right handed

iii right handed, given that the student is female.

116 Find the value of k if the lines $4x - 5y = 11$ and $2x + ky = -8$ are:

a parallel **b** perpendicular.

117 The scatter diagram displays the amount James spends on coffee in the cafeteria against the number of hours he works in the week.



a Over a period of weeks, James worked an average of 32 hours, and his average expenditure was \$56 per week. Plot the mean point P(32, 56) on the graph.

b Draw the line of best fit passing through P.

c Use this line to predict the amount James will spend on coffee if he works a 35 hour week.

d Describe the nature and strength of the linear relationship between the length of time James works and the amount he spends on coffee. Comment on whether the predicted quantity found in **c** is a reliable estimate.

118 A straight line L has equation $54x - 2y = 17$.

a Find the gradient of the line.

b A curve has equation $y - x^3 = -12$, $0 \leqslant x \leqslant 5$. Find the gradient function for the curve.

c A tangent to the curve at point P is parallel to the straight line L. Find the coordinates of P.

119 Consider the function $f(x) = x^3 + \dfrac{5}{x}$.

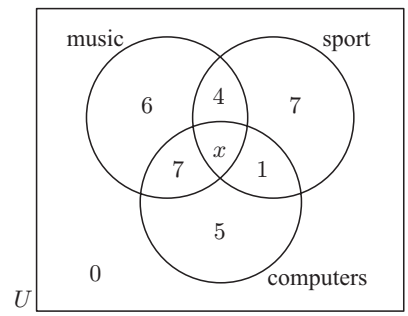
a Find $f'(x)$. **b** Find $f'(1)$, and interpret your answer.

c Hence, find the equation of the tangent to $f(x)$ when $x = 1$.

d Find the equation of the normal to $f(x)$ when $x = 1$.

e Use technology to find the point where the normal meets the graph of $y = f(x)$ again.

120 A group of 50 employees were surveyed regarding their interest in music, sport, and computers. The number of employees interested in each area is shown in the Venn diagram.



a Write down the value of x.

b If an employee is selected at random, determine the probability that they are:

i interested in music

ii interested in music, sport, and computers

iii not interested in computers

iv interested in sport, given that they are interested in music.

121 The tangent to the curve $y = ax^2 - \frac{b}{x}$ at $x = -0.5$ is horizontal.

a Show that $a = 4b$.

b Given that the tangent at $x = -0.5$ has the equation $y = 3$, show that $a + 8b = 12$.

c Use the equations from **a** and **b** to determine the values of a and b.

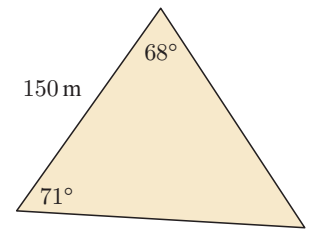
122 Brian deposits £x into his daughter's bank account on her 1st birthday. On her second birthday, he deposits £$1.5x$ into the account, and he continues to add £$0.5x$ to the amount deposited for each subsequent birthday. The final deposit is made on her 20th birthday.

a Write down, in terms of x, the amount Brian deposits into his daughter's bank account on her 3rd birthday.

b Show that the total amount of birthday money Brian deposits is £$115x$.

c For the total amount of birthday money to reach £20 000, find the value of x. Round your answer to the next whole number.

123 Consider the triangle alongside.

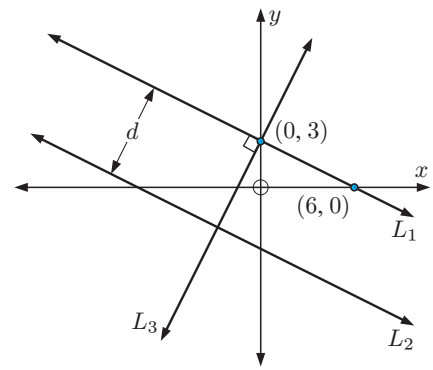


a Find the perimeter of the triangle.

b Find the area of the triangle, giving your answer in:

i square metres **ii** hectares.

124 Line L_1 passes through $(0, 3)$ and $(6, 0)$.
Line L_2 has equation $y = -\frac{1}{2}x - 4$.



a Show that L_1 is parallel to L_2.

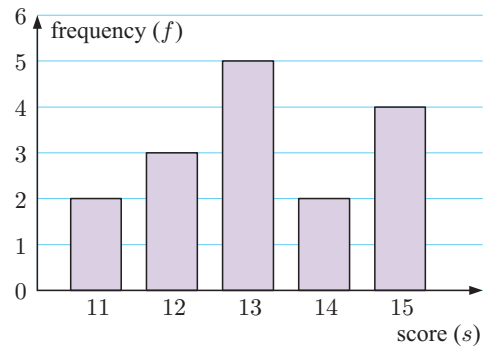
b Line L_3 is perpendicular to L_1 and meets L_1 at $(0, 3)$. Write down the equation of L_3, giving your answer in the form $y = ax + b$.

c At what point does line L_2 meet L_3?

d Calculate the perpendicular distance d between L_1 and L_2.

125 The table and histogram below represent the same set of data.

Score (s)	*Frequency* (f)	$s \times f$
11	2	22
12	w	x
13	5	65
14	2	28
15	4	60
Totals	y	z



a Determine the mode of the data.

b Determine the value of:

i w **ii** x **iii** y **iv** z

c Evaluate $\frac{z}{y}$ and interpret its meaning.

126 ¥600 000 is to be invested for 6 years.

a Society Bank offers a return of 7% p.a. interest compounding annually. Calculate the amount of interest this account will generate over the 6 year period.

b Corporate Credit Union offers a return of $r\%$ p.a. interest compounded monthly. What rate r is needed for Corporate Credit Union to match the amount of interest offered by Society Bank?

127 The nth term of an arithmetic sequence is given by $u_n = 63 - 4n$.

a Calculate the first two terms of this sequence.

b Which term of the sequence is -13?

c Two consecutive terms of this sequence, u_k and u_{k+1}, have the sum 34. Find k.

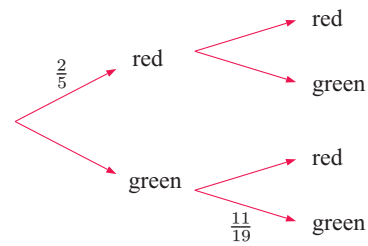
128 A box contains 8 red pens and 12 green pens. Juanita takes one pen from the box and notes its colour. She then takes a second pen from the box without replacing the first pen.

a Complete the tree diagram alongside to show all probabilities.

b Find the probability that she chooses:

i two red pens

ii at least one green pen.



129 A flu virus spreads in a school according to the exponential model $N = 4 \times 1.332^t$ where N is the number of people who have caught the virus, and t is the number of days after the virus was first detected, $t \geqslant 0$.

a Find the number of people who were initially infected.

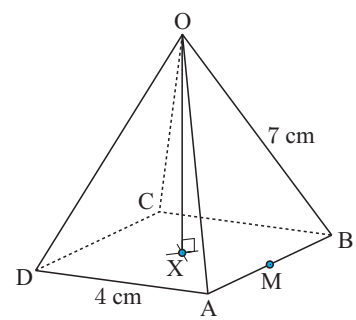
b Calculate the number of people who were infected after 16 days.

c There are 1200 people in the school. Estimate the time it will take for everybody in the school to catch the flu.

