ACTIVITY 2 MEASURING DEVICES

Examine a variety of measuring instruments at school and at home. Make a list of the names of these instruments, what they measure, what their units are, and the degree of accuracy to which they can measure.

For example:

A ruler measures length. In the Metric System it measures in centimetres and millimetres, and can measure to the nearest millimetre. Its accuracy is $\pm\frac{1}{2}$ mm.

Example 5 **Self Tutor**

Ling uses a ruler to measure the length l of her pencil case. She records the length as 18.7 cm.

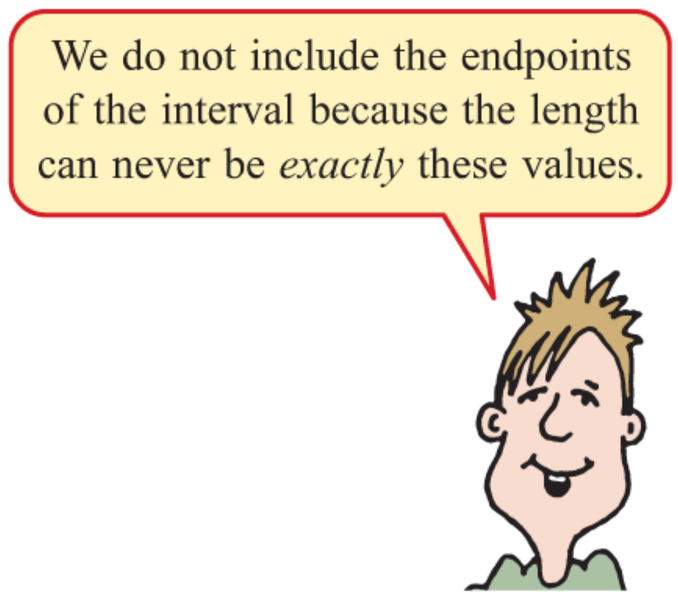
Find the range of values in which the length may lie.

18.7 cm is 187 mm, so the measuring device must be accurate to the nearest half mm.

$\therefore$ the range of values is $187 \pm \frac{1}{2}$ mm

The actual length is in the range $186\frac{1}{2}$ mm to $187\frac{1}{2}$ mm.

$\therefore$ 18.65 cm $< l <$ 18.75 cm.



EXERCISE 1C

1 State the accuracy of the following measuring devices:

a a tape measure marked in cm

b a measuring cylinder with 1 mL graduations

c a beaker with 100 mL graduations

d a set of scales with marks every 500 g

e a thermometer with marks every 0.1°C.

2 Roni checks his weight every week using scales with 1 kg graduations. This morning he recorded a weight of 68 kg. In what range of values does Roni's actual weight w lie?

3 Find the range of possible values corresponding to the following measurements:

a 27 mm **b** 38.3 cm **c** 4.8 m

d 1.5 kg **e** 25 g **f** 3.75 kg

4 Tom's digital thermometer said his temperature was 36.4°C. In what range of values did Tom's actual temperature T lie?

5 Joanne's exercise watch displays the distance she has run to 3 significant figures. State the *least* distance Joanne could have run, if the watch displays:

a 1.06 km **b** 9.72 km **c** 10.1 km

Comment on the accuracy of the watch.

6 Four students measured the width of their classroom using the same tape measure. The measurements were 6.1 m, 6.4 m, 6.0 m, 6.1 m.

a Which measurement is likely to be incorrect? Explain your answer.

b What answer would you give for the width of the classroom? Explain your answer.

c What graduations do you think were on the tape measure?

7 Hasan has many lengths of rope. He has measured each length to be 2.4 m.

a In what range of values does the actual length of a rope l lie?

b If Hasan carefully places n of his ropes end to end, in what range of values will the total length of rope L lie?

8 In the 800 m race at the sports carnival, the times recorded for Jiao and Liang were 2 min 8 s and 2 min 13 s respectively.

Find the range of possible values for the time t by which Jiao beat Liang.

Example 6 — Self Tutor

A rectangular board was measured as 78 cm by 24 cm. Find the boundary values for its perimeter.

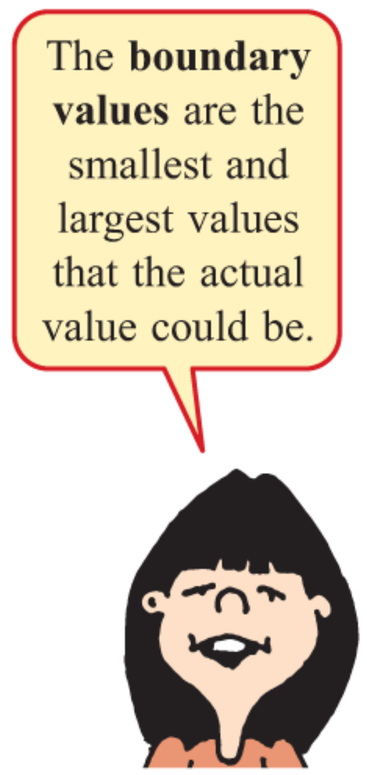
The length of the board could be from $77\frac{1}{2}$ cm to $78\frac{1}{2}$ cm.

