EXAM-STYLE QUESTIONS

3 Let $t = 22.05 \times 10^8$ and $q = 3.15 \times 10^6$

a Write down t in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

b Calculate $\frac{t}{q}$.

c Write your answer to part **b** in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

4 Let $x = 225 \times 10^8$.

a Write x in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

b State whether the following statement is true: $x^2 > 10^{20}$
Justify your answer.

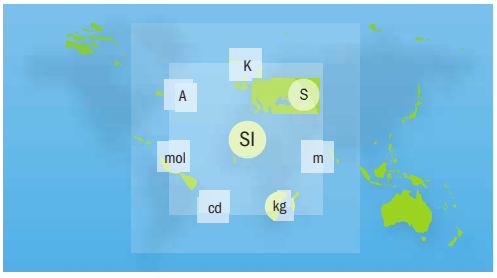
c **i** Calculate $\frac{x}{\sqrt{x}}$.

ii Give your answer to part **i** in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

1.4 SI units of measurement

Ariel is baking a tuna pie.
He needs a tuna can whose net weight is 180 ***g****.*
Another ingredient is 240 ***ml*** *of milk.*
He bakes the pie in a preheated oven to 200 ***°C*** *for 20* ***minutes****.*
Ariel recycles material. He has decided to use the metal from the can so he needs to take some measurements:
The height of the tuna can is 4 ***cm****.*
The total area of metal used to make the tuna can was 219 ***cm²****.*
The volume of the tuna can is 314 ***cm³****.*

Here we have seen how in an everyday situation we deal with different kinds of units such as g, ml, °C, minutes, cm, cm^2, cm^3. These units are internationally accepted and have the same meaning in any part of the world.



SI is the international abbreviation for the *International System of Units* (in French, ***S****ystème* ***I****nternational d'Unités*). There are seven **base units** (see table). Each unit is accurately defined and the definition is independent from the other six units.

The 11th General Conference on Weights and Measures, CGPM, held in 1960, adopted the name Système International d'Unités. The CGPM is made up of representatives from 54 member states and 31 associate states and economies.

The seven base units and their respective quantities are given in the following table.

Base quantity	Base unit name	Base unit symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Intensity of light	candela	cd

One metre is defined in the SI as the distance travelled by light in a vacuum in $\frac{1}{299\,792\,458}$ of a second.

In the **SI** there are other units, the **derived units**. These units are expressed in terms of the base units. Some of these units along with their quantities are listed below:

- The **square metre** (m^2) for area
- The **cubic metre** (m^3) for volume
- The **metre per second** ($m\,s^{-1}$) for speed or velocity
- The **kilogram per cubic metre** ($kg\,m^{-3}$) for density or mass density

Derived units are products of powers of **base units**.

→ In Mathematical Studies, the most common **SI base units** used are m, kg and s, and **derived units** are m^2 (area), m^3 (volume), $km\,h^{-1}$ (velocity), $kg\,m^{-3}$ (density).

Example 23

Write down the symbol used for the quantities in bold:

a The **velocity** of an object that travels 1000 km in 3 hours.

b The **density** of an object with a mass of 550 g and a volume of 400 cm^3.

Answers

a $km\,h^{-1}$

b $g\,cm^{-3}$

Velocity is kilometres per hour. Density is grams per cubic centimetre.

SI prefixes

To avoid writing very small or very large quantities we use prefix names and prefix symbols. Some of these are shown in this table.

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^3	kilo	k	10^{-3}	milli	m
10^2	hecto	h	10^{-2}	centi	c
10^1	deca	da	10^{-1}	deci	d

The kilogram is the only SI base unit with a prefix as part of its name.

Investigation – SI units

a How many prefix names and symbols are there nowadays?
b Six prefix names and their symbols are listed in the table. Find the others.
c Choose at least two of them and describe situations where they are used.

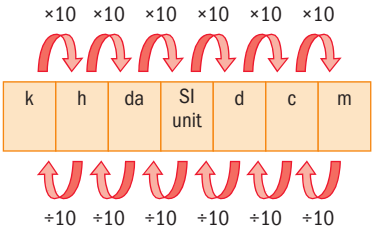
Does the use of SI notation help us to think of mathematics as a 'universal language'?