130 OABCD is a square based pyramid with base sides of length 4 cm. Each slant edge of the pyramid is 7 cm long. OX is the perpendicular height of the pyramid.

a Calculate the length of BX.

b Calculate the angle between the line OB and the base ABCD. Give your answer to the nearest degree.



131 A 60 cm length of wire is bent into a rectangle with length x cm and width y cm.

a Write an expression for y in terms of x.

b Write an expression for the area $A(x)$ of the rectangle enclosed by the wire.

c Find $A'(x)$.

d Hence determine the value of x which maximises the area. What are the dimensions of the rectangle in this case?

132 The table shows the sizes of drinks purchased from a store at different times of the day.

	Small	*Medium*	*Large*
Morning	15	23	4
Afternoon	24	24	11
Evening	6	18	8

a Write a suitable null hypothesis for a χ^2 test on this data.

b Use technology to find the value of χ^2.

c Determine the number of degrees of freedom.

d Given that the critical value at the 5% level of significance is 9.488, what conclusion can be drawn from this test? Justify your answer.

133 A car was purchased 5 years ago for \$21 000. It depreciated in value by 15% in the first year, 12.5% in the second year, and 10% in the third year.

a Find the value of the car after 3 years.

b Find the average annual rate of depreciation for the first 3 years. Give your answer correct to 3 decimal places.

c If the car depreciated over the 5 years by the average rate found in **b**, find the current value of the car.

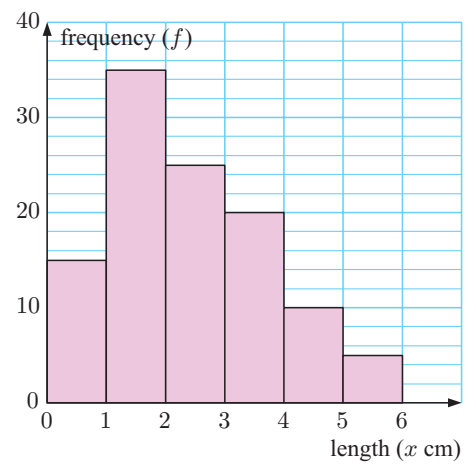
134 Consider the following argument:

Jane ate an apple or a banana, but not both.
Jane did not eat a banana.
Therefore, Jane ate an apple.

a Use the propositions a: Jane ate an apple and b: Jane ate a banana to write the argument in logical form.

b Construct a truth table to show that the argument is valid.

135 The histogram shows the frequencies of a set of lengths.



a Copy and complete the table below using information from the histogram.

Length x (cm)	*Frequency*	*Cumulative frequency*
$0 \leqslant x < 1$	15	15
$1 \leqslant x < 2$	35	50
$2 \leqslant x < 3$	25	75
$3 \leqslant x < 4$	20	95
$4 \leqslant x < 5$		
$5 \leqslant x < 6$		

b Draw a cumulative frequency graph to represent this data.

c Hence, estimate Q_3, the 75th percentile.

136 Suppose A and B are two events such that $P(A) = 0.7$, $P(B) = 0.5$, and $P(A \cup B) = 0.9$.

a Find:

i $P(A \cap B)$ **ii** $P(A \mid B)$ **iii** $P(B \mid A)$

b Are A and B independent?

137 The statement "*If I watch a movie then I will relax*" consists of two propositions p and q such that $p \Rightarrow q$.

a State p and q in words.

b Under what conditions is the statement $p \Rightarrow q$ false?

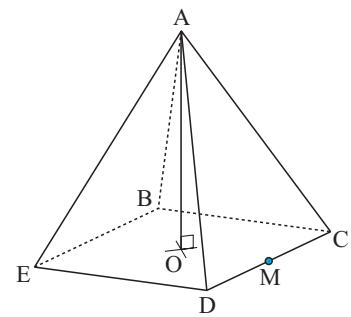
c In words, write down the contrapositive to $p \Rightarrow q$.

d By completing the truth table below, show that the implication $p \Rightarrow q$ is logically equivalent to its contrapositive.

Contrapositive to $p \Rightarrow q$

p	q	$\neg p$	$\neg q$	$p \Rightarrow q$	
T	T				
T	F				
F	T				
F	F				

138 BCDE is the square base of the pyramid ABCDE. O is the centre of BCDE, and the vertex A sits directly above O. M is the midpoint of CD.



a If A sits 12 cm directly above O, and AM is 13 cm in length, calculate the length of OM.

b Calculate the volume of the pyramid.

c What angle do the triangular sides of the pyramid make with the square base?

139 An arithmetic sequence begins with the terms 21, 29, 37, 45, ...

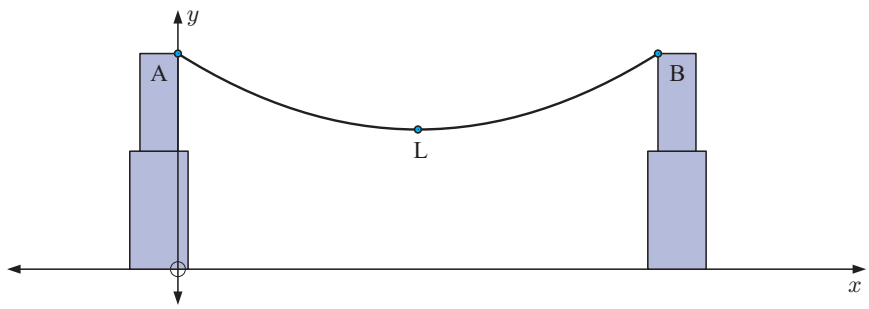
a Write an expression for:

i the nth term **ii** the sum of the first n terms.

b Hence, determine the exact value of:

i the 50th term **ii** the sum of the first 50 terms.

140 A tightrope connects two elevated platforms A and B. The curve of a tightrope between these platforms is given by the equation $y = 0.008x^2 - 0.8x + 50$. The units are metres.



a Find the height of the platform at A.

b Given that platform B has the same height as platform A, determine:

i the distance between the two platforms

ii the domain for which $y = 0.008x^2 - 0.8x + 50$ represents the tightrope.

c L is the lowest point along the tightrope. Determine the coordinates of L.

B LONG QUESTIONS

EXERCISE 22B

1 A competition offers three options for the first prize, each of which pays the winner a monthly sum for 24 months.

Option 1: \$8000 per month.

Option 2: \$1000 in the first month, then each successive month pays \$600 more than the previous month.

Option 3: \$500 in the first month, then each successive month pays 20% more than the previous month.

a Calculate the total prize value for *Option 1*.

b For *Option 2*:

i write down the amount won in each of the first three months

ii calculate the total amount won over the 24 month period.

c For *Option 3*:

i write down the amount won in each of the first three months

ii calculate the total amount won over the 24 month period.

d Which option is worth the greatest amount of money overall?

e The amount won in the first month under *Option 3* is to be altered so that the total prize over 24 months is \$250 000. Calculate the new initial amount, writing your answer to the nearest cent.

2 A jukebox contains 100 different songs, 60 of which are classified as rock songs. Songs from the jukebox are played at random and can be repeated.

a If two songs are played, what is the probability that:

i both are rock songs **ii** one is a rock song **iii** neither is a rock song?

b Explain why the sum of the results in **a** is 1.

c If three songs are played consecutively, determine the probability that they are all rock songs.

d The jukebox is altered so that once a song has been played, it cannot be repeated until all the songs are played. Using R to represent the event of a rock song being played, complete a tree diagram showing the possible outcomes for the first three songs.

Hence, or otherwise, determine the probability that:

i exactly one of the first three songs is a rock song

ii at least one of the first three songs is a rock song.