The width of the board could be from $23\frac{1}{2}$ cm to $24\frac{1}{2}$ cm.

$\therefore$ the lower boundary of the perimeter is $2 \times 77\frac{1}{2} + 2 \times 23\frac{1}{2} = 202$ cm

and the upper boundary of the perimeter is $2 \times 78\frac{1}{2} + 2 \times 24\frac{1}{2} = 206$ cm

The perimeter is between 202 cm and 206 cm, which is 204 ± 2 cm.



9 A rectangular bath mat was measured as 86 cm by 38 cm. Find the boundary values of its perimeter.

10 A rectangular garden bed is measured as 252 cm by 143 cm. Find the range of possible values for the total length of edging l required to border the garden bed.

Example 7 — Self Tutor

A paving brick is measured as 18 cm $\times$ 10 cm. What are the boundary values for its actual area?

The length of the paving brick could be from $17\frac{1}{2}$ cm to $18\frac{1}{2}$ cm.

The width of the paving brick could be from $9\frac{1}{2}$ cm to $10\frac{1}{2}$ cm.

$\therefore$ the lower boundary of the area is $17\frac{1}{2} \times 9\frac{1}{2} = 166.25$ cm^2

and the upper boundary of the area is $18\frac{1}{2} \times 10\frac{1}{2} = 194.25$ cm^2.

The area is between 166.25 cm^2 and 194.25 cm^2.

This could also be represented as $\frac{166.25 + 194.25}{2} \pm \frac{194.25 - 166.25}{2}$ cm^2 which is 180.25 ± 14 cm^2.

11 A rectangle is measured to be 6 cm by 8 cm. Find:

a the largest area it could have **b** the smallest area it could have.

12 Find the boundary values for the actual area of a glass window measured as 42 cm by 26 cm.

13 The base of a triangle is measured as 9 cm and its height is measured as 8 cm. In what range of values does its actual area A lie?

14 Find the boundary values for the actual volume of a box measuring 4 cm by 8 cm by 6 cm.

15 Find the range of values in which the actual volume V of a house brick measuring 21.3 cm by 9.8 cm by 7.3 cm must lie.

16 A cylinder is measured to have radius 5 cm and height 15 cm. Find the boundary values for the cylinder's volume.

17 A cone is measured to have radius 8.4 cm and height 4.6 cm. Find the boundary values for the cone's volume.

18 Eko measures the diameter of a ball to be 18.2 cm. Do you expect the rounding in Eko's measurement to have more effect on a calculation of the ball's surface area, or a calculation of its volume? Explain your answer.

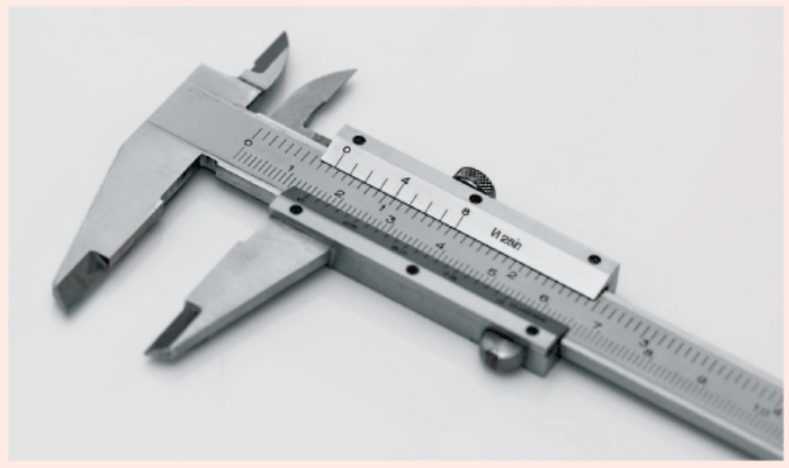
19 Rachel measures the base side lengths of a square-based pyramid to be 4.6 cm, and its height to be 5.2 cm. Find the boundary values for the pyramid's:

a volume **b** surface area.

RESEARCH **VERNIER SCALES**

A **vernier scale** is used to measure the length of objects with a high degree of accuracy.

Research how vernier scales work.



