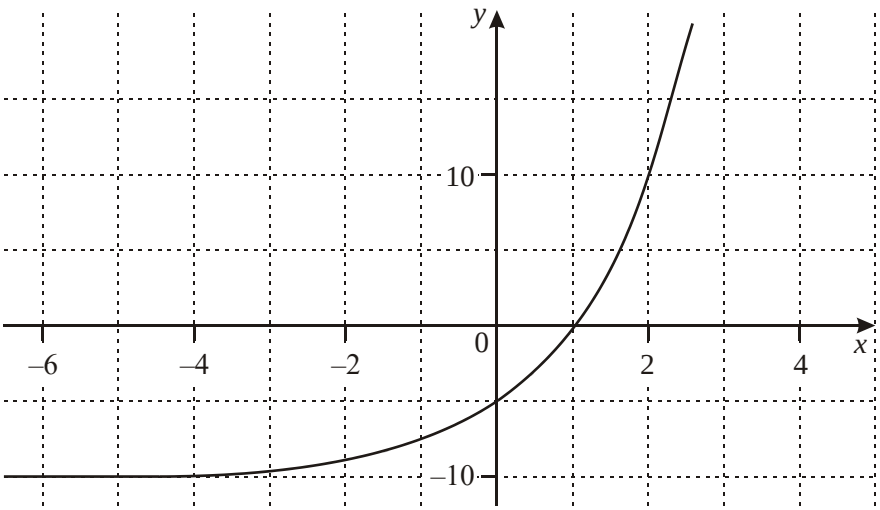
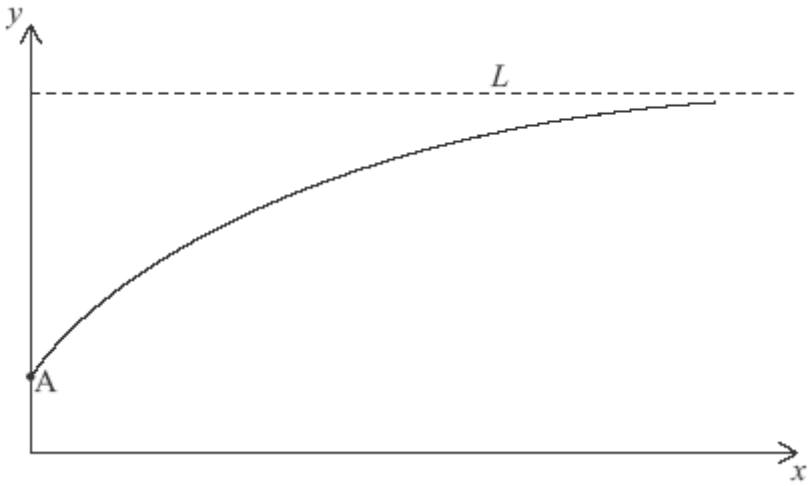
1. The graph below shows the curve $y = k(2^x) + c$, where k and c are constants.



Find the values of c and k.

(Total 4 marks)

2. Consider the function $f(x) = 1.25 - a^{-x}$, where a is a positive constant and $x \geq 0$.
The diagram shows a sketch of the graph of f. The graph intersects the y-axis at point A and the line L is its horizontal asymptote.



(a) Find the y-coordinate of A.

(2)

The point (2, 1) lies on the graph of $y = f(x)$

(b) Calculate the value of a.

(2)

(c) Write down the equation of L.

(2)
(Total 6 marks)

3. Shiyun bought a car in 1999. The value of the car V, in USD, is depreciating according to the exponential model

$$V = 25\,000 \times 1.5^{-0.2t},\ t \geq 0,$$

where t is the time, in years, that Shiyun has owned the car.

(a) Write down the value of the car when Shiyun bought it.

(1)

(b) Calculate the value of the car three years after Shiyun bought it. Give your answer correct to **two decimal places**.

(2)

(c) Calculate the time for the car to depreciate to half of its value since Shiyun bought it.

(3)

(Total 6 marks)

4. The function $f(x) = 5 - 3(2^{-x})$ is defined for $x \geq 0$.

(a) (i) On the axes below sketch the graph of $f(x)$ and show the behaviour of the curve as x increases.

(ii) Write down the coordinates of any intercepts with the axes.

(4)

(b) Draw the line $y = 5$ on your sketch.

(1)

(c) Write down the number of solutions to the equation $f(x) = 5$.

(1)

(Total 6 marks)

5. The value of a car decreases each year. This value can be calculated using the function

$$v = 32\,000r^t,\ t \geq 0,\ 0 < r < 1,$$

where v is the value of the car in USD, t is the number of years after it was first bought and r is a constant.