3 Consider the function $f(x) = \dfrac{4}{x-1} + 2$.

a Find the axes intercepts for the graph $y = f(x)$.

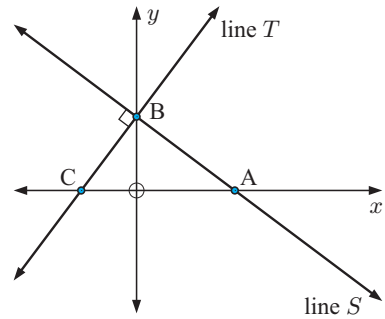
b Determine the equations of the horizontal and vertical asymptotes of the function.

c Hence, write down the domain and range of the function.

d Draw the graphs of $y = f(x)$ and $y = x + 1$ on the same set of axes.

e Hence solve the equation $\dfrac{4}{x-1} + 2 = x + 1$.

4 Line S has equation $3x + 4y = 24$. It intersects the x-axis at A and the y-axis at B.



a Write down the coordinates of:
- **i** A
- **ii** B.

b Determine the:
- **i** gradient of AB
- **ii** distance between A and B.

c Line T is drawn through point B perpendicular to line S. Determine the equation of line T.

d Line T cuts the x-axis at C. Calculate the area of triangle ABC.

e Point D lies on the line segment AC such that the area of triangle ABD is 15 units^2. Determine the coordinates of D.

5 To test the difficulty level of a new computer game, a company measures the time taken for a group of players to complete the game. Their results are displayed in the table opposite.

Completion time (minutes)	*Number of players*
$0 \leqslant t < 30$	1
$30 \leqslant t < 60$	4
$60 \leqslant t < 90$	12
$90 \leqslant t < 120$	18
$120 \leqslant t < 150$	7
$150 \leqslant t < 180$	2

a How many players were surveyed?

b Write down the modal class.

c Using graph paper, draw a cumulative frequency graph for the data. Use 1 cm to represent 15 minutes on the horizontal axis, and 1 cm to represent 4 individuals on the vertical axis.

d The game is considered too easy if either the mean or median completion time is below 90 minutes.
- **i** Estimate the median completion time using your cumulative frequency graph.
- **ii** Estimate the mean completion time using your calculator.
- **iii** Hence, comment on whether the game is too easy.

e Complete the sentence below:
The middle 50% of players completed the game in times between .... and .... minutes.

6 Let $f(x) = x^3 - 6x^2 + px + q$, where p and q are real constants.

a Find $f'(x)$.

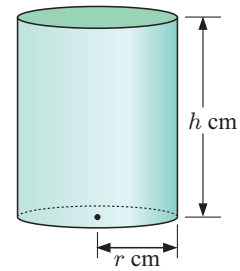
b The graph $y = f(x)$ has a local maximum at $(1, 7)$. Find the values of p and q.

c By solving $f'(x) = 0$, determine the coordinates of a local minimum of the function.

d Sketch $y = f(x)$ on the region $-1 \leqslant x \leqslant 5$, $-10 \leqslant y \leqslant 25$. Clearly show all turning points and axes intercepts.

e Consider the tangent to $y = f(x)$ at $(1, 7)$.
- **i** Add this tangent to your sketch from **d**.
- **ii** Write down the equation of this tangent.
- **iii** At what other point does the tangent meet the curve?

7 A cylinder with base radius r cm has volume 250π cm^3.



a Show that $h = \dfrac{250}{r^2}$.

b Hence show that the surface area of the cylinder is $S = 2\pi r^2 + \dfrac{500\pi}{r}$ cm^2.

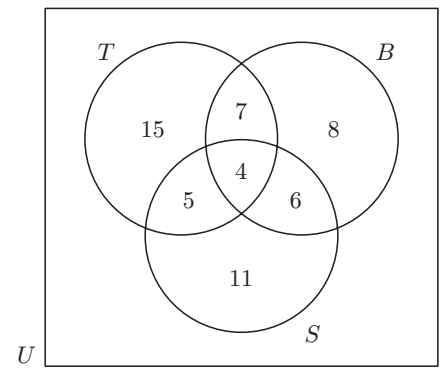
c Find $\dfrac{dS}{dr}$.

d Find r such that $\dfrac{dS}{dr} = 0$.

e Hence, find the radius which minimises the surface area of the cylinder.

f Find the surface area of the cylinder in this case.

8 The racquet sports offered at a local club are tennis (T), badminton (B), and squash (S). The Venn diagram shows the number of members involved in these activities. All of the members play at least one racquet sport.



a Write down the number of members in the club.

b Write down the number of members who:

i only play badminton

ii do not play tennis

iii play both tennis and squash, but not badminton.

c Is $\{\ \} \subset B$?

d Copy the diagram above, and shade the region that represents $S \cap (T \cup B)$.

e Write down the number of members in $S \cap (T \cup B)'$.

9 A bag contains 5 red marbles and 3 green marbles.

a One marble is taken out of the bag. Find the probability that the marble is:

i not red **ii** red or green.

b One marble is taken out and its colour noted before it is returned to the bag. A second marble is then taken out of the bag. Find the probability that:

i both marbles are red **ii** at least one marble is red.

c Two marbles are taken out of the bag at the same time. Find the probability that:

i both marbles are green **ii** exactly one marble is green.

d Three marbles are taken out of the bag at the same time. Find the probability that exactly two of them are red.

10 The vertices of a triangle are P(3, 5), Q(10, 6), and R(a, 4). R is in the first quadrant. The distance PR equals the distance PQ.

a Calculate the distance PQ. **b** Find the value of a.

c Find the gradient of the line PQ. **d** Find the equation of the line PQ.

e The line PQ meets the x-axis at S. Write down the coordinates of S.

f Find the angle between the x-axis and the line RS.

11 Scientists are monitoring a population of wild ferrets. Their lengths are normally distributed with mean 50 cm and standard deviation 2.2 cm.

a A scientist captures one of the ferrets at random. Find the probability that the ferret is no more than 45 cm long.

b Find the proportion of ferrets measuring between 52 cm and 56 cm long.

c In a colony of 150 ferrets, how many would you expect to measure at least 48 cm in length?

d The longest 10% of ferrets are at least k cm long. Find k.

12 The lengths and weights of 10 melons are shown in the table below.

Length x (cm)	32	40	43	36	42	35	38	46	36	44
Weight y (kg)	1.9	2.8	2.8	2.4	2.5	2.3	2.6	2.8	2.0	2.5

a Plot this information on a scatter diagram. Use a scale of 1 cm to represent 5 cm on the x-axis, and 1 cm to represent 0.25 kg on the y-axis.

b Use your graphics calculator to calculate the linear correlation coefficient r. What does this value tell you about the relationship between the two variables?

c Use your graphics calculator to determine the equation of the least squares regression line. Draw this line on your graph.

d Use the line on your graph to estimate:

i the weight of a melon of length 35 cm **ii** the length of a melon of weight 2.5 kg.

13 Consider the function $f(x) = x^3 - 4x^2 + 3x - 4$.

a Evaluate $f(0)$, $f(1)$, and $f(2)$. **b** Find $f'(x)$.

c Write down the coordinates of the points where $f'(x) = 0$.

d Describe the nature of each point found in **c**.

e Sketch the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 4$.

14 A group of friends meet each week for a games night. The number of people attending ranges from 4 to 8, with the probabilities shown alongside.