Example 24

Convert each measurement to the stated unit.
a 1 dm to m **b** 1 das to s **c** 1 hg to g

Answers
a $1\,\text{dm} = 10^{-1}\,\text{m}$
b $1\,\text{das} = 10^{1}\,\text{s}$
c $1\,\text{hg} = 10^{2}\,\text{g}$

Use the information on prefixes given in the table on the previous page.
dm *reads decimetre*
das *reads decasecond*
hg *reads hectogram*



This diagram will help you to convert between SI units.

Example 25

Convert each measurement to the stated unit. Give your answers in standard form.
a 2.8 m to hm **b** 3200 s to ms **c** 0.5 kg to dg

Answers
a $1\,\text{m} = 10^{-2}\,\text{hm}$
$2.8\,\text{m} = 2.8 \times 10^{-2}\,\text{hm}$

In this example replace 'SI unit' in the diagram with m.
To convert from m to hm divide by 10 twice therefore $1\,\text{m} = 10^{-2}\,\text{hm}$.

b $1\,\text{s} = 10^{3}\,\text{ms}$
$3200\,\text{s} = 3200 \times 10^{3}\,\text{ms}$
$= 3.2 \times 10^{6}\,\text{ms}$

In this example replace 'SI unit' in the diagram with s.
To convert from s to ms multiply by 10 three times therefore $1\,\text{s} = 10^{3}\,\text{ms}$.

c $1\,\text{kg} = 10^{4}\,\text{dg}$
$0.5\,\text{kg} = 0.5 \times 10^{4}\,\text{dg}$
$= 5 \times 10^{3}\,\text{dg}$

In this example replace 'SI unit' in the diagram with g.
To convert from kg to dg multiply by 10 four times therefore $1\,\text{kg} = 10^{4}\,\text{dg}$.

Exercise 10

1 Write down the symbol used for the quantities in bold.

a The **acceleration** of an object that has units measured in kilometres per hour squared.

b The **density** of an object with a mass of 23 kg and a volume of 1.5 m^3.

c The average **speed** of an object that travels 500 m in 70 seconds.

2 Write down these units in words.

a dag **b** cs **c** mm **d** dm

3 Convert each of these to the stated unit.

a 32 km to m **b** 0.87 m to dam **c** 128 cm to m

4 Convert each of these to the stated unit.

a 500 g to kg **b** 357 kg to dag **c** 1080 dg to hg

5 Convert each of these to the stated unit.

a 0.080 s to ms **b** 1200 s to das **c** 0.8 hs to ds

6 **a** Convert 67 800 000 mg to kg. Give your answer correct to the nearest kg.

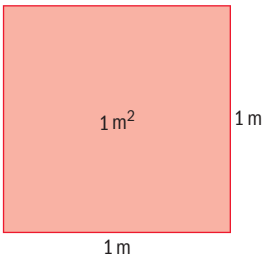
b Convert 35 802 m to km. Give your answer correct to the nearest km.

c Convert 0.654 g to mg. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

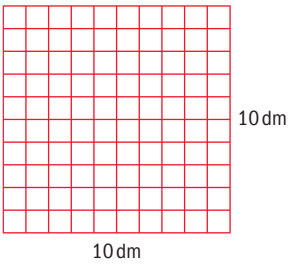
Area and volume SI units

Area

The diagrams show two different ways of representing 1 m^2.



▲ A square metre is equal to the area of a square with sides of length 1 m.

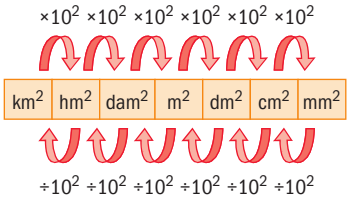


▲ 1 m^2 = 100 dm^2

1 m^2 = 1 m × 1 m = 10 dm × 10 dm = 100 dm^2

To convert from m^2 to dm^2 we multiply by **100** or 10^2.
You can use the same method to convert from

- km^2 to hm^2
- hm^2 to dam^2
- dam^2 to m^2
- dm^2 to cm^2
- cm^2 to mm^2



Example 26

Convert each measurement to the stated unit.
Give your answers in full.

