A number written in **exponent form** is one which explicitly looks like:

$$a^n$$

n is referred to as the **exponent** or **power**
a is referred to as the **base**

a^n *is* pronounced 'a to the exponent n' or, more simply, 'a to the n'.

To investigate the rules of exponents let us consider an example:

Worked example 2.1

Simplify.

(a) $a^3 \times a^4$ (b) $a^3 \div a^4$ (c) $(a^4)^3$ (d) $a^4 + a^3$

(a) $a^3 \times a^4 = (a \times a \times a \times a) \times (a \times a \times a) = a^7$

(b) $a^3 \div a^4 = \dfrac{\cancel{a \times a \times a}}{\cancel{a \times a \times a} \times a} = \dfrac{1}{a} = a^{-1}$

Use the idea from part (a)

(c) $(a^4)^3 = a^4 \times a^4 \times a^4 = a^{12}$

(d) $a^4 + a^3 = a^3(a+1)$

The example above suggests some rules of exponents.

KEY POINT 2.1

$$a^m \times a^n = a^{m+n}$$

KEY POINT 2.2

$$a^m \div a^n = a^{m-n}$$

KEY POINT 2.3

$$(a^m)^n = a^{m \times n}$$

We can use Key point 2.3 to justify the interpretation of $a^{\frac{1}{n}}$ as the nth root of a, since $\left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n} \times n} = a$. This is exactly the property we require of the nth root of a. So, we get the rule: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$.

Mathematics is often considered a subject without ambiguity. However, the value of 0^0 is undetermined; it depends upon how you get there!

It is questionable whether in part (d) we have actually simplified the expression. Sometimes the way mathematicians choose to simplify expressions is governed by how it looks as well as how it is used.

EXAM HINT

These rules are NOT given in the formula booklet. Make sure that you can use them in both directions, e.g. if you see 2^6 you can rewrite it as $(2^3)^2$ and if you see $(2^3)^2$ you can rewrite it as 2^6. Both ways will be important!

Worked example 2.2

Evaluate $64^{\frac{2}{3}}$.

Use $a^{m/n} = \left(a^{\frac{1}{n}}\right)^m$ to split the calculation into two steps

$$64^{\frac{2}{3}} = \left(64^{\frac{1}{3}}\right)^2$$
$$= (4)^2$$

So, $64^{\frac{2}{3}} = 16$

You must take care when expressions with *different* bases are to be combined by multiplication or division, for example $2^3 \times 4^2$. The rules such as 'multiplication means add the exponents together' are only true *when the bases are the same*. You cannot use this rule to simplify $2^3 \times 4^2$.

There is however, a rule that works when the bases are different *but* the exponents are the *same*.

Consider the following example:

$$\begin{aligned} 3^2 \times 5^2 &= 3\times3\times5\times5 \\ &= 3\times5\times3\times5 \\ &= 15\times15 \\ &= 15^2 \end{aligned}$$

This suggests the following rules:

