Sometimes we approximate a quantity because we don't need the exact value, as in the following examples.

- India's population is about 1 800 000 000.
- I run for about 3 hours every Sunday.
- China's economy grew at an average rate of 10% per year during the period 1990–2004.

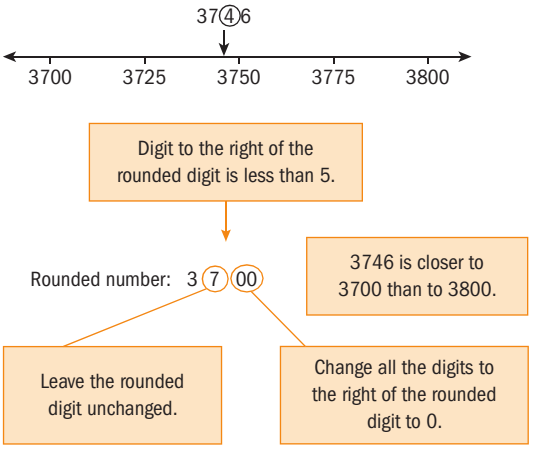
Rounding a number is the process of approximating this number to a given degree of accuracy.

Rounding numbers to the nearest unit, nearest 10, nearest 100, nearest 1000, etc.

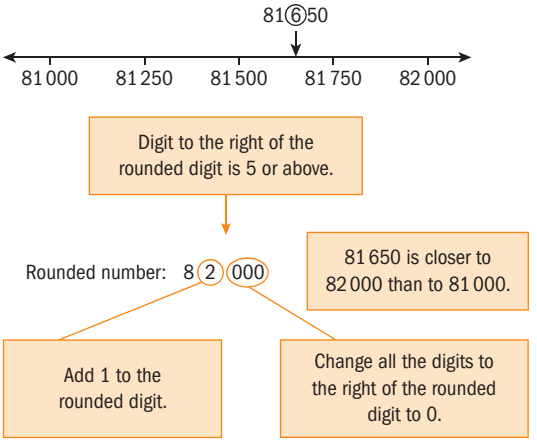
➔ Rounding a number to the **nearest 10** is the same as rounding it to the **nearest multiple of 10**.

Rounding a number to the **nearest 100** is the same as rounding it to the **nearest multiple of 100**.

To round 3746 to the nearest hundred:



To round 81 650 to the nearest thousand:



➔ **Rules for rounding**

If the digit after the one that is being rounded is less than 5 then keep the rounded digit unchanged and change all the remaining digits to the right of this to 0.

If the digit after the one that is being rounded is 5 or more then add 1 to the rounded digit and change all remaining digits to the right of this to 0.

Example 9

a Write down 247 correct to the nearest ten.
b Write down 1050 correct to the nearest hundred.

Answers

a 250

Both 240 and 250 are multiples of 10 but 250 is closer to 247.

b 1100

Both 1000 and 1100 are multiples of 100 and 1050 is exactly in the middle. Because the digit after the one being rounded is 5, round up.

Exercise 1G

1 Write these numbers correct to the nearest unit.
a 358.4 **b** 24.5 **c** 108.9 **d** 10 016.01

2 Write these numbers correct to the nearest 10.
a 246.25 **b** 109 **c** 1015.03 **d** 269

3 Write these numbers correct to the nearest 100.
a 140 **b** 150 **c** 1240 **d** 3062

4 Write these numbers correct to the nearest 1000.
a 105 607 **b** 1500 **c** 9640 **d** 952

5 Write down a number that correct to the nearest 100 is 200.

6 Write down a number that correct to the nearest 1000 is 3000.

7 Write down a number that correct to the nearest unit is 6.

Rounding numbers to a given number of decimal places (dp)

This is rounding numbers to the nearest tenth, to the nearest hundredth, etc.

→ Rounding a number **correct to one decimal place** is the same as rounding it to the **nearest tenth**.

Rounding a number **correct to two decimal places** is the same as rounding it to the **nearest hundredth**.