D ABSOLUTE AND PERCENTAGE ERROR

Whenever we measure a quantity there is almost always a difference between our measurement and the actual value. We call this difference the **error**.

The *size* or *magnitude* of the error, whether the measured or estimated value is too high or too low, is called the **absolute error**.

If the actual or exact value is V_E and the approximate value is V_A then the

$$\textbf{absolute error} = |V_A - V_E|$$

Error is often expressed as a percentage of the exact value:

$$\textbf{percentage error} = \frac{|V_A - V_E|}{V_E} \times 100\%$$

Example 8 **Self Tutor**

Suppose you estimate a fence's length to be 70 m, whereas its true length is 78.3 m. Find, correct to one decimal place, the:

a absolute error **b** percentage error.

a absolute error $= |V_A - V_E|$
$= |70 - 78.3|$ m
$= |-8.3|$ m
$= 8.3$ m

b percentage error $= \dfrac{|V_A - V_E|}{V_E} \times 100\%$
$= \dfrac{8.3}{78.3} \times 100\%$
$\approx 10.6\%$

EXERCISE 1D

1 Find the absolute error and percentage error in rounding:

a the yearly profit of €1 367 540 made by a company to €1.37 million
b a population of 31 467 people to 31 000 people
c a retail sales figure of \$458 110 to \$460 000
d the number of new cars sold in a year from 2811 to 3000.

2 Find the absolute error and percentage error if you estimate:

a the mass of a brick to be 5 kg when its actual mass is 6.238 kg
b the perimeter of a property to be 100 m when its actual perimeter is 97.6 m
c the capacity of a container to be 20 L when its actual capacity is 23.8 L
d the time to write a computer program to be 50 hours when it actually takes 72 hours.

Example 9 **Self Tutor**

Alan wants to lay carpet on his 4.2 m by 5.1 m lounge room floor. He estimates the area of the lounge room by rounding each measurement to the nearest metre.

a Find Alan's estimate of the lounge room area.
b The carpet costs \$39 per square metre. Find the cost of the carpet using Alan's estimate of the area.
c Find the actual area of Alan's lounge room.
d Find the percentage error in Alan's estimation.
e Will Alan have enough carpet to cover his lounge room? How should he have rounded the measurements?

a Area ≈ 4 m $\times 5$ m
≈ 20 m^2

b Cost $= 20 \times \$39$
$= \$780$

c Actual area $= 4.2 \times 5.1$
$= 21.42$ m^2

d Percentage error $= \dfrac{|V_A - V_E|}{V_E} \times 100\%$
$= \dfrac{|20 - 21.42|}{21.42} \times 100\%$
$\approx 6.63\%$

e Alan only has 20 m^2 of carpet, so he will not have enough to cover his lounge room. He should have rounded the measurements *up* to make sure he had enough carpet.

3 Jon's apartment is a 10.3 m by 9.7 m rectangle.

a Find the actual area of the apartment.

b Estimate the floor area by rounding each length to the nearest metre.

c Find the absolute error and percentage error in your estimate.

4 The cost of freight for a parcel is dependent on its volume. Justine lists the dimensions of a parcel as 24 cm by 15 cm by 9 cm on the consignment note. The actual dimensions are 23.9 cm $\times$ 14.8 cm $\times$ 9.2 cm.

a Calculate the actual volume of the parcel.

b Estimate the volume using the dimensions given on the consignment note.

c Find the absolute error and percentage error in the calculation using the consignment note.

5 A hotel wants to cover an 8.2 m by 9.4 m rectangular courtyard with synthetic grass. The manager estimates the area by rounding each measurement to the nearest metre.

a Find the manager's estimate of the area.

b The synthetic grass costs \$85 per square metre. Find its cost using the manager's estimate.

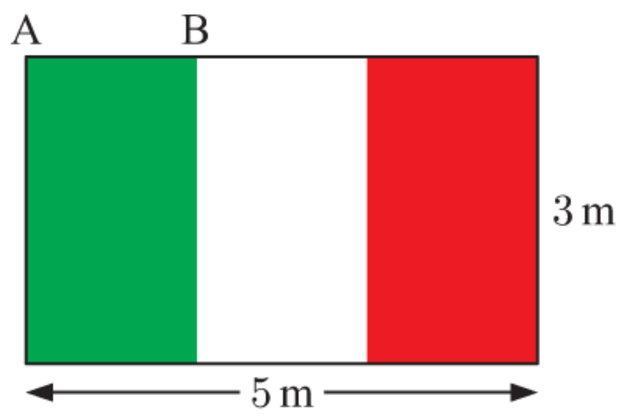
c Find the actual area of the rectangle.

d Calculate the percentage error in the manager's estimate.

e Will the hotel have enough grass to cover the courtyard?

f Find the cost of the grass if the manager had rounded each measurement *up* to the next metre.