Number of people	4	5	6	7	8
Probability	0.1	0.19	0.35	a	0.08

a Find a.

b Find the probability that on a randomly chosen week, less than 6 people attend.

c Over a period of 25 weeks, how many times would you expect at least 7 people to attend?

d Find the average number of people attending the games night each week.

15 **a** The 5th term of an arithmetic sequence is 50, and the sum of the first 15 terms is 1200.

i Determine two linear equations involving the first term u_1 and the common difference d.

ii Solve these equations simultaneously to find u_1 and d.

iii Write down the first 5 terms of the arithmetic sequence.

b A geometric sequence begins at 100, and each subsequent term is half of the previous one.

i Write down the first 5 terms of the geometric sequence.

ii Find the sum of the first 10 terms of the sequence correct to 3 significant figures.

iii Find the sum of the first 15 terms of the sequence correct to 3 significant figures.

iv Explain why your answers to parts **ii** and **iii** are so similar.

c Hence, or otherwise, determine the sum of the first 15 terms of the sequence $110 + 70 + 55 + 52.5 + ...$ Round your answer to the **nearest hundred**.

16 Consider the universal set $U = \{x \mid 1 \leqslant x \leqslant 15,\ x \in \mathbb{Z}\}$.
P, Q, and R are subsets of U, where $P = \{\text{factors of } 12\}$, $Q = \{\text{multiples of } 3\}$, and $R = \{\text{prime numbers}\}$.

a Construct a Venn diagram showing the relationship between P, Q, and R. Show each element of U in an appropriate position.

b Consider the following propositions: p: x is a factor of 12.
q: x is a multiple of 3.
r: x is a prime number.

For what values of $x \in U$ are the following statements true:

i $p \wedge q$ **ii** $q \veebar r$ **iii** $q \wedge \neg r$ **iv** $\neg p \wedge \neg q \wedge \neg r$?

c Consider the statement $(p \wedge q) \wedge \neg r$.

i Write the statement in words.

ii Using an appropriate truth table, show that there is only one set of truth values for p, q, and r which make this statement true. State the truth values of p, q, and r in this case.

iii For what values of x is $(p \wedge q) \wedge \neg r$ true?

17 A function f is defined by $f(x) = x^2 + x - 2$.

a Find the x-intercepts of $y = f(x)$.

b Determine the coordinates of the vertex of $y = f(x)$.

c Draw the graph $y = f(x)$ for $-5 \leqslant x \leqslant 5$.

d On the same axes, draw the graph of $y = 6 - x$.

e Use your graph to solve $x^2 + x - 2 = 6 - x$.

18 A university lecturer is investigating the association between attendance at lectures and performance in first year mathematics examinations. The lecturer collects the following information:

		Percentage attendance at lectures		
		0 - 39	40 - 79	80 - 100
Exam result	*Pass*	12	50	78
	Fail	18	20	22

a What proportion of students:

i attended less than 80% of lectures and failed the examination

ii missed more than 60% of the lectures but still passed?

b The university lecturer performs a χ^2 test for independence using this data. Write down the:

i null and alternative hypotheses **ii** number of degrees of freedom

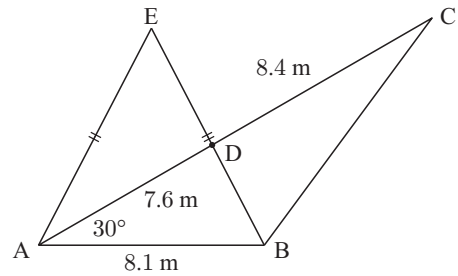
iii critical value if the test is run at a 10% significance level.

c Construct the expected frequency table.

d Find χ^2_{calc}.

e What conclusion should the lecturer draw from these results? Explain your response.

19 Consider the figure shown.



a Find the length of:

i DB **ii** BC

b Find the measure of:

i $\widehat{ABE}$ **ii** $\widehat{DBC}$

c Find the area of triangle BCD.

d Find the length of AE.

20 Consider the function $y = \dfrac{3}{2-x} + 1$.

a Determine the axes intercepts of the graph of the function.

b Sketch the graph of $y = \dfrac{3}{2-x} + 1$.

c Write down the equation of the:

i horizontal asymptote **ii** vertical asymptote.

d Points $A(3, -2)$ and $B(p, q)$ lie on $y = \dfrac{3}{2-x} + 1$ for some $p > 3$.

Suppose the gradient of line segment AB is m_{AB}.

i Using your graph, explain why $m_{AB} > 0$.

ii Show that $m_{AB} = \dfrac{-3}{2-p}$ for $p > 3$.

iii Calculate the missing values a and b in the table below.

p	5	4	3.5	3.1	3.01	3.001
m_{AB}	1.0000	a	2.0000	2.7273	2.9703	b

iv Hence, or otherwise, comment on the likely value of $\dfrac{dy}{dx}$ at $x = 3$.

21 The table opposite shows exchange rates between Japanese yen (JPY), Swiss francs (CHF), and British pounds (GBP).

The shaded box shows 1 GBP = 1.6000 CHF.

	GBP	CHF	JPY
1 GBP =	1	1.6000	c
1 CHF =	b	1	80.000
1 JPY =	d	0.0125	a

a Using this table, convert:

i 10 000 GBP to CHF **ii** 2500 CHF to JPY.

b Determine the value of:

i a **ii** b

c Write down the value of:

i 1 CHF in JPY **ii** 1 CHF in GBP.

d Calculate the value of c.

e Convert 9000 British pounds into Japanese yen assuming there is a charge of 1.5% commission on the transaction.

f Find d correct to 4 significant figures. Give your answer in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$.

22 **a** Consider $P = \{x \mid 3 \leqslant x < 10,\ x \in \mathbb{Z}\}$, $Q = \{2, 9, 15\}$, and $R = \{\text{multiples of 3 less than 12}\}$.

i List the elements of P.

ii Write down $n(P)$.

iii State whether P is finite or infinite.

iv Explain why:

(1) $Q \not\subset P$ **(2)** $R \subset P$

v List the elements of:

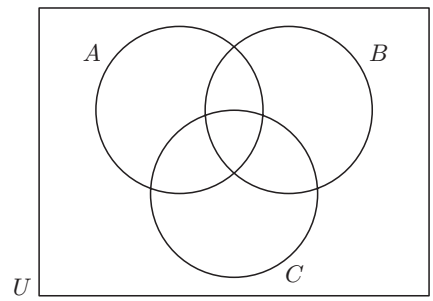
(1) $P \cap Q$ **(2)** $R \cap Q$ **(3)** $R \cup Q$

b On Venn diagrams like the one shown, shade the regions which are described by:

i $(A \cup B)' \cap C$

ii $C' \cap B$

iii $B' \cap (A \cap C)$.



23 Consider the following propositions:

p: Pepin lives in Jakarta.

q: Pepin rides a motorbike.

r: Pepin plays the guitar.

a Write each of the following statements in symbols:

i If Pepin does not ride a motorbike, then he does not live in Jakarta.

ii If Pepin rides a motorbike, then he either lives in Jakarta or plays the guitar.

b Consider the compound proposition $\neg r \Rightarrow \neg(q \wedge p)$.

i Write the compound proposition in words.

ii Complete the following truth table for the compound proposition.

p	q	r	$\neg r$	$(q \wedge p)$	$\neg(q \wedge p)$	$\neg r \Rightarrow \neg(q \wedge p)$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

iii State whether the compound statement is a tautology.

iv Describe in words, for the given propositions p, q, and r, the situation where the compound proposition is false.