Rounding a number **correct to three decimal places** is the same as rounding it to the **nearest thousandth**.

To write 3.021 correct to 1 dp:

		Rounded digit	First digit to the right is less than 5	
NUMBER	3	• 0	2	1
ROUNDED NUMBER	3	• 0	.....	.....
		Rounded digit remains unchanged	Digits to the right of rounded digit are deleted	Digits to the right of rounded digit are deleted

3.021 = 3.0 (1 dp)

To write 10.583 correct to 2 dp:

NUMBER	1	0	• 5	8	3
ROUNDED NUMBER	1	0	• 5	8	.....
				Rounded digit remains unchanged	Digits to the right of rounded digit are deleted

10.583 = 10.58 (2 dp)

To write 4.371 to 1 dp:

		Rounded digit	First digit to the right is more than 5	
NUMBER	4	• 3	7	1
ROUNDED NUMBER	4	• 4	.....	.....
		Rounded digit is changed to 1 more	Digits to the right of rounded digit are deleted	Digits to the right of rounded digit are deleted

4.371 = 4.4 (1dp)

→ Rounding rules for decimals

- If the digit after the one that is being rounded is less than 5 keep the rounded digit unchanged and delete all the following digits.
- If the digit after the one that is being rounded is 5 or more then add 1 to the rounded digit and delete all the following digits.

Example 10

a Write down 10.045 correct to 2 dp.
b Write down 1.06 correct to 1 dp.

Answers

a 10.045 = 10.05 (2 dp) *10.045 Next digit is 5, so round up: 10.05*

b 1.06 = 1.1 (1 dp) *1.06 Next digit is 6, so round up: 1.1*

Exercise 1H

1 Write these numbers correct to 1 dp.

a 45.67 **b** 301.065 **c** 2.401 **d** 0.09

2 Write these numbers correct to 2 dp.

a 0.0047 **b** 201.305 **c** 9.6201 **d** 28.0751

3 Write these numbers correct to the nearest thousandth.

a 10.0485 **b** 3.9002 **c** 201.7805 **d** 0.008 41

4 Calculate $\frac{\sqrt{1.8}}{3.08 \times 0.012^2}$; use your GDC.
Give your answer correct to

a 1 dp **b** 2 dp **c** 3 dp **d** nearest 100 **e** nearest 1000.

5 Given that $p = 3.15$ and $q = 0.8$, find the value of $\frac{(p+q)^3}{p+q}$ giving your answer correct to

a 2 dp **b** 3 dp **c** nearest unit **d** nearest ten.

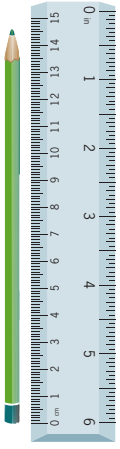
6 Write down a number that correct to 2 dp is 2.37.

7 Write down a number that correct to 1 dp is 4.1.

Rounding numbers to a given number of significant figures (sf)

→ The number of **significant figures** in a result is the number of figures that are known with some degree of reliability.

This sometimes depends on the measurement that is being taken. For example, if the length of a pencil is measured with a ruler whose smallest division is 1 mm, then the measurement is only accurate to the nearest millimetre.



You can say: *The length of this pencil is 14.6 cm.*
But you cannot say: *The length of this pencil is 14.63 cm.*
The length of the pencil can be given correct to 3 sf but cannot be given correct to 4 sf.

Rules for significant figures:	
• All non-zero digits are significant.	2578 kg has 4 sf
• Zeros between non-zero digits are significant.	20 004 km has 5 sf
• Zeros to the left of the first non-zero digit are **not** significant.	0.023 g has 2 sf
• Zeros placed after other digits but to the right of the decimal point are significant.	0.100 ml has 3 sf

Make sure you understand when a digit is significant.

The rules for rounding to a given number of significant figures are similar to the ones for rounding to the nearest 10, 1000, etc. or to a number of decimal places.