a $1.5\,m^2$ to cm^2

b $3240\,m^2$ to km^2

Answers

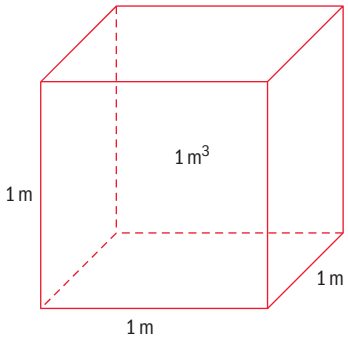
a $1\,m^2 = 10^4\,cm^2$
Therefore
$1.5\,m^2 = 1.5 \times 10^4\,cm^2$
$= 15\,000\,cm^2$

b $1\,m^2 = 10^{-6}\,km^2$
Therefore
$3240\,m^2 = 3240 \times 10^{-6}\,km^2$
$= 0.003\,240\,km^2$

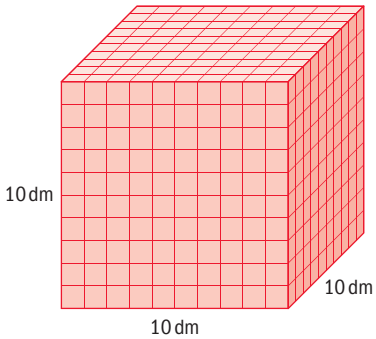
To convert from m^2 to cm^2 multiply by 10^2 twice; this is multiply by 10^4.
$(10^2)^2 = 10^4$
To convert from m^2 to km^2 divide by 10^2 three times; this is divide by 10^6 or multiply by 10^{-6}.
$(10^2)^3 = 10^6$

Volume

The diagrams show two different ways of representing $1\,m^3$.



▲ A cubic metre is equal to the volume of a cube with sides of length 1 m.

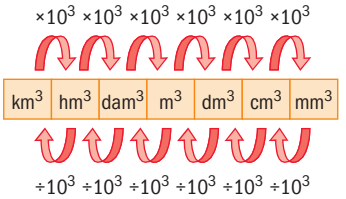


▲ $1\,m^3 = 1000\,dm^3$

$1\,m^3 = 1\,m \times 1\,m \times 1\,m = 10\,dm \times 10\,dm \times 10\,dm = 1000\,dm^3$

To convert from m^3 to dm^3 we multiply by **1000** or 10^3.
You can use the same method to convert from

- km^3 to hm^3
- hm^3 to dam^3
- dam^3 to m^3
- dm^3 to cm^3
- cm^3 to mm^3



Example 27

Convert each measurement to the stated unit.
Give your answers in standard form.

a $0.8\,m^3$ to cm^3

b $15\,900\,cm^3$ to dam^3

Answers

a $1\,m^3 = 10^6\,cm^3$
Therefore
$0.8\,m^3 = 0.8 \times 10^6\,cm^3$
$= 8 \times 10^5\,cm^3$

To convert from m^3 to cm^3 multiply by 10^3 twice; this is multiply by 10^6. $(10^3)^2 = 10^6$

b $1\,cm^3 = 10^{-9}\,dam^3$
Therefore
$15\,900\,cm^3$
$= 15\,900 \times 10^{-9}\,dam^3$
$= 1.59 \times 10^{-5}\,dam^3$

To convert from cm^3 to dam^3 divide by 10^3 three times; this is multiply by 10^{-9}.

Exercise 1P

1 Convert these measurements to the stated unit.
Give your answers in full.

a $2.36\,m^2$ to cm^2 **b** $1.5\,dm^2$ to dam^2
c $5400\,mm^2$ to cm^2 **d** $0.06\,m^2$ to mm^2
e $0.8\,km^2$ to hm^2 **f** $35\,000\,m^2$ to km^2

2 Convert these measurements to the stated unit.
Give your answers in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

a $5\,m^3$ to cm^3 **b** $0.1\,dam^3$ to m^3
c $3\,500\,000\,mm^3$ to dm^3 **d** $255\,m^3$ to mm^3
e $12\,000\,m^3$ to dam^3 **f** $0.7802\,hm^3$ to dam^3

3 The side length of a square is 13 cm. Find its area in

a cm^2 **b** m^2

4 The side length of a cube is 0.85 m. Find the volume of the cube in

a m^3 **b** cm^3

5 Write these measurements in order of size starting from the smallest.
$0.081\,dam^2$, $8\,000\,000\,mm^2$, $82\,dm^2$, $7560\,cm^2$, $0.8\,m^2$

Convert all to the same unit.

