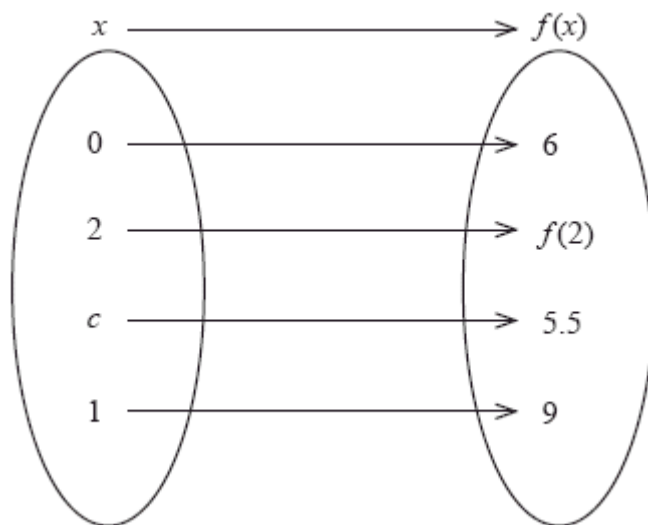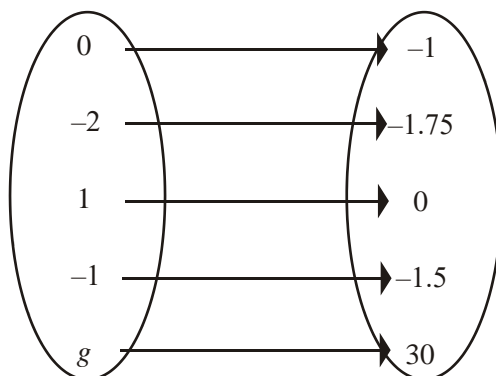**1.** The function $f(x) = a^x + b$ is defined by the mapping diagram below.

(a)   Find the values of a and b.

(3)

(b)   Write down the image of 2 under the function f.

(1)

(c)   Find the value of c.
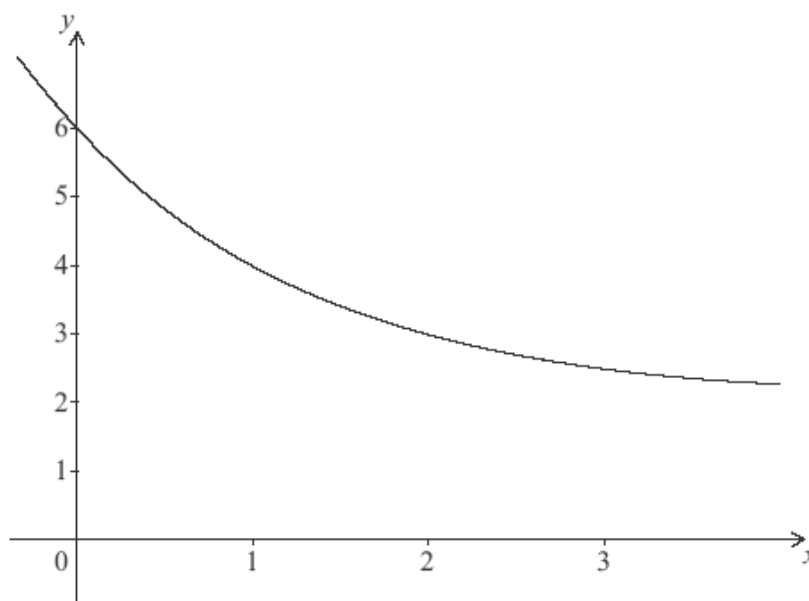
(2)

(Total 6 marks)

2. The mapping below is of the form $f : x \mapsto a \times 2^x + b$ and maps the elements of x to elements of y.

(a)   (i)   List the elements in the domain of f.

(ii)   List the elements in the range of f.

(b)   Find a and b.

(c)   Find the value of g.

(Total 6 marks)

3. Consider the function $f(x) = p(0.5)^x + q$ where p and q are constants. The graph of $f(x)$ passes through the points $(0, 6)$ and $(1, 4)$ and is shown below.

(a) Write down two equations relating p and q.

(2)

(b) Find the value of p and of q.

(2)

(c) Write down the equation of the horizontal asymptote to the graph of $f(x)$.

(2)
(Total 6 marks)

4. The number of cells, C, in a culture is given by the equation $C = p \times 2^{0.5t} + q$, where t is the time in hours measured from 12:00 on Monday and p and q are constants.

The number of cells in the culture at 12:00 on Monday is 47.
The number of cells in the culture at 16:00 on Monday is 53.

Use the above information to

(a) write down two equations in p and q;

(2)

(b) calculate the value of p and of q;

(2)

(c) find the number of cells in the culture at 22:00 on Monday.