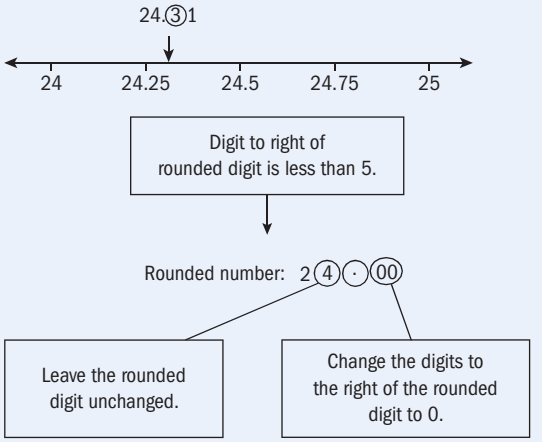
This example shows you the method.

Example 11

a Write down 24.31 correct to 2 sf.
b Write down 1005 correct to 3 sf.
c Write down 0.2981 correct to 2 sf.

Answers

a 24.31 = 24 (2 sf)



b 1005 = 1010 (3 sf)

Digit to right of rounded digit is equal to 5. Add 1 to the rounded digit. Change all digits to the right of the rounded digit to 0.

9 + 1 = 10 Replace the rounded digit with 0. Add 1 to the digit to the left of the rounded digit.

c 0.2981 = 0.30 (2 sf)

Digit to right of rounded digit is greater than 5. Add 1 to the rounded digit. Change all digits to the right of the rounded digit to 0.

→ Rounding rules for significant figures

- If the $(n+1)$th figure is less than 5 then keep the nth figure unchanged.
- If the $(n+1)$th figure is 5 or more then add 1 to this figure.
- In both cases all the figures to the right of figure n should be deleted if they are to the right of the decimal point and should be replaced by zeros if they are to the left of the decimal point.

Example 12

Let $t = \dfrac{12.4^3}{2.1 + \sqrt{3}}$.

a Write down the value of t giving the full calculator display.
b Write the answer to part **a** correct to
i 3 significant figures **ii** 2 significant figures.

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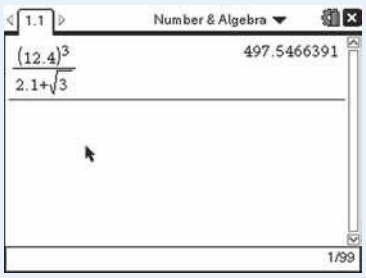
Answers

a 497.5466391



b i 498 — *497.54 3 = 498 (3 sf)*

ii 500 — *497.54 2 = 500 (2 sf)*

Exercise 1I

1 Write the number of significant figures of each of these numbers.
a 106 **b** 200 **c** 0.02 **d** 1290 **e** 1209

2 Write these numbers correct to 1 sf.
a 280 **b** 0.072 **c** 390.8 **d** 0.001 32

3 Write these numbers correct to 2 sf.
a 355 **b** 0.0801 **c** 1.075 **d** 1560.03

4 Write these numbers correct to 3 sf.
a 2971 **b** 0.3259 **c** 10 410 **d** 0.5006

5 Calculate $\frac{\sqrt{8.7 + 2 \times 1.6}}{0.3^4}$.
Give your answer correct to
a 1 sf **b** 3 sf **c** 1 dp **d** nearest hundredth.

6 Write the value of π correct to
a nearest unit **b** 2 dp **c** 2 sf **d** 3 dp.

7 Write down these numbers to the accuracy stated.
a 238 (1 sf) **b** 4609 (3 sf) **c** 2.7002 (3 sf)

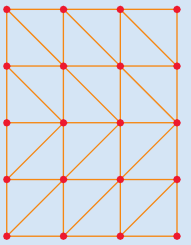
8 **a** Calculate $\frac{\sqrt[3]{3.375}}{1.5^2 + 1.8}$. Write down the full calculator display.
b Give your answer to part **a** correct to
i 2 sf **ii** 3 sf **iii** 4 sf.

