Divisibility proofs

We will discuss some proofs, where we show that a number is divisible by a certain other number.

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Remember that if we want to show that a number is divisible by k, then we need to show that it can be written as $k \times m$, where m is an integer.

Example 1

Show that a sum of 3 consecutive integers is divisible by 3.

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Three consecutive integers: $n, n+1, n+2$.

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$$n+n+1+n+2=3n+3=3(n+1)=3m \qquad \text{where } \; m\in\mathbb{Z}$$

$\square$

Example 2

Show that a sum of squares of 5 consecutive integers is divisible by 5.

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Five consecutive integers: $n-2, n-1, n, n+1, n+2,$.

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$$
\begin{aligned}
&(n-2)^2+(n-1)^2+n^2+(n+1)^2+(n+2)^2= \\
=&n^2-4n+4+n^2-2n+1+n^2+n^2+2n+1+n^2+4n+4= \\
=&5n^2+10=5(n^2+2)=5m \qquad \text{where } \ m\in\mathbb{Z}
\end{aligned}
$$

□

Example 3

Show that a sum of squares of 4 consecutive even numbers is divisible by 8.

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Four consecutive even numbers: $2n-2, 2n, 2n+2, 2n+4,$.

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$$\begin{aligned} &(2n-2)^2+(2n)^2+(2n+2)^2+(2n+4)^2= \\ =&4n^2-8n+4+4n^2+4n^2+8n+4+4n^2+16n+16= \\ =&16n^2+16n+24=8(2n^2+2n+3)=8m \qquad \text{where} \;\; m\in\mathbb{Z} \end{aligned}$$

□

Example 4

Show that the number $5^{2019} + 5^{2020} + 5^{2021}$ is divisible by 31.

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$$5^{2019}+5^{2020}+5^{2021} = 5^{2019}(1+5+5^2) = 5^{2019}\times 31 = 31m \qquad \text{where} \quad m \in \mathbb{Z}$$

□

Example 5

Show that the number $3^{200} + 2 \times 3^{199} - 4 \times 3^{198}$ is divisible by 11.

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The common term is 3^{198}, we factor it out and get:

$3^{200}+2\times3^{199}-4\times3^{198} = 3^{198}(3^2+2\times3-4) = 3^{198}\times11 = 11m,$ where $m \in \mathbb{Z}$

$\square$

Example 6

Show that the number $2^{30} + 2^{29} + 2^{27}$ is divisible by 13.

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The common term is 2^{27}, when we factor it out we get:

$$2^{30} + 2^{29} + 2^{27} = 2^{27}(2^3 + 2^2 + 1) = 2^{27} \times 13 = 13m \qquad \text{where } \ m \in \mathbb{Z}$$

$\square$

Example 7

Show that the number $2^{100} + 3 \times 2^{99} - 2^{98}$ is divisible by 18.

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Example 7

Show that the number $2^{100} + 3 \times 2^{99} - 2^{98}$ is divisible by 18.

The common term is 2^{98}, when we factor it out we get:

$$\begin{aligned} &2^{100} + 3 \times 2^{99} - 2^{98} = 2^{98}(2^2 + 3 \times 2 - 1) = \\ =&2^{98} \times 9 = 2^{97} \times 2 \times 9 = 2^{97} \times 18 = 18m \qquad \text{where} \;\; m \in \mathbb{Z} \end{aligned}$$

□

The short test at the beginning of the class will be similar to the examples above.