Exponential equations

We need to be able to solve simple exponential equations.

Exponential equations are of the form.

$$a^{f(x)} = b^{g(x)}$$

where $a, b > 0$ and f, g are functions. In our examples f and g will be simple linear functions.

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Now we compare the exponents:

$$2x - 1 = 5$$
$$x = 3$$

Example 1

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$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$

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We write everything as a power of 2:

$$\begin{aligned}\left(\frac{1}{2}\right)^{x+1} &= 4^{x+2}\\ (2^{-1})^{x+1} &= (2^2)^{x+2}\\ 2^{-x-1} &= 2^{2x+4}\end{aligned}$$

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Now we equate the exponents:

$$-x-1 = 2x+4$$

$$x = -\frac{5}{3}$$

Example 2

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$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$

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We write both sides as powers of 3:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$

$$(3^{-2})^{x-2} = (3^{\frac{1}{2}})^{x+6}$$

$$3^{-2x+4} = 3^{\frac{x}{2}+3}$$

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We write both sides as powers of 3:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$
$$(3^{-2})^{x-2} = (3^{\frac{1}{2}})^{x+6}$$
$$3^{-2x+4} = 3^{\frac{x}{2}+3}$$

Now compare the exponents:

$$-2x+4 = \frac{x}{2}+3$$
$$x = \frac{2}{5}$$

Example 3

Solve:

$$4 \times 8^{x} = (2\sqrt{2})^{-x}$$

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$$4 \times 8^x = (2\sqrt{2})^{-x}$$
$$2^2 \times (2^3)^x = (2^1 \times 2^{\frac{1}{2}})^{-x}$$
$$2^2 \times 2^{3x} = (2^{\frac{3}{2}})^{-x}$$
$$2^{3x+2} = 2^{-\frac{3}{2}x}$$
$$3x + 2 = -\frac{3}{2}x$$
$$x = -\frac{4}{9}$$

Example 4

Solve:

$$3 \times 81^{x-1} = (\sqrt[3]{3})^{-2x}$$

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Solution (try yourself first):

$$\begin{aligned} 3 \times 81^{x-1} &= (\sqrt[3]{3})^{-2x} \\ 3^1 \times (3^4)^{x-1} &= (3^{\frac{1}{3}})^{-2x} \\ 3^1 \times 3^{4x-4} &= 3^{-\frac{2x}{3}} \\ 3^{4x-3} &= 3^{-\frac{2x}{3}} \\ 4x - 3 &= -\frac{2x}{3} \\ x &= \frac{9}{14} \end{aligned}$$

Example 5

Solve:

$$4 \times \left(\frac{1}{\sqrt{2}}\right)^x = \frac{1}{2} \times 16^{x-1}$$

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$$2^2 \times (2^{-\frac{1}{2}})^x = 2^{-1} \times (2^4)^{x-1}$$

$$2^2 \times 2^{-\frac{x}{2}} = 2^{-1} \times 2^{4x-4}$$

$$2^{2-\frac{x}{2}} = 2^{4x-5}$$

$$2 - \frac{x}{2} = 4x - 5$$

$$x = \frac{14}{9}$$

The short test will include example similar to the above.