6 Write these measurements in order of size starting from the smallest.
$11.2\,m^3$, $1200\,dm^3$, $0.01\,dam^3$, $11\,020\,000\,000\,mm^3$, $10\,900\,000\,cm^3$

Convert all to the same unit.

Extension material on CD:
Worksheet 1 - Calculations with measures

Non-SI units accepted in the SI

→ There are some units that are **non-SI** units but are accepted for use with the SI because they are widely used in everyday life, for example, min, h, ℓ.

Each of these non-SI units has an exact definition in terms of an SI unit. The table below shows some of these units along with their equivalents in SI units.

Quantity	Name of unit	Symbol	Equivalents in SI units
time	minute	min	1 min = 60 s
	hour	h	1 h = 60 min = 3600 s
	day	d	1 d = 24 h = 86 400 s
area	hectare	ha	$1\,ha = 1\,hm^2 = 10^4\,m^2$
volume	litre	L, ℓ	$1\,\ell = 1\,dm^3$
mass	tonne	t	$1\,t = 10^3\,kg$

The SI prefixes are used with ℓ, but *not* used with min, h and d.

Example 28

a Convert 3 d 15 h 6 min to seconds.
b Convert the average speed of $12\,km\,h^{-1}$ to $m\,s^{-1}$.

Answers

a 1 d = 86 400 s
⇒ 3 d = 259 200 s
1 h = 3600 s ⇒ 15 h = 54 000 s
1 min = 60 s ⇒ 6 min = 360 s
Therefore
3 d 15 h 6 min = 259 200 s
+ 54 000 s + 360 s
= 313 560 s

1 day = 24 hours
= 24 × 60 min
= 24 × 60 × 60 s

'⇒' means 'therefore' or 'implies'.

b Average speed = $12\,km\,h^{-1}$
⇒ in 1 h the object moved 12 km
⇒ in 3600 s it moved 12 000 m

$$\text{Average speed} = \frac{12000\text{ m}}{3600\text{ s}}$$
$$= 3.33\,m\,s^{-1}\,(3\,sf)$$

1 h = 60 min
= 60 × 60 s
12 km = 12 000 m

Average speed = $\frac{\text{distance traveled}}{\text{time taken}}$

Example 29

Convert
a 120 hl to cl
b 5400 ℓ to m^3

Answers

a $120\,\text{hl} = 120 \times 10^4\,\text{cl}$
$= 1\,200\,000\,\text{cl}$

To convert from hl to cl, multiply by 10 four times, i.e. multiply by 10^4.

b $1\,\ell = 1\,\text{dm}^3$
$\Rightarrow 5400\,\ell = 5400\,\text{dm}^3$
$5400\,\text{dm}^3 = 5400 \times 10^{-3}\,\text{m}^3$
$= 5.4\,\text{m}^3$

To convert from dm^3 to m^3 we divide by 10^3 once; this is multiply by 10^{-3}.

Exercise 1Q

1 **a** Convert 1 d 2 h 23 m to seconds.
b Give your answer to part **a** correct to the nearest 100.

2 **a** Convert 2 d 5 m to seconds.
b Give your answer to part **a** in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

3 Convert these measurements to the stated unit.
Give your answers in full.
a 5 ℓ to ml **b** 0.56 ml to hl **c** 4500 dal to cl

4 Convert these measurements to the stated unit. Give your answers in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.
a 500 ℓ to cm^3 **b** 145.8 dl to dm^3 **c** 8 hl to cm^3

5 Convert these measurements to the stated unit.
Give your answers to the nearest unit.
a 12.5 dm^3 to ℓ **b** 0.368 m^3 to hl **c** 809 cm^3 to cl

6 A particle travels 3000 m at an average speed of 40 m min^{-1}.
a Find in minutes the time travelled by the particle.
b Give your answer to part **a** in seconds.