6 The Italian flag has three different regions of equal size. Consider the flag alongside.



a Find the area of the green section exactly.

b Find the length AB correct to 1 decimal place.

c Use your rounded value in **b** to estimate the area of the green section.

d Find the percentage error in your estimate.

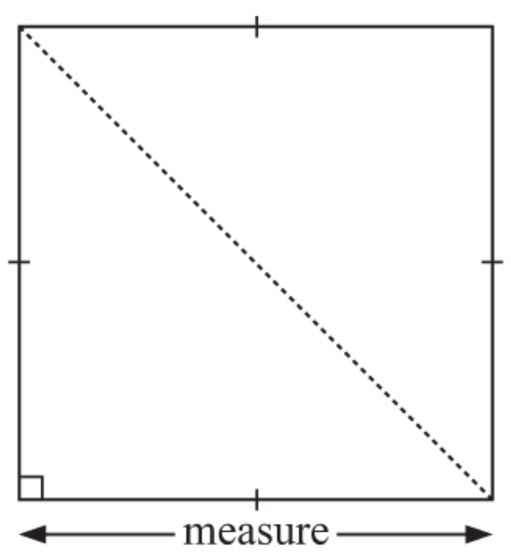
7 Hemi estimates that she can drive at an average speed of 70 km h^{-1} between her house and the beach, 87 km away. One particular journey took her 1 hour and 20 minutes.

a Calculate Hemi's average speed for this journey.

b Find the absolute error and percentage error in her estimate.

8 The sentence below is translated from an ancient Indian text, the *Śulba sūtra*:

The measure is to be increased by its third and this (third) again by its own fourth less the thirtyfourth part (of that fourth); this is (the value of) the diagonal of a square (whose side is the measure).



a Use Pythagoras' theorem to show that the diagonal of a square is $\sqrt{2}$ times the measure of its side.

b Hence show that the text estimates the value of $\sqrt{2}$ as $\frac{577}{408}$.

c Find the percentage error for this estimate, giving your answer in scientific notation.

Example 10 **Self Tutor**

The side length of a square is measured as 22 cm, rounded to the nearest centimetre.

a Use this measurement to estimate the area of the square.

b Find the boundary values for the area of the square.

c Hence find the maximum percentage error in the estimate.

a Area ≈ 22 cm $\times$ 22 cm ≈ 484 cm^2

b The side length of the square could be from $21\frac{1}{2}$ cm to $22\frac{1}{2}$ cm.

$\therefore$ the lower boundary of the area is $21\frac{1}{2} \times 21\frac{1}{2} = 462.25$ cm^2

and the upper boundary of the area is $22\frac{1}{2} \times 22\frac{1}{2} = 506.25$ cm^2.

c If the exact area V_E was 462.25 cm^2, the

$$\text{percentage error} = \frac{|V_A - V_E|}{V_E} \times 100\%$$
$$= \frac{|484 - 462.25|}{462.25} \times 100\%$$
$$\approx 4.71\%$$

If the exact area V_E was 506.25 cm^2, the

$$\text{percentage error} = \frac{|V_A - V_E|}{V_E} \times 100\%$$
$$= \frac{|484 - 506.25|}{506.25} \times 100\%$$
$$\approx 4.40\%$$

$\therefore$ the maximum percentage error in the estimate $\approx 4.71\%$.

9 The side lengths of a rectangle are measured as 2.3 m and 1.4 m, rounded to one decimal place.

a Use these measurements to estimate the area of the rectangle.

b Find the boundary values for the area of the rectangle.

c Hence find the maximum percentage error in the estimate.

10 Jasper measured the dimensions of his cylindrical can of tuna. He found the radius was 4 cm and the height was 5 cm, rounded to the nearest centimetre.

a Use these measurements to estimate the volume of the can.

b Find the boundary values for the volume of the can.

c Hence find the maximum percentage error in the estimate.

11 Carolina completed a 250 km car trip (rounded to the nearest km). The GPS in her car displays the average speed for the trip as 56.8 km h^{-1} (rounded to 1 decimal place).

a Estimate the time it took Carolina to complete the trip.

b Find the maximum possible:

i absolute error ii percentage error in the estimate.

DISCUSSION

Why is it important to understand errors?

What things can go wrong if people measure inaccurately or round off incorrectly?

You may wish to consider the examples in the previous Exercise, and also cases such as:

- If a pilot flies off-course by 0.1° for 1000 km, how far away from his target will he be?
- What happens to a patient if a doctor injects 2 mg of a drug instead of 2 μg?