24 The daily profit made by a local baker selling x homemade pies is given by

$$P = -0.05x^2 + 9x - 60 \text{ dollars.}$$

a Copy and complete the table alongside.

x	0	20	40	60	80	100
P		100		300		340

b Plot the points in **a** with x on the horizontal axis and P on the vertical axis. Use these points to sketch the graph of P against x.

c Find:

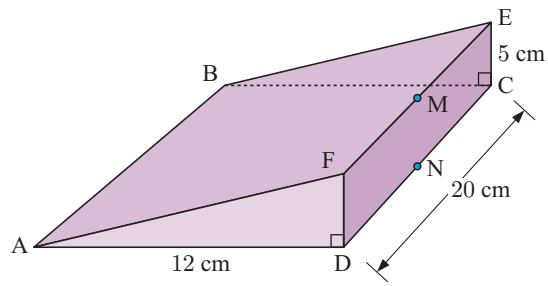
i the number of pies that need to be sold to maximise the profit

ii the maximum possible daily profit

iii the number of pies that need to be sold to make a profit of \$200

iv the amount of money the baker loses if no pies are sold.

25 In the diagram opposite, ABEF, ABCD, and CDFE are all rectangles.
AD = 12 cm, DC = 20 cm, and DF = 5 cm.
M is the midpoint of EF, and N is the midpoint of CD.



a Calculate the length of AN.

b Calculate the length of AM.

c Calculate the size of the angle AM makes with the base ABCD.

d Calculate the area of triangle AMN.

e Find the surface area of the solid.

f Find the volume of the solid.

26 The maximum daily temperature and the number of cups of coffee sold at a kiosk that day are given in the following table:

Temperature T (°C)	21	20.7	20	19	18	17.3	17	17.3	18	19
Coffees sold, n	120	110	105	125	120	150	140	130	120	120

a Find the mean temperature.

b Find the standard deviation of the temperatures.

c Find the correlation coefficient r and interpret this value.

d Find the equation of the least squares regression line.

e Hence estimate how many cups of coffee will be sold when the temperature is 19.6°C.

f On a day when the temperature is forecast to be 30°C, the owner estimates that only 40 cups of coffee will be sold. Discuss this estimate by comparison with the regression line.

27 **a** The curve $y = x^3 + ax + b$ has a local minimum at the point $(2, -10)$.
Find the values of a and b.

b The function f is defined by $f(x) = px^2 + qx + c$ where $p, q, c \in \mathbb{R}$.
$f(x)$ has a minimum value of -6.8 at the point A.

i Given that $f'(x) = 10x - 4$, find the values of p and q.

ii Find the x-coordinate of A.

iii Find the value of c.

28 Consider the propositions p: Peter is going to the zoo.
q: Quentin is going to the zoo.
r: Ryan is going to the zoo.

a Write the statement "If Peter is going to the zoo, then Ryan is going to the zoo" in symbolic form.

b Write, in words, the converse of the statement in **a**.

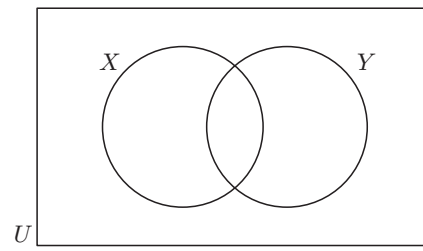
c Consider the following argument:

Peter is going to the zoo if and only if Ryan is going to the zoo.
Either Peter or Quentin is going to the zoo, but not both.
Therefore, Ryan is not going to the zoo.

i Write the argument in logical form.

ii Use a truth table to determine whether the argument is valid.

29 Let $U = \{\text{positive integers} \leqslant 20\}$,
$X = \{\text{factors of 24 which are} \leqslant 20\}$,
and $Y = \{\text{multiples of 4 which are} \leqslant 20\}$.



a List the elements of: **i** X **ii** Y

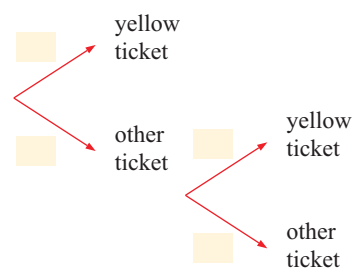
b Complete the Venn diagram alongside by placing each element of U in an appropriate position.

c Hence, write down the elements of:

i $X \cap Y$ **ii** $(X \cup Y)'$

30 A game is played where tickets are randomly selected from a bag. To begin with the bag contains five tickets, one of which is yellow. Tickets are selected one at a time, without replacement, until the yellow ticket is selected. At this point, the game ends.

a A tree diagram is used to illustrate the first two possible selections in the game. Copy and complete the tree diagram by filling in the probability values.



b Use the tree diagram to determine the probability that the game ends in two selections or less.

c If the game was played 250 times, how many times would you expect the game to end in two selections or less?

d Determine the probability that the game ends within:

i four selections **ii** five selections.

e The game is altered so that tickets are now **replaced** after each selection. Determine the probability that the game ends on the:

i 1st selection **ii** 2nd selection **iii** 3rd selection.

f Using the results from **e**, determine the probability that the game ends within three selections.

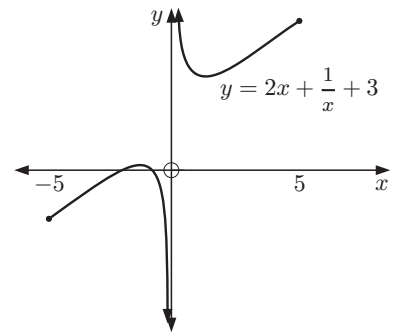
g The values found in **e** form a geometric sequence. For this geometric sequence:

i write down the common ratio

ii show that the sum of the first n terms is $1 - \left(\frac{4}{5}\right)^n$.

h Hence, determine the probability that the game ends within fifteen selections. Write your answer as a percentage, correct to 3 significant figures.

31 The graph of $y = 2x + \frac{1}{x} + 3$ is shown for $-5 \leqslant x \leqslant 5$.



a Write down the equation of the vertical asymptote.

b Find $\frac{dy}{dx}$.

c Hence find the gradient of the tangent to the curve at the point where $x = 1$.

d Write down the coordinates of the points where the tangent to the curve is horizontal.

e State the range of the function.

32 A group of 250 IB students were given the option to participate in a survey. The table alongside shows participation levels according to gender.

	Male	*Female*
Participated	69	81
Did not participate	61	39

A χ^2 test at a 5% significance level is performed to investigate the following hypotheses:

H_0: Participation in the survey was independent of gender.

H_1: Participation in the survey was not independent of gender.

a Show that the expected number of male students to participate in the survey is 78.

b Hence, complete the table of expected values below:

	Male	*Female*	
Participated	78		150
Did not participate			100
	130	120	

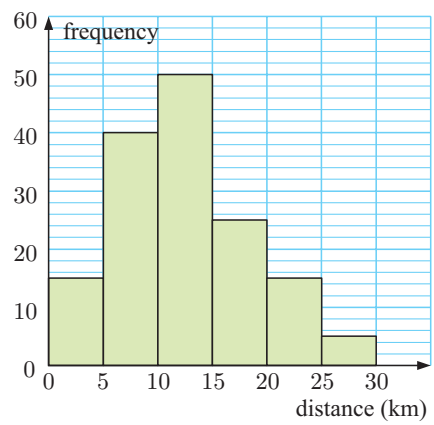
c Write down the number of degrees of freedom.

d Determine the value of χ^2_{calc}.

e Given that the critical value is ≈ 3.84, discuss whether H_0 or H_1 is rejected.

f What conclusion can be drawn from this χ^2 test?