EXAM-STYLE QUESTION

7 A cubic container has sides that are 1.5 m long.
a Find the volume of the container. Give your answer in m^3.
b Give your answer to part **a** in dm^3.
c Decide whether 4000 ℓ of water can be poured in the container.
Justify your answer.

EXAM-STYLE QUESTIONS

8 The volume of a tea cup is 220 cm^3. Mercedes always serves a tea cup to $\frac{4}{5}$ of its capacity to avoid spilling any.

a Find, in ℓ, the amount of tea that Mercedes serves in a tea cup.

The volume of Mercedes' teapot is 1.5 ℓ.

b Find the maximum number of tea cups that Mercedes can serve from one teapot.

9 The distance by air from Buenos Aires to Cape Town is 6900 km. An airplane flies at an average speed of 800 $km\,h^{-1}$.

a Find the time it takes for this airplane to fly from Buenos Aires to Cape Town.

Abouo takes this flight and then flies to Johannesburg, which is 1393 km from Cape Town. The flight is 2 hours long.

b Find the average speed of this second airplane.

Abouo leaves Buenos Aires at 10:00 a.m. When he arrives at Cape Town he waits 1.5 hours until the next flight.

c Find the time at which he arrives at Johannesburg.

Temperature

➔ There are three temperature scales:

- **kelvin** (K)
- **Celsius** (°C)
- **Fahrenheit** (°F)

Fahrenheit 451 is the name of a book written by Ray Bradbury. The title refers to the temperature at which paper combusts. This temperature is also known as the flashpoint of paper.

The kelvin (K) is the only SI base unit of temperature and is mainly used by scientists. The °C is an SI derived unit. The Celsius scale is used in most countries but not in the United States, where the Fahrenheit scale is used. In the following table the freezing and boiling points of water for each of the three scales are shown.

Scale	Freezing point of water	Boiling point of water
Fahrenheit (°F)	32	212
Celsius (°C)	0	100
Kelvin (K)	273.15	373.15

In this formula t_C represents temperature in °C and t_F represents temperature in °F.

The formula used to convert from °C to °F is

$$t_F = \frac{9}{5} \times t_C + 32$$

In this formula t_C represents temperature in °C and t_K represents temperature in K.

The formula used to convert from K to °C is

$$t_C = t_K - 273.15$$

Example 30

Convert
a 25 °C to °F **b** 300 K to °C **c** 200 °F to °C

Answers

a $\frac{9}{5} \times 25 + 32 = 77$ °F — *Use the formula* $t_F = \frac{9}{5} \times t_C + 32$

b $300 - 273.15 = 26.85$ °C — *Use the formula* $t_C = t_K - 273.15$

c $200 = \frac{9}{5} \times t_C + 32$ — *Rearrange to make* t_C *the subject of the formula.*

$t_C = (200 - 32) \times \frac{5}{9}$

$t_C = 93.3$ °C (3 sf)

You will derive formulae like this to model real-life situtations in chapter 6.

Exercise 1R

1 Convert into °C. Give your answer correct to one tenth of a degree.
a 280 K **b** 80 °F

2 Convert into °F. Give your answer correct to the nearest degree.
a 21 °C **b** 2 °C

3 **a** Convert 290 K to °C.
b Hence convert 290 K to °F.

4 **a** The formula to convert from K to °C is $t_C = t_K - 273.15$.
Find the formula used to convert from °C to K.
b The formula to convert from °C to °F is $t_F = \frac{9}{5} \times t_C + 32$.
Find the formula used to convert from °F to °C.

Review exercise

Paper 1 style questions

EXAM-STYLE QUESTION

1 Consider the numbers $5, \frac{\pi}{2}, -3, \frac{5}{4}, 2.\dot{3}$ and the number sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.
Complete the following table by placing a tick (✓) in the appropriate box if the number is an element of the set.

	5	$\frac{\pi}{2}$	-3	$\frac{5}{4}$	$2.\dot{3}$
$\mathbb{N}$					
$\mathbb{Z}$					
$\mathbb{Q}$					
$\mathbb{R}$					