KEY POINT 2.4

$$a^n \times b^n = (ab)^n$$

KEY POINT 2.5

$$a^n \div b^n = \left(\frac{a}{b}\right)^n$$

Exercise 2A

1. Simplify the following, leaving your answer in exponent form.

(a) (i) $6^4 \times 6^3$ (ii) $5^3 \times 5^5$

(b) (i) $a^3 \times a^5$ (ii) $x^6 \times x^3$

(c) (i) $7^{11} \times 7^{-14}$ (ii) $5^7 \times 5^{-2}$

(d) (i) $x^4 \times x^{-2}$ (ii) $x^8 \times x^{-3}$

(e) (i) $g^{-3} \times g^{-9}$ (ii) $k^{-2} \times k^{-6}$

2. Simplify the following, leaving your answer in exponent form.

(a) (i) $6^4 \div 6^3$ (ii) $5^3 \div 5^5$

(b) (i) $a^3 \div a^5$ (ii) $x^6 \div x^3$

(c) (i) $5^7 \div 5^{-2}$ (ii) $7^{11} \div 7^{-4}$

(d) (i) $x^4 \div x^{-2}$ (ii) $x^8 \div x^{-3}$

(e) (i) $2^{-5} \div 2^{-7}$ (ii) $3^{-6} \div 3^8$

(f) (i) $g^{-3} \div g^{-9}$ (ii) $k^{-2} \div k^6$

3. Express the following in the form required.

(a) (i) $(2^3)^4$ as 2^n (ii) $(3^2)^7$ as 3^n

(b) (i) $(5^{-1})^4$ as 5^n (ii) $(7^{-3})^2$ as 7^n

(c) (i) $(11^{-2})^{-1}$ as 11^n

(ii) $(13^{-3})^{-5}$ as 13^n

(d) (i) $4 \times (2^5)^3$ as 2^n

(ii) $3^{-5} \times (9^{-1})^{-4}$ as 3^n

(e) (i) $(4^2)^3 \times 3^{12}$ as 6^n

(ii) $(6^3)^2 \div (2^2)^3$ as 3^n

4. Simplify the following, leaving your answer in exponent form with a prime number as the base.

(a) (i) 4^5 (ii) 9^7

(b) (i) 8^3 (ii) 16^5

(c) (i) $4^2 \times 8^3$ (ii) $9^5 \div 27^2$

(d) (i) $4^{-3} \times 8^5$ (ii) $3^7 \div 9^{-2}$

(e) (i) $\left(\frac{1}{4}\right)^3$ (ii) $\left(\frac{1}{9}\right)^3$

(f) (i) $\left(\frac{1}{8}\right)^2 \div \left(\frac{1}{4}\right)^4$ (ii) $9^7 \times \left(\frac{1}{3}\right)^4$

5. Write the following without brackets or negative exponents:

(a) (i) $(2x^2)^3$ (ii) $(3x^4)^2$

(b) (i) $2(x^2)^3$ (ii) $3(x^4)^2$

(c) (i) $\dfrac{(3a^3)^4}{9a^2}$ (ii) $\dfrac{(4x)^4}{8(2x)^4}$

(d) (i) $(2x)^{-1}$ (ii) $\left(\dfrac{3}{y}\right)^{-2}$

(e) (i) $2x^{-1}$ (ii) $\dfrac{3}{y^{-2}}$

(f) (i) $5 \div \left(\dfrac{3}{xy^2}\right)^2$ (ii) $\left(\dfrac{ab}{2}\right)^3 \div \left(\dfrac{a}{b}\right)^2$

(g) (i) $\left(\dfrac{2}{q}\right)^2 \div \left(\dfrac{p}{2}\right)^{-3}$ (ii) $\left(\dfrac{6}{x}\right)^4 \div \left(2 \times \dfrac{3^2}{x}\right)^{-3}$

6. Simplify the following:

(a) (i) $(x^6)^{\frac{1}{2}}$ (ii) $(x^9)^{\frac{4}{3}}$

(b) (i) $(4x^{10})^{0.5}$ (ii) $(8x^{12})^{-\frac{1}{3}}$

(c) (i) $\left(\dfrac{27x^9}{64}\right)^{-\frac{1}{3}}$ (ii) $\left(\dfrac{x^4}{y^8}\right)^{-1.5}$

7. Solve for x, giving your answer as a rational value:

(a) (i) $8^x = 32$ (ii) $25^x = \dfrac{1}{125}$

(b) (i) $\dfrac{1}{49^x} = 7$ (ii) $\dfrac{1}{16^x} = 8$

(c) (i) $2 \times 3^x = 162$ (ii) $3 \div 5^x = 0.12$

(d) (i) $2 \times 5^{x-1} = 250$ (ii) $5 + 3^{x+2} = 14$

(e) (i) $16 + 2^x = 2^{x+1}$ (ii) $100^{x+5} = 10^{3x-1}$

(f) (i) $6^{x+1} = 162 \times 2^x$ (ii) $4^{1.5x} = 2 \times 16^{x-1}$

In Section 2G you will see that there is an easier way to solve equations like this when you have a calculator and can use logarithms.

8. Any simple computer program is able to sort n input values in $k \times n^{1.5}$ microseconds. Observations show that it sorts a million values in half a second. Find the value of k. *[3 marks]*

9. A square-ended cuboid has volume xy^2, where x and y are lengths. A cuboid for which $x = 2y$ has volume 128 cm³. Find x. *[3 marks]*