The 150 completed surveys provided information on how far students travelled to get to school each day. This information is shown in the histogram alongside.



g Is *distance* a discrete or continuous variable?

h Using your calculator, estimate the:

i mean

ii standard deviation of the data.

i A student is selected at random. Given that this student completed a survey, determine the probability the student travels no more than 10 kilometres to school.

33 The points A$(0, 3)$ and B$(4, 0)$ form two vertices of triangle ABC.

a On a grid showing the region $-2 \leqslant x \leqslant 12$, $-2 \leqslant y \leqslant 12$, plot points A and B.

b Calculate the distance between A and B.

c Find the gradient of the line segment AB.

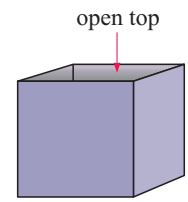
d Given that $\widehat{ABC} = 90°$, calculate:
- **i** the gradient of line BC
- **ii** the equation of line BC.

e Point C has coordinates $(11.2, b)$.
- **i** Find b.
- **ii** Add point C to your diagram from **a**.

f Calculate the area of triangle ABC.

g Determine the measure of $\widehat{ACB}$.

34 4800 cm^2 of material is used to construct a hollow, square based cuboid with an open top. The cuboid has base width x cm and height h cm.



a Show that $h = \dfrac{4800 - x^2}{4x}$.

b Hence, show that the volume of the cuboid is given by $V = 1200x - \frac{1}{4}x^3$ cm^3.

c The table alongside shows values of x and V. Find the value of:

x	10	30	60
V	a	b	18 000

- **i** a
- **ii** b

d Find $\dfrac{dV}{dx}$.

e Determine the dimensions x and y which maximise the volume of the cuboid.

f Write down the maximum volume of the cuboid.

35 Marinda invests €12 000 in a bank account offering 4.2% p.a. interest, compounded quarterly.

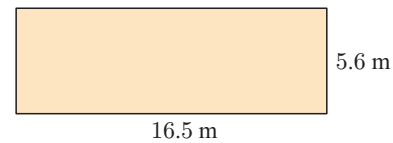
a Calculate the interest earned after:
- **i** 1 year
- **ii** 3 years.

b Write a formula for the value of Marinda's investment after x years.

c How long will it take for Marinda's investment to reach €30 000? Give your answer to the nearest year.

d What rate of interest compounded annually would be needed for Marinda's investment to reach €30 000 in the same time as found in **c**?

36 A rectangular field needs to be top-dressed. We need to know the area of the field so we can order the correct amount of soil. The tape measure used to measure the field has marks on it every 10 cm.



a Use the measurements given to estimate the area of the field.

b The measurements in the diagram involved some inaccuracy. Write down the maximum possible length and width of the field correct to 2 decimal places.

c Calculate the maximum possible area of the field, correct to 3 significant figures.

d The top-dressing of soil will be 2 cm thick across the field. Calculate the amount of soil that needs to be ordered to make sure there is sufficient. Give your answer in cubic metres.

e The soil costs \$120 per cubic metre, plus a delivery fee of \$27.50 per kilometre travelled. The labour for spreading the soil costs \$60 per cubic metre. The field is 5 km from the supplier's depot. Find the maximum total cost for top-dressing the field.

f Find the difference in cost if the amount of soil ordered was based on your answer to **a**.

37 There are 60 senior students in a school. Each of these students studies History, Geography, or both of these subjects. 38 students study History, 31 study Geography, and n study both.

a Find the value of n.

b Draw a fully labelled Venn diagram to illustrate this information.

c Find the probability that a student selected at random studies only one of these subjects.

d 28 of the 60 students are female. 17 females study History, and 15 females study Geography. Find the probability that a student selected at random:

i is female and studies exactly one of these subjects

ii is male and studies both of these subjects.

e One of the History students is randomly selected. Calculate the probability that this student:

i is female **ii** also studies Geography.

38 The intensity of light L diminishes below the surface of the sea according to the formula $L = L_0 \times (0.95)^d$ units, where d is the depth in metres measured from the surface of the sea.

a If the intensity of light at the surface is 10 units, calculate the value of L_0.

b Find the intensity of light 25 m below the surface.

c A light intensity of 4 units is considered adequate for divers to be able to see clearly. Calculate the depth corresponding to this intensity of light.

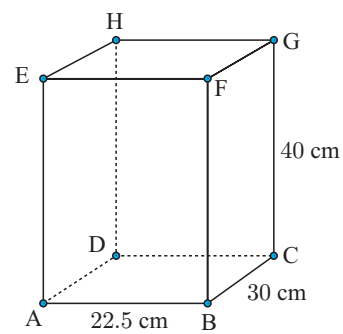
d The table gives some values for the intensity of light at different depths.

Depth d (metres)	10	20	30	50
Intensity L (units)	5.99	3.58	2.15	0.769

Using these values and your answers to **a**, **b**, and **c**, graph the intensity of light against depth for $0 \leqslant d \leqslant 50$.

e Calculate the range of depths for which the light intensity is between 1 and 3 units.

39 The diagram shows a cuboid which measures 22.5 cm by 30 cm by 40 cm.



a Find the length of AC.

b Find the area of the plane ACGE.

c Find the volume of the triangular prism ACGEFB.

d Find the length of CE.

e Find $\widehat{ACE}$.

f Let M be the midpoint of CE. Find the area of triangle AMC.

40 The table shows the time taken for a group of runners to finish a cross country race.

Time (min)	*Number of runners*
$20 \leqslant t < 25$	5
$25 \leqslant t < 30$	8
$30 \leqslant t < 35$	15
$35 \leqslant t < 40$	15
$40 \leqslant t < 45$	5
$45 \leqslant t < 50$	2

a Construct a cumulative frequency table for this data.

b Draw the corresponding cumulative frequency curve.

c Use the graph to estimate the:

i median finishing time **ii** interquartile range

iii number of runners who finished in under 38 minutes

iv time it took for the fastest 40% of the runners to complete the course.

d The first 3 runners will be awarded medals. Estimate the time that was needed to be awarded a medal.

41 The cost of producing a pair of shoes is \$15, and they are then sold for \$$x$.

a Write an expression for the profit made on each pair sold.

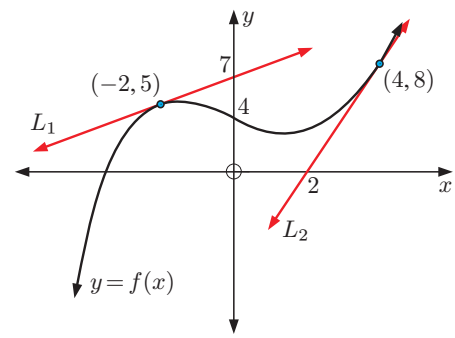
b In a given period, a total of $(1500 - 9x)$ pairs of shoes are sold. Show that the total profit made on the shoes sold is $P = 1635x - 9x^2 - 22\,500$ dollars.

c Calculate the profit made when the shoes are sold for \$100 per pair.

d Find $\dfrac{dP}{dx}$.

e Calculate to the nearest dollar, the selling price that will maximise the profit, and find the maximum profit in this case.

42 The lines L_1 and L_2 are tangents to the graph of $y = f(x)$.



a Find:

i $f(0)$ **ii** $f'(-2)$ **iii** $f'(4)$

b Find the equation of:

i L_1 **ii** L_2

c Find where the lines L_1 and L_2 intersect.

d Find the equation of the normal to $y = f(x)$ at the point where $x = -2$.

