Approximations and errors

Things you need to learn

- How to approximate calculations.
- How to specify lower and upper boundaries of approximated answers/calculations.
- How to calculate absolute and percentage errors in approximations.

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The exact average speed was $v_{ave} = \frac{325km}{4.1h} = 79.268...\frac{km}{h}$.

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i. length l has to be such that $55cm \leqslant l < 65cm$
width w has to be such that $85cm \leqslant w < 95cm$,
ii. perimeter $P = 2(l + w)$, so the lower bound for P is $2(55 + 85) = 280cm$ and the upper bound is $2(65 + 95) = 320cm$. We have $280cm \leqslant P < 320cm$.
iii. area is given by $A = l \times w$, so the lower bound for A is $55 \times 85 = 4675cm^2$ and the upper bound is $65 \times 95 = 6175cm^2$. We have $4675cm^2 \leqslant A < 6175cm^2$.

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So we have $43.125cm^2 \leqslant A < 53.125cm^2$

Errors

Definition

Let v_e denote the exact value of a certain quantity and v_a denote its approximated value. Then the absolute error ϵ of the approximation is given by

$$\epsilon = \left| v_e - v_a \right|$$

And the percentage error $\epsilon_{\%}$ is given by:

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$$\epsilon = |1223337 - 1200000| = 23337$$

and the percentage error:

$$\epsilon_{\%} = \frac{23337}{1223337} \times 100\% = 1.90765...\%$$

Example 2

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$$\epsilon = |v_e - v_a| = 356m^2$$

The percentage error is:

$$\epsilon_{\%} = \frac{356}{5356} \times 100\% \approx 6.64675...\%$$

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The approximated area is given by $v_a = 3 \times 3^2 = 27$

So we have the absolute error:

$$\epsilon = |v_e - v_a| = 0.33971...$$

$$\epsilon_{\%} = \frac{0.33971...}{8.7025\pi} \times 100\% \approx 1.24255...\%$$

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This gives the absolute error:

$$\epsilon = |v_e - v_a| \approx 0.230769...h$$

$$\epsilon_{\%} = \frac{0.230769...}{3.76923...} \times 100\% \approx 6.122489...\%$$

The short test will include example similar to the above.