Example 12 – Applying laws of exponents

Evaluate and/or simplify each of the following expressions. Leave only positive exponents.

a) $(3a^2b)^3$ b) $3(a^2b)^3$ c) $(-2)^{-3}$

d) $(x+y)^0$ e) $(3^3)^{\frac{1}{2}} \cdot 9^{\frac{3}{4}}$ f) $\dfrac{m^2n^{-3}}{m^{-5}n^3}$

g) $(-27)^{-\frac{2}{3}}$ h) $8^{\frac{2}{3}}$ i) $(2^x)(2^{3-x})$

j) $(0.04)^{-2}$ k) $\dfrac{\sqrt{a}\sqrt{a^3}}{a^3}\ (a>0)$ l) $\dfrac{x^{-2}y^3z^{-4}}{(2x^2)^3} \times \dfrac{8}{y^{-2}z^4}$

m) $\sqrt[4]{81a^8b^{12}}$ n) $\dfrac{x^{\frac{3}{2}}+x^{\frac{1}{2}}}{x^{\frac{1}{2}}}\ (x>0)$ o) $2^{n+3}-2^{n+1}$

p) $\dfrac{\sqrt{a+b}}{a+b}$ q) $\dfrac{(x+y)^2}{(x+y)^{-2}}$ r) $\dfrac{x^2+2^{\frac{3}{2}}-2(x^2+2)^{\frac{1}{2}}}{x^2}$

• **Hint for (o):** apply $b^m b^n = b^{m+n}$ in other direction.

Solution

a) $(3a^2b)^3 = 3^3(a^2)^3b^3 = 27a^6b^3$

b) $3(a^2b)^3 = 3(a^2)^3b^3 = 3a^6b^3$

c) $(-2)^{-3} = \dfrac{1}{(-2)^3} = -\dfrac{1}{8}$

d) $(x+y)^0 = 1$

e) $(3^3)^{\frac{1}{2}} \cdot 9^{\frac{3}{4}} = 3^{\frac{3}{2}}(3^2)^{\frac{3}{4}} = 3^{\frac{3}{2}} \cdot 3^{\frac{3}{2}} = 3^{\frac{6}{2}} = 3^3 = 27$

f) $\dfrac{m^2n^{-3}}{m^{-5}n^3} = \dfrac{m^2}{m^{-5}} \cdot \dfrac{n^{-3}}{n^3} = \dfrac{m^{2-(-5)}}{1} \cdot \dfrac{1}{n^{3-(-3)}} = \dfrac{m^7}{n^6}$

g) $(-27)^{-\frac{2}{3}} = [(-3)^3]^{-\frac{2}{3}} = (-3)^{3(-\frac{2}{3})} = (-3)^{-2} = \dfrac{1}{(-3)^2} = \dfrac{1}{9}$

h) $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$ or $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = (2)^2 = 4$ or $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$

i) $(2^x)(2^{3-x}) = 2^{x+3-x} = 2^3 = 8$

j) $(0.04)^{-2} = \left(\dfrac{4}{100}\right)^{-2} = \left(\dfrac{1}{25}\right)^{-2} = \left(\dfrac{25}{1}\right)^2 = 625$

k) $\dfrac{\sqrt{a}\sqrt{a^3}}{a^3} = \dfrac{a^{\frac{1}{2}} \cdot a^{\frac{3}{2}}}{a^3} = \dfrac{a^{\frac{1}{2}+\frac{3}{2}}}{a^3} = \dfrac{a^2}{a^3} = \dfrac{1}{a}$

l) $\dfrac{x^{-2}y^3z^{-4}}{(2x^2)^3} \times \dfrac{8}{y^{-2}z^4} = \dfrac{x^{-2}y^3z^{-4}}{8x^6} \times \dfrac{8}{y^{-2}z^4} = \dfrac{y^3}{x^2x^6z^4} \times \dfrac{y^2}{z^4} = \dfrac{y^5}{x^8z^8}$

m) $\sqrt[4]{81a^8b^{12}} = \sqrt[4]{81} \cdot \sqrt[4]{a^8} \cdot \sqrt[4]{b^{12}} = 3a^{\frac{8}{4}}b^{\frac{12}{4}} = 3a^2b^3$

n) $\dfrac{x^{\frac{3}{2}}+x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} + \dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} = \dfrac{x^{\frac{3}{2}-\frac{1}{2}}}{1} + 1 = x+1$