(2)
(Total 6 marks)

5. Given the function $f(x) = 2 \times 3^x$ for $-2 \le x \le 5$,

(a) find the range of f;

(4)

(b) find the value of x given that $f(x) = 162$.

(2)
(Total 6 marks)

6. It is thought that a joke would spread in a school according to an exponential model $N = 4 \times (1.356)^{0.4t}$, $t \geq 0$; where N is the number of people who have heard the joke, and t is the time in minutes after the joke is first told.

(a) How many people heard the joke initially?

(b) How many people had heard the joke after 16 minutes?

There are 1200 people in the school.

(c) Estimate how long it would take for everybody in the school to hear this joke.

(Total 6 marks)

7. The equation $M = 90 \times 2^{-t/20}$ gives the amount, in grams, of radioactive material held in a laboratory over t years.

(a) What was the original mass of the radioactive material?

The table below lists some values for M.
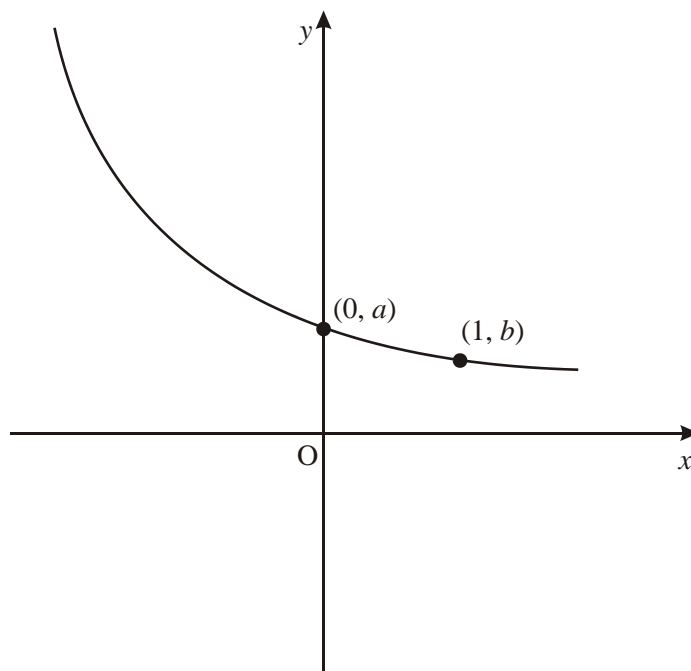
t	60	80	100
M	11.25	v	2.8125

(b) Find the value of v.

(c) Calculate the number of years it would take for the radioactive material to have a mass of 45 grams.

(Total 8 marks)

8. The following diagram shows the graph of $y = 3^{-x} + 2$. The curve passes through the points $(0, a)$ and $(1, b)$.
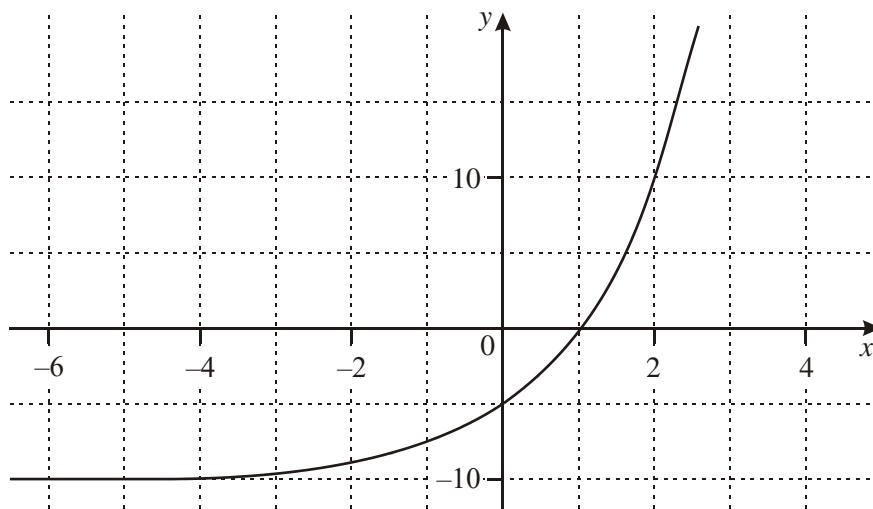
Diagram not to scale

(a) Find the value of

(i) a;

(ii) b.

(b) Write down the equation of the asymptote to this curve.

(Total 8 marks)

9. The graph below shows the curve $y = k(2^x) + c$, where k and c are constants.

Find the values of c and k.

(Total 4 marks)

10. In an experiment researchers found that a specific culture of bacteria increases in number according to the formula

$$N = 150 \times 2^t,$$

where N is the number of bacteria present and t is the number of hours since the experiment began.

Use this formula to calculate

(a) the number of bacteria present at the start of the experiment;

(b) the number of bacteria present after 3 hours;

(c) the number of hours it would take for the number of bacteria to reach 19 200.

(Total 4 marks)