43 **a** Consider the sum of the first n positive integers $1 + 2 + 3 + ... + n$.

i Find the sum of the first 10 positive integers.

ii Show that the sum of the first n positive integers is given by the formula $\dfrac{n(n+1)}{2}$.

iii Using this formula, calculate the sum of the first 1000 integers.

iv Determine n such that the sum of the first n positive integers is 11 325.

b Consider the sequence 7, 14, 21, 28, ...

i Write an expression for the nth term.

ii Find the largest term which is no greater than 1000.

iii Determine a formula for the sum of the first n terms.

c Calculate the sum of all positive integers which are no greater than 1000 and which are *not* divisible by 7.

44 A survey is conducted regarding the types of cardio equipment used by the 300 members of a fitness club.

134 members use rowers (R), 92 use treadmills (T), and 144 use spin-bikes (S).
19 members use rowers, treadmills, and spin-bikes.
23 members use rowers and treadmills only.
28 members use treadmills and spin-bikes only.
41 members use spin-bikes and rowers only.

a Construct a Venn diagram to represent this information.

b Determine the number of fitness club members who do not use any type of cardio equipment.

c Find the proportion of the members that use:

i all three types of cardio equipment

ii exactly two of the three types of cardio equipment.

d Of the members who use at least one type of cardio equipment, what proportion use spin-bikes?

45 A study measured the bone density (in g/cm^3) of a group of 25 year old adults, and a group of 60 year old adults. The results are shown below.

25 year old group:	1.35	1.15	1.3	1.3	1.15	1.25	1.3	1.15	1.3	1.45
	1.25	1.4	1.45	1.35	1.2	1.2	1.3	1.45	1.2	1.3
60 year old group:	0.85	0.9	0.85	1.0	0.95	1.05	0.9	0.9	0.95	0.95
	1.2	1.2	0.9	1.0	1.1	1.05	1.1	0.95	1.0	0.95

a Calculate the five-number summary for each of the data sets.

b Display the data in a parallel boxplot.

c Compare the measures of the centre of each distribution.

d Compare the measures of spread of each distribution.

e What conclusions can be drawn from the data?

46 Consider the quadratic function $f(x) = x^2 - 8x + 12$.

a Find the axes intercepts.

b Find the equation of the axis of symmetry.

c Find the coordinates of the vertex.

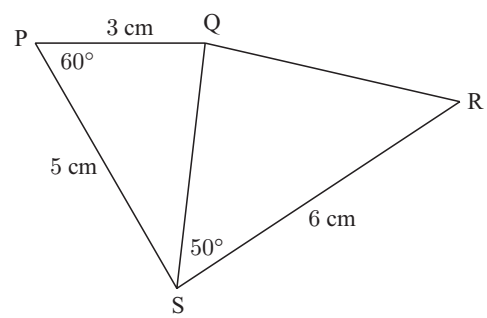
d Sketch the graph of the function, showing the features found in **a**, **b**, and **c**.

e State the intervals where the function is increasing or decreasing.

f Find the equation of the tangent to $y = f(x)$ at $x = 3$.

g Find the equation of the normal to $y = f(x)$ at $x = 6$.

47 QS is a diagonal of quadrilateral PQRS where PQ $= 3$ cm, RS $= 6$ cm, PS $= 5$ cm, $\widehat{\text{QSR}} = 50°$, and $\widehat{\text{QPS}} = 60°$.



a Show that QS $= \sqrt{19}$ cm.

b Find the length of QR correct to 4 significant figures.

c Hence find, correct to 3 significant figures, the:

i perimeter of PQRS **ii** area of PQRS.

48 Aaron purchased a new car for \$40 000. The car depreciates at 25% p.a.

a Let V_n represent the value of the car after n years.

i Find V_1.

ii Show that $V_3 = \$16\,875$.

iii Deduce a formula for V_n using the sequence of values $V_1, V_2, V_3, ...$

iv Plot V_n for $0 \leqslant n \leqslant 15$.

b Aaron insured his car immediately after it was purchased. Initially, the annual cost of the insurance was \$1200. One year later, the annual insurance cost increased to \$1260.

i Find the percentage increase in the annual insurance cost.

ii Let P_n represent the annual insurance cost after n years, so $P_1 = \$1260$. Assuming the insurance cost increases by the same percentage each year, calculate P_2 and P_3.

iii Write down a formula for P_n in terms of n.

c After how many whole years will the annual cost of car insurance be more than the depreciated value of the car?

49 A group of students compare their average test results for Physics (x) and Chemistry (y).

Physics Test ($x\%$)	43	45	50	51	55	56	59	63	65	72	77	93
Chemistry Test ($y\%$)	52	53	57	57	58	62	63	70	72	87	88	100

a Draw a scatter diagram for this data.

b Find the mean point $(\overline{x}, \overline{y})$.

c Draw a line of best fit by eye on the scatter diagram drawn in **a**.

d Using your graphics calculator, determine:

i the product-moment correlation coefficient r

ii the equation of the least squares regression line for y on x.

e Hence, predict to the nearest 1% the average test result in Chemistry for a student who achieved an average test result of 85% in Physics.

50 The table alongside shows the number of balloons in a giant party pack.

	Red	*Yellow*	*Blue*
Large	12	5	9
Medium	15	8	10
Small	24	11	6

a State the:

i total number of balloons in the pack.

ii number of medium balloons in the pack.

b One balloon is chosen at random from the pack. Find the probability that:

i the balloon is not yellow

ii the balloon is either medium or small.

c Two balloons are selected at random from the pack. Find the probability that:

i both balloons are red

ii neither of the balloons are large

iii exactly one of the balloons is blue

iv at least one of the balloons is blue.

d Three balloons are selected at random from the pack. Find the probability that:

i all three balloons are small and yellow

ii exactly two balloons are medium and red.

51 Consider the function $f(x) = 3x^2 - 5x - 2^x - 10$.

a Using technology:

i find the axes intercepts

ii find the position and nature of any turning points

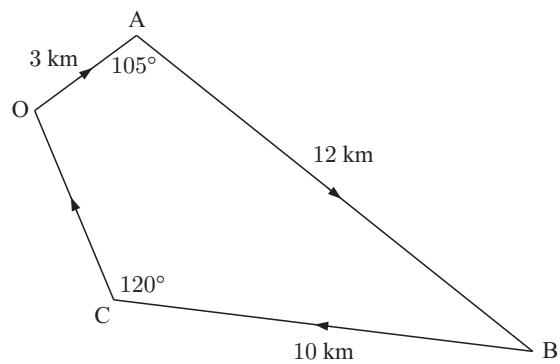
iii discuss the behaviour of the function as $x \to \infty$ and $x \to -\infty$

iv sketch the function.

b Add the graph of $g(x) = 3^{-x}$ to your graph in **a iv**.

c Solve for x: $3x^2 - 5x - 2^x - 10 = 3^{-x}$.

52 A yachting course is illustrated in the diagram alongside. The yachts start and finish at O, and travel in the direction indicated.



a Find the distance from O to B in a straight line.

b Find $\widehat{BOC}$.

c Find the length of OC.

d Calculate the area enclosed by the course OABC.

e The course designer stated the length of the course is 30 km. Calculate the percentage error in this approximation.

53 Felicity is offered a new job and is given two salary options to choose from:

Option A: \$40 000 in the first year, and 5% extra each subsequent year.