o) $2^{n+3} - 2^{n+1} = (2^n)(2^3) - (2^n)(2^1) = 8(2^n) - 2(2^n) = 6(2^n)$

p) $\dfrac{\sqrt{a+b}}{a+b} = \dfrac{(a+b)^{\frac{1}{2}}}{(a+b)^1} = \dfrac{1}{(a+b)^{1-\frac{1}{2}}} = \dfrac{1}{(a+b)^{\frac{1}{2}}} = \dfrac{1}{\sqrt{a+b}}$

q) $\dfrac{(x+y)^2}{(x+y)^{-2}} = (x+y)^{2-(-2)} = (x+y)^4$

Although $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$, merely expanding is not 'simplifying'.

r) $\dfrac{(x^2+2)^{\frac{3}{2}} - 2(x^2+2)^{\frac{1}{2}}}{x^2} = \dfrac{(x^2+2)^{\frac{1}{2}}[(x^2+2)^1 - 2]}{x^2} = \dfrac{(x^2+2)^{\frac{1}{2}}[x^2]}{x^2}$

$= (x^2+1)^{\frac{1}{2}}$ or $\sqrt{x^2+1}$

● **Hint:** Note that in Example 12 q) that the square of a sum is **not** equal to the sum of the squares. That is, avoid the error $(x+y)^2 \neq x^2 + y^2$, and in general $(x+y)^n \neq x^n + y^n$.

Exercise 1.3

In questions 1–6, simplify (without your GDC) each expression to a single integer.

1 $16^{\frac{1}{4}}$ **2** $9^{\frac{3}{2}}$ **3** $64^{\frac{2}{3}}$

4 $8^{\frac{4}{3}}$ **5** $32^{\frac{3}{5}}$ **6** $(\sqrt{2})^6$

In questions 7–9, simplify each expression (without your GDC) to a quotient of two integers.

7 $\left(\dfrac{8}{27}\right)^{\frac{2}{3}}$ **8** $\left(\dfrac{9}{16}\right)^{\frac{1}{2}}$ **9** $\left(\dfrac{25}{4}\right)^{\frac{3}{2}}$

In questions 10–13, evaluate (without your GDC) each expression.

10 $(-3)^{-2}$ **11** $(13)^0$ **12** $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$ **13** $\left(-\dfrac{3}{4}\right)^{-3}$

In questions 14–34, simplify each exponential expression (leave only positive exponents).

14 $(-xy^3)^2$ **15** $-(xy^3)^2$ **16** $(-2xy^3)^3$

17 $(2x^3y^{-5})(2x^{-1}y^3)^4$ **18** $(4m^2)^{-3}$ **19** $\dfrac{3k^3p^4}{(3k^3)^2p^2}$

20 $(-32)^{\frac{3}{5}}$ **21** $(125)^{\frac{2}{3}}$ **22** $\dfrac{x\sqrt{x}}{\sqrt[3]{x}}$

23 $\dfrac{4a^3b^5}{(2a^2b)^4} \cdot \dfrac{b^{-1}}{a^{-3}}$ **24** $\dfrac{(\sqrt[3]{x})(\sqrt[3]{x^4})}{\sqrt[3]{x^2}}$ **25** $\dfrac{6(a-b)^2}{3a-3b}$

26 $\dfrac{(x+4y)^{\frac{1}{2}}}{2(x+4y)^{-1}}$ **27** $\dfrac{p^2+q^2}{\sqrt{p^2+q^2}}$ **28** $\dfrac{5^{3x+1}}{25}$

29 $\dfrac{x^{\frac{1}{3}} + x^{\frac{1}{4}}}{x^{\frac{1}{2}}}$ **30** $3^{n+1} - 3^{n-2}$ **31** $\dfrac{8^{k+2}}{2^{3k+2}}$

32 $\sqrt[3]{24x^6y^{12}}$ **33** $\dfrac{1}{n}\sqrt{n^2+n^4}$ **34** $\dfrac{x+\sqrt{x}}{1+\sqrt{x}}$

● **Hint:** In question 34 it is incorrect to 'cancel' the term of $\sqrt{x}$ from the numerator and denominator. That is, remember $\dfrac{a+b}{c+b} \neq \dfrac{a}{c}$.