Often in exams you need to do multi-step calculations.
In those situations, *keep at least one more significant digit in intermediate results* than needed in your final answer.
For instance, if the final answer needs to be given correct to 3 sf, then carry at least 4 sf in the intermediate calculations, or store the unrounded values in your GDC.

The general rule in Mathematical Studies is *Unless otherwise stated in the question answers must be given exactly or correct to three significant figures.*

Example 13

The diagram represents a window grille made of wire, to keep pigeons out of the house. The small triangles are right-angled triangles and are all **congruent**. Their hypotenuse is 15 cm long. The other two sides are equal lengths. Find the total length of the wire, L. Give your answer correct to 3 significant figures.

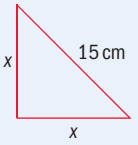


'Congruent' means exactly the same shape and size.

Answers

Let x be the side length of the triangles.

First find the length of the shorter sides using Pythagoras.



$x^2 + x^2 = 15^2$

$2x^2 = 225$

$x^2 = 112.5$

$x = \sqrt{112.5}$

$x = 10.6066\ldots$

Keep this value either exact or with more than three significant figures as this is just an intermediate value.

$L = 31 \times x + 12 \times 15$

$L = 31 \times 10.6066\ldots + 12 \times 15$

$L = 508.804\ldots$

$L = 509$ cm (3 sf)

In the grille there are 31 sides of triangles with length x and 12 sides with length 15.

Do not forget to write down the units in your answers.

Exercise 1J

EXAM-STYLE QUESTIONS

1 The area of a circle is 10.5 cm^2.

a Find the length of its radius. Give your answer correct to four significant figures.

b Find the length of its circumference. Give your answer correct to two significant figures.

2 Let the numbers $p = \sqrt{2}$ and $q = \sqrt{10}$.

a Find the arithmetic mean of p and q. Give your answer correct to 4 sf.

b Find the value of $(p + q)^2$. Give your answer correct to 3 sf.

c Find the area of a rectangle whose sides are p cm and q cm long. Give your answer correct to 2 sf.

1.3 Standard form

- The number of internet users in the world up to June 2010 was $\mathbf{2 \times 10^9}$.
- The mass of the Earth is about $\mathbf{5.97 \times 10^{24}}$ kg.
- An estimate for the average mass of a human cell is about $\mathbf{10^{-9}}$ g.

These numbers are either very large or very small.

They are written in **standard form**: a way of writing very large or very small numbers without writing a lot of zeros.

If we did not use standard form, we would write the mass of the Earth as 5 970 000 000 000 000 000 000 000 kg

When numbers are written in standard form it is easier to

- compare them
- calculate with them.

→ A number is written in standard form if it is in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

A **googol** is the number 1 followed by 100 zeros. In standard form it is 10^{100}. The name googol was invented by a 9-year-old, who was asked by his uncle, the American mathematician Edward Kasner, to think up a name for a very large number.

The name of the company Google comes from a misspelling of the word googol and is related to the amount of information that the company handles.

Example 18

These numbers are written in standard form ($a \times 10^k$).
For each of them state the value of a and of k.

a 2×10^9 **b** 5.97×10^{24} **c** 10^{-9}

Answers

a $a = 2$; $k = 9$
b $a = 5.97$; $k = 24$
c $a = 1$; $k = -9$

Compare with $a \times 10^k$

Example 19

Decide which of these numbers are *not* written in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. Justify your decisions.

a 2.06×10^{-5} **b** 13×10^{-1} **c** $6.13 \times 10^{\frac{1}{3}}$
d 7.05 **e** 0.12×10^6

Answers

b 13×10^{-1} is not written in standard form as 13 is greater than 10.
c $6.13 \times 10^{\frac{1}{3}}$ is not written in standard form as $\frac{1}{3}$ is not an integer.
e 0.12×10^6 is not written in standard form as 0.12 is smaller than 1.

Compare with $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$

Abu Kamil Shuja (c. 850–c. 930), also know as al-Hasib al-Misri, meaning 'the calculator from Egypt', was one of the first to introduce symbols for indices, such as $x^m \, x^n = x^{m+n}$, in algebra.