Option B: \$60 000 in the first year, and \$1000 more each subsequent year.

a If Felicity believed that she would work for 3 years in this new job, explain why *Option B* would be best for her.

b Write down a formula for the amount of money earned in the nth year if she selects:

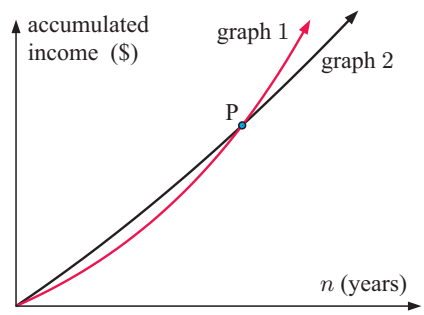
i *Option A* **ii** *Option B*

c Determine the minimum length of time Felicity would need to work before the amount of money earned per year from *Option A* exceeds that of *Option B*.

d Felicity decides that the best way to compare the two options is to consider the total income accumulated after the first n years in each case. If T_A and T_B represent the total income earned over n years for *Options A* and *B* respectively, show that:

i $T_A = 800\,000(1.05^n - 1)$ dollars **ii** $T_B = 500n^2 + 59\,500n$ dollars

e The graph alongside shows T_A and T_B graphed against n.



i Write down which graph represents T_A and which graph represents T_B.

ii Find the coordinates of the point P, where T_A meets T_B.

f Hence, write down a time interval, in whole years, for which *Option B* provides the greater accumulated income.

54 A club of beagle owners records the number of pups born per litter over a one year period. The results are shown in the frequency table.

Number of pups per litter (x)	*Frequency* (f)	$x \times f$
2	1	2
3	3	9
4	7	28
5	15	s
6	21	126
7	17	t
8	9	72
9	4	36
10	2	20

a Is this data discrete or continuous?

b Calculate the total number of litters for this one year period.

c Determine the value of: **i** s **ii** t

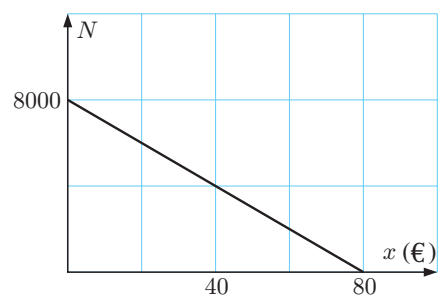
d How many beagle pups were born over this one year period?

e Calculate the average number of pups per litter.

f Using your graphics calculator, draw a boxplot to represent this data.

g Hence, determine the: **i** range **ii** interquartile range **iii** median.

55 A furniture manufacturer constructs and sells wooden stools. The number N of stools sold depends on the selling price per stool, €x. The relationship between N and x is illustrated alongside.



a Calculate the gradient of this line, and interpret its meaning.

b Write down the equation of the line, giving your answer in the form $N = ax + b$.

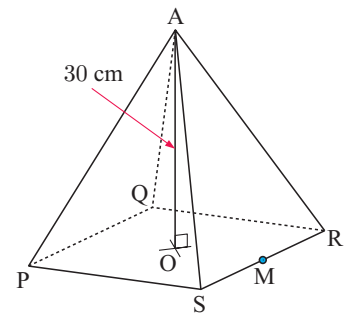
c If each wooden stool costs €50 to build, write an expression for the manufacturer's profit per stool.

d Using the results of **b** and **c**, show that the total profit for the sale of N chairs is given by $P = -100x^2 + 13\,000x - 400\,000$ euros.

e Find $\frac{dP}{dx}$.

f Calculate the selling price required to produce the maximum profit, and the maximum profit in this case.

56 PQRS is a square and forms the base of a pyramid. O is the centre of the base, and the vertex A is 30 cm directly above O. M is the midpoint of RS. The volume of the pyramid is 4000 cm^3.



a Calculate the width of the square base.

b Calculate the distance OR.

c The triangular face ARS forms an angle θ with the square base.

i Using a well labelled diagram, draw a triangle showing the correct position of angle θ.

ii Hence, calculate the measure of θ.

d Calculate the total surface area of the pyramid.

e A wire frame is placed along the edges of the pyramid. If the wire costs \$0.12 per cm, find the total cost of the frame.

57 Michael is saving to buy a house and needs \$200 000.

a Three years ago, he invested a sum of money in an account paying 6.5% p.a. interest compounded half-yearly. This investment has just matured at \$50 000. How much did Michael invest three years ago?

b Michael decides to reinvest his \$50 000 lump sum into an account for a period of n years at 6.0% p.a. interest compounded annually.

Copy and complete the table below showing the value V_n of Michael's investment after n years.

n (years)	0	1	2	3	4
V_n (\$)	50 000	53 000	56 180		

c Write a formula for V_n in terms of n.

d Michael also decides to start an additional saving plan, whereby he deposits \$3000 into a safe at the end of each year. Write down a formula for S_n, the amount of money in Michael's safe after n years.

e The total amount of money Michael has for his house after n years is given by $T_n = V_n + S_n$. Calculate the missing values in the table below.

n (years)	0	1	2	3	4
T_n (\$)	50 000	56 000	62 180		

f After how many whole years will Michael have the \$200 000 needed to buy his house?

58 Mr Hilditch makes a cup of coffee in the staff room of his school, but accidentally leaves it behind. The approximate temperature of the coffee t minutes after it is made is $A(t) = 95 - 3t$ °C for $0 \leqslant t \leqslant 30$ minutes.

a Based on this model, estimate the temperature of Mr Hilditch's coffee:

i initially **ii** after 30 minutes.

b Sketch $A(t)$ for $0 \leqslant t \leqslant 30$. Clearly show the information obtained in **a**.

c Discuss whether the use of $A(t)$ to represent the temperature of the coffee could be valid beyond $t = 30$ minutes. Give evidence to support your response.

d The actual temperature of the coffee t minutes after being made is $C(t) = 80 \times (0.8)^t + 15$ °C, $t \geqslant 0$.

Determine the actual temperature of the coffee:

i initially **ii** after 30 minutes.

e Add the graph of $C(t)$ to your graph from **b**.

f Mr Hilditch will drink his coffee provided its temperature does not drop below 20°C. How long will it take for the coffee to become too cold to drink?

g If left for a long time, the temperature of the coffee will eventually reach the temperature of the surrounding environment. Based on the model $C(t)$, find the temperature of the staff room where Mr Hilditch left his cup of coffee.

h Consider the function $D(t) = A(t) - C(t)$.

i Solve $D(t) = 0$, and interpret this result.

ii Determine the maximum value of $D(t)$ for $0 \leqslant t \leqslant 30$.

iii Interpret the value found in **ii**.

59 Jose conducts a survey of 200 people to see which type of movie they prefer to watch. The results are shown in the table. Jose will conduct a χ^2 test at the 5% level of significance to determine whether the preferred movie type is independent of gender.

	Adventure	*Comedy*	*Action*	*Drama*
Men	25	25	40	15
Women	18	34	12	31

a State the null and alternative hypotheses.

b Calculate the expected frequency for the number of females who prefer comedies. Give your answer to the nearest whole number.

c Using your graphics calculator or otherwise, find the χ^2 statistic for Jose's data.

d Determine the number of degrees of freedom available for this calculation.

e Write down the critical value for the test.

f Give a conclusion for Jose's test, including reasons for your decision.

g Jose realised after he had completed the test that he entered some information incorrectly. The adventure and drama numbers for males had been reversed. Perform the test again with the correct data, and state whether the conclusion drawn in **f** is still valid.