SET 1A

1. Write as a product of prime factors:

(a) 72 (b) 50 (c) 32

(d) 27 (e) 29 (f) 1001

2. Find the gcd and lcm of:

(a) 12 and 25 (b) 17 and 16 (c) 18 and 27

(d) 42 and 56 (e) 26 and 39 (f) 14 and 32

(g) 50 and 25

3. Prove that a prime number can only give remainder 1 or 5 when divided by 6. *[5 marks]*

4. (a) Show that $5 \mid a$ if and only if $5 \mid a^2$.

(b) Show that if $3 \mid b$ then $9 \mid b^2$. *[7 marks]*

5. Show that if n is an even number greater than 2 then $2^n - 1$ cannot be prime. *[4 marks]*

6. Show that there is no prime number p for which $2p+1$ is a square number. *[6 marks]*

SET 1B

3. Let $l = \text{lcm}(a,b)$ and write $l = pa$ and $l = qb$. Prove that $\gcd(p,q) = 1$. *[7 marks]*

4. If $\gcd(a,b) = 1$ prove that $\gcd(a+b, a-b)$ is either 1 or 2. *[6 marks]*

5. Prove that if $a \mid (bc)$ and $\gcd(a,b) = 1$ then $a \mid c$. *[5 marks]*

6. If $d = \gcd(a,b)$ and f is any other common divisor of a and b, prove that $f \mid d$. *[5 marks]*

Divisibility tests using last digits:

- A number is divisible by 2 if its last digit is even.
- A number is divisible by 5 if its last digit is 0 or 5.
- A number is divisible by 10 if its last digit is 0.
- A number is divisible by 4 if the number formed by its last two digits is divisible by 4.
- A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Divisibility test using all the digits:

- A number is divisible by 3 if the sum of its digits is divisible by 3.
- A number is divisible by 9 if the sum of its digits is divisible by 9.
- A number is divisible by 11 if the alternating sum of its digits is divisible by 11. (So if the number has digits $a_1, a_2, a_3 \ldots$ we calculate the sum $a_1 - a_2 + a_3 - \ldots$)

SET 2

Find the possible values of digits p and q such that the five-digit number $(386pq)$ is divisible by 18.

Q6. Without a calculator state, which of the following numbers are divisible by 2, 3, 4, 5, 6, 9, 10.

(1) 5303145	(4) 2316537	(7) 1167424	(10) 7094372
(2) 517380	(5) 33698267	(8) 24760990	(11) 60445656
(3) 4849845	(6) 3964502	(9) 26640625	(12) 6885714

1. Check whether each of these numbers is divisible by 3, 4 and 11:

(a) (i) 333 444 (ii) 33 334 444

(b) (i) 515 151 (ii) 5 151 515

(c) (i) 123 456 (ii) 8 765 432

(d) (i) 515 152 (ii) 747 472

2. Find the missing digits so that the given number is divisible by 36:

(a) (i) $(32a4b)$ (ii) $(11a65b)$

(b) (i) $(613ab)$ (ii) $(2213ab)$

3. Find digits a and b so that the number $(2006ab)$ is divisible by 33. *[7 marks]*

4. Show that the number 19581958…….1958 (400 digits) is divisible by 22 but not by 44. *[6 marks]*

9. Find all three digit numbers which have the hundreds and the units digits equal and which are divisible by 15. *[5 marks]*

10. For which values of n is the number $\underbrace{111\cdots11}_{n \text{ digits}}$ divisible by

(a) 9?

(b) 11? *[4 marks]*