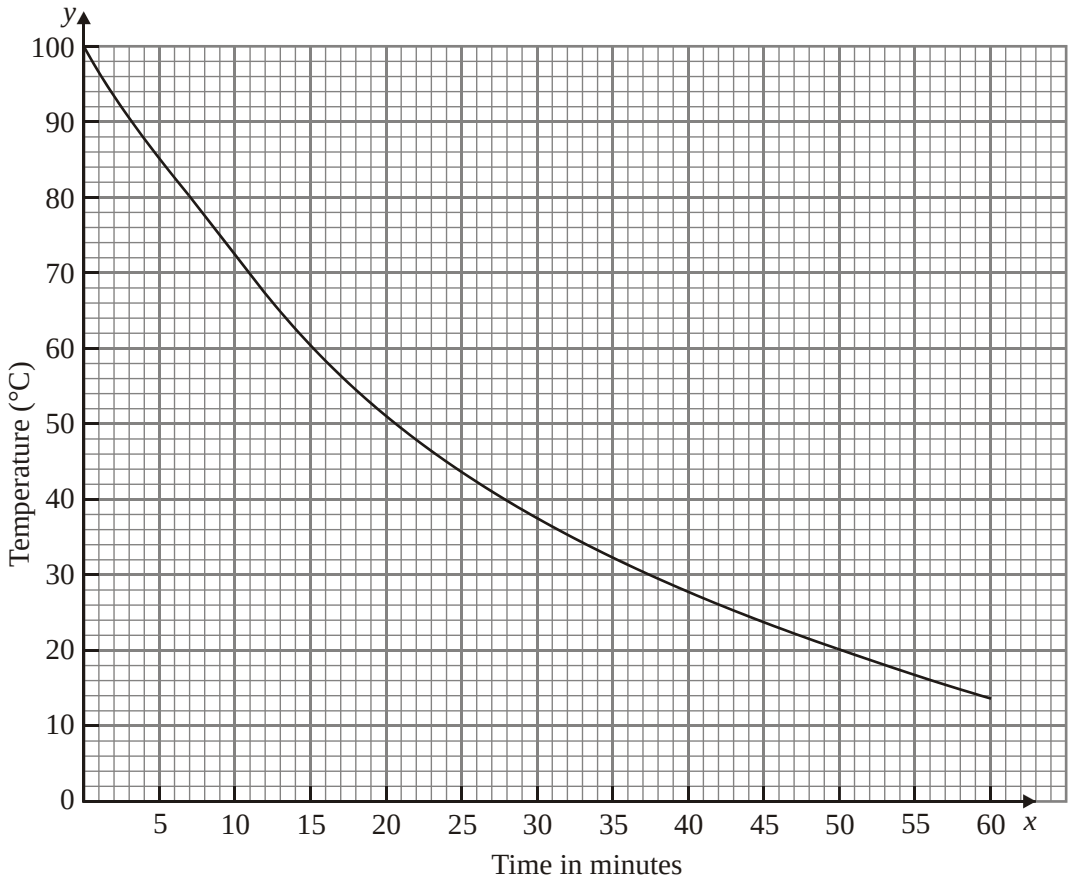
(a) (i) Write down the value of the car when it was first bought.

(ii) One year later the value of the car was 27 200 USD. Find the value of r.

(b) Find how many years it will take for the value of the car to be less than 8000 USD.

(Total 6 marks)

6. The graph below shows the temperature of a liquid as it is cooling.



(a) Write down the temperature after 5 minutes.

(b) After how many minutes is the temperature 50°C?

The equation of the graph for all positive x can be written in the form $y = 100(5^{-0.02x})$.

(c) Calculate the temperature after 80 minutes.

(d) Write down the equation of the asymptote to the curve.

(Total 8 marks)

7. A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

where N is the number of teenagers who have heard the rumour t hours after it is first started.

(a) Find the number of teenagers who started the rumour.

(2)

(b) Write down the number of teenagers who have heard the rumour five hours after it is first started.

(1)

(c) Determine the length of time it would take for 150 teenagers to have heard the rumour. **Give your answer correct to the nearest minute.**

(3)

(Total 6 marks)

8. A function is represented by the equation $f(x) = 3(2)^x + 1$.

The table of values of $f(x)$ – $3 \le x \le 2$ is given below.

x	–3	–2	–1	0	1	2
$f(x)$	1.375	1.75	a	4	7	b

(a) Calculate the values for a and b.

(2)

(b) On graph paper, draw the graph of $f(x)$, for $-3 \le x \le 2$, taking 1 cm to represent 1 unit on both axes.

