Introduction to sequences

Things you need to learn to do

- Give a recursive formula or general formula of a given sequence.
- Find some terms of the sequence given a recursive or general formula.

Formulae for sequences

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- using a general (explicit) formula.

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We could write this mathematically as:

$$F_1 = 1, \qquad F_2 = 1, \qquad F_n = F_{n-1} + F_{n-2} \quad \text{for} \quad n > 2$$

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$$a_1 = 7, \qquad a_n = a_{n-1} + 2 \quad \text{for} \quad n > 1$$

Recursive formula

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$$2, 6, 18, 54, 162, \dots$$

Can you think of recursive formula for this one? The first term is 2 and then to get to the next term we multiply the previous one by 3, so we can write this mathematically as:

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Can you think of recursive formula for this one? The first term is 2 and then to get to the next term we multiply the previous one by 3, so we can write this mathematically as:

$$b_1 = 2, \qquad b_n = 3 \times b_{n-1} \quad \text{for} \quad n > 1$$

Recursive formula

Another one:

$$-5, 5, -5, 5, 5....$$

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$$c_1 = -5, \qquad c_n = -c_{n-1} \quad \text{for} \quad n > 1$$

General formula

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$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}...$$

This sequence can be nicely defined using the general formula as $d_n = \frac{1}{n}$.

The difference between the recursive and general formula is that:

- in the recursive formula we have a term defined in terms of previous terms,
- in the general formula a term is defined in terms of n (its number).

General formula

Consider:

$$2, 4, 8, 16, 32, \ldots$$

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What is the general formula here? We have consecutive powers of 2, so the formula is $e_n = 2^n$.

General formula

Consider:

$$3, 6, 9, 12, 15, \ldots$$

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Consider:

$$3, 6, 9, 12, 15, \ldots$$

What is the general formula here? We have consecutive multiples of 3, so the formula is $f_n = 3n$.

General formula

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$$1, 4, 9, 16, 25, \ldots$$

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$$1, 4, 9, 16, 25, \ldots$$

What is the general formula here? We have consecutive square numbers, so the formula is $g_n = n^2$.

General formula

Note that the formula may be more complicated, here:

$$9, 15, 21, 27, 33, \ldots$$

We have $h_n = 6n + 3$.

Now what we want to do is the opposite, given a formula, find some terms.

Recursive formula

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$$7, 4, 1, -2, -5, ...$$

Recursive formula

We have $k_1 = 8$ and $k_n = \frac{k_{n-1}}{2}$.

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We have $k_1 = 8$ and $k_n = \frac{k_{n-1}}{2}$. So we start with 8 and then to get to the next term we divide the previous one by 2.

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$$8, 4, 2, 1, 0.5, \ldots$$

Recursive formula

We have $l_1 = 3$ and $l_n = 2l_{n-1} - 1$.

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We have $l_1 = 3$ and $l_n = 2l_{n-1} - 1$. So we start with 3 and then to get to the next term we multiply the previous one by 2 and subtract 1.

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$$3, 5, 9, 17, 33, ..$$

Recursive formula

We have $m_1 = 2$ and $m_1 = m_{n-1}^2 - 2$.

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We have $m_1 = 2$ and $m_1 = m_{n-1}^2 - 2$. So we start with 2 and then to get to the next term we square the previous term and subtract 2.

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$$2, 2, 2, 2, 2, ..$$

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$$9, 11, 13, 15, 17$$

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General formula $w_n = 3^n$,

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General formula $v_n = 2n^2 - 3$, first five terms:

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General formula $u_n = n^2 - n$, first five terms:

$$0, 2, 6, 12, 20, ...$$

The short test at the beginning of the next class will consist of finding some terms of a sequence given its formula.

If you have any questions or doubts email me at T.J.Lechowski@gmail.com