Example 20

Write these numbers in standard form, showing your working.

a 257 000 000 **b** 0.000 43

Answers

a 257 000 000

so $k = 8$

$257\,000\,000 = 2.57 \times 10^8$

First significant figure of 257 000 000 is 2. Place the decimal point immediately after 2.
Moving the decimal point 8 places to the right is the same as multiplying by 10^8.

b 0.000 43

so $k = -4$

$0.000\,43 = 4.3 \times 10^{-4}$

First significant figure of 0.000 43 is 4. Place the decimal point immediately after 4.
Moving the decimal point 4 places to the left is the same as multiplying by 10^{-4}.

Tips to write a number in standard form:

1 Write down *a*: write down all the significant figures of the number and place the decimal point immediately after the first significant figure.

2 Find *k*.

Exercise 1M

1 Which of these numbers are written in standard form?

2.5×10^{-3} 12×10^5 10^{10} $3.15 \times 10^{\frac{1}{2}}$ 0.81×10^2

2 Write these numbers in standard form.

a 135 600 **b** 0.002 45 **c** 16 000 000 000

d 0.000 108 **e** 0.23×10^3

3 Write these numbers in ascending order.

2.3×10^6 3.4×10^5 0.21×10^7 215×10^4

4 Write these numbers in descending order.

3.621×10^4 31.62×10^2 0.3621×10^4 3.261×10^3

Change them to decimal numbers,
e.g 2.3×10^6
$= 2\,300\,000$.
A decimal number is a 'normal' number written to base 10. It doesn't necessarily have a decimal point or decimal places.

Example 21

Let $x = \dfrac{-5 + \sqrt{121}}{(7-1)^2}$.

a Calculate the value of x. Write down the full calculator display.

b Write your answer to part **a** correct to 3 sf.

c Write your answer to part **b** in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

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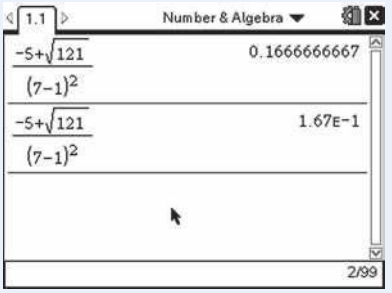
Answers

a 0.1666666667

Use your GDC.



b 0.167

c 1.67×10^{-1}

0.166 666...

3 sf, round up

Careful!
1.67 E-1 is calculator notation and is *not* accepted as an answer. You must interpret it as 1.67×10^{-1}.

Calculations with numbers expressed in standard form

You can use your GDC for calculations in standard form.

Example 22

Let $x = 2.4 \times 10^4$ and $y = 5.10 \times 10^5$.

a Find the value of $3x + y$.

b Write your answer to part **a** correct to 2 sf.

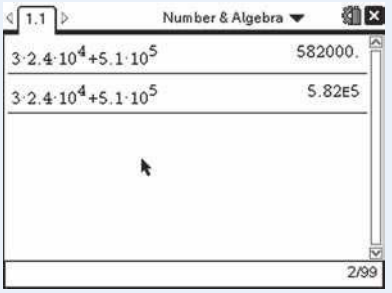
c Write your answer to part **b** in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

Answers

a $3 \times 2.4 \times 10^4 + 5.10 \times 10^5$
$= 582\,000$

b 580 000

c 5.8×10^5



Always use a GDC in this type of question, but show the working as shown in **a**.

Exercise 1N

1 Given that $x = 6.3 \times 10^6$ and $y = 2.8 \times 10^{10}$, calculate the following. Give your answers in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

a $x \times y$ **b** $\frac{x}{y}$ **c** $\sqrt{\frac{x}{y}}$

2 Let $x = 2.5 \times 10^6$ and $y = 3.48 \times 10^6$.

a Find the arithmetic mean of x and y. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

b Give your answer to part **a** correct to the nearest million.