(4)

The domain of the function $f(x)$ is the real numbers, $\mathbb{R}$.

(c) Write down the range of $f(x)$.

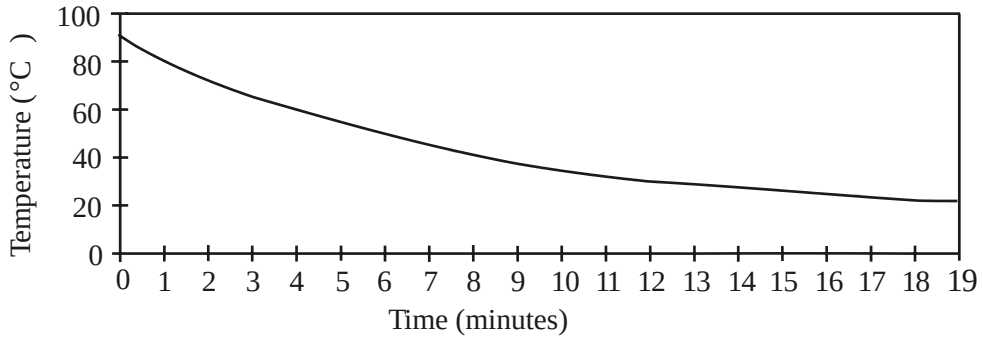
(2)

(d) Using your graph, or otherwise, find the approximate value for x when $f(x) = 10$.

(2)

(Total 10 marks)

9. The following graph shows the temperature in degrees Celsius of Robert's cup of coffee, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 16 + 74 \times 2.8^{-0.2t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the coffee out.



(a) Find the initial temperature of the coffee.

(1)

(b) Write down the equation of the horizontal asymptote.

(1)

(c) Find the room temperature.

(1)

(d) Find the temperature of the coffee after 10 minutes.

(1)

If the coffee is not hot enough it is reheated in a microwave oven. The liquid increases in temperature according to the formula

$$T = A \times 2^{1.5t}$$

where T is the final temperature of the liquid, A is the initial temperature of coffee in the microwave and t is the time in minutes after switching the microwave on.

(e) Find the temperature of Robert's coffee after being heated in the microwave for **30 seconds** after it has reached the temperature in part (d).

(3)

(f) Calculate the length of time it would take a similar cup of coffee, initially at 20°C, to be heated in the microwave to reach 100°C.

(4)

(Total 11 marks)

10. The population of fleas on a dog after t days, is modelled by

$$N = 4 \times (2)^{\frac{t}{4}}, t \geq 0,$$

Some values of N are shown in the table below.

t	0	4	8	12	16	20
N	p	8	16	32	q	128

(a) Write down the value of

(i) p;

(ii) q.

(3)

(b) Using the values in the table above, draw the graph of N for $0 \leq t \leq 20$.
Use 1 cm to represent 2 days on the horizontal axis and 1 cm to represent 10 fleas on the vertical axis.

(6)

(c) **Use your graph** to estimate the number of days for the population of fleas to reach 55.

(2)

(Total 11 marks)

11. The temperature in °C of a pot of water removed from the cooker is given by $T(m) = 20 + 70 \times 2.72^{-0.4m}$, where m is the number of minutes after the pot is removed from the cooker.

(a) Show that the temperature of the water when it is removed from the cooker is 90 °C.

(2)

The following table shows values for m and $T(m)$.

m	1	2	4	6	8	10
$T(m)$	66.9	51.4	34.1	26.3	22.8	s

(b) (i) Write down the value of s.

(ii) Draw the graph of $T(m)$ for $0 \leq m \leq 10$. Use a scale of 1 cm to represent 1 minute on the horizontal axis and a scale of 1 cm to represent 10 °C on the vertical axis.

(iii) **Use your graph** to find how long it takes for the temperature to reach 56 °C. Show your method clearly.

(iv) Write down the temperature approached by the water after a long time. Justify your answer.

(9)

Consider the function $S(m) = 20m - 40$ for $2 \leq m \leq 6$.
The function $S(m)$ represents the temperature of soup in a pot placed on the cooker two minutes after the water has been removed. The soup is then heated.

(c) Draw the graph of $S(m)$ on the same set of axes used for part (b).

(2)

(d) Comment on the meaning of the constant **20** in the formula for $S(m)$ in relation to the temperature of the soup.

(1)

(e) (i) **Use your graph** to solve the equation $S(m) = T(m)$. Show your method clearly.

(ii) Hence describe by using inequalities the set of values of m for which $S(m) > T(m)$.

(4)
(Total 18 marks)