

1. The function  $Q(t) = 0.003t^2 - 0.625t + 25$  represents the amount of energy in a battery after  $t$  minutes of use.
- State the amount of energy held by the battery immediately before it was used.
  - Calculate the amount of energy available after 20 minutes.
  - Given that  $Q(10) = 19.05$ , find the average amount of energy produced per minute for the interval  $10 \leq t \leq 20$ .
  - Calculate the number of minutes it takes for the energy to reach zero.

(Total 6 marks)

2. (a) A function  $f(x)$  is defined by  $f(x) = 2x^2 - 10x + 60$ ,  $-5 \leq x \leq 8$ .

$x$	-5	0	2	5	8
$f(x)$	160	$a$	$b$	60	108

- Write down the values of  $a$  and  $b$ . (2)
- Using the values in the above table, draw the graph of  $f(x)$  on a set of coordinate axes. Use a scale of 1 cm to represent 1 unit on the horizontal axis and 1 cm to represent 20 units on the vertical axis. (4)
- Show that the coordinates of the vertex of the graph are  $(2.5, 47.5)$ . (3)
- State the values of  $x$  for which the function is increasing. (2)

- (b) A second function  $h(x)$  is defined by:

$$h(x) = 80, 0 \leq x \leq 8.$$

- On the same axes used for part (a), draw the graph of  $h(x)$ . (2)
- Find the coordinates of the point at which  $f(x) = h(x)$ . (2)
- Find the vertical distance from the vertex of the graph of  $f(x)$  to the line  $h(x)$ . (2)

(Total 17 marks)

3. The graph of a quadratic function  $f(x)$  intersects the horizontal axis at  $(1, 0)$  and the equation of the axis of symmetry is  $x = -1$ .
- (a) Write down the  $x$ -coordinate of the other point where the graph of  $y = f(x)$  intersects the horizontal axis.
- (b)  $y = f(x)$  reaches its maximum value at  $y = 5$ .
- (i) Write down the value of  $f(-1)$ .
- (ii) Find the range of the function  $y = f(x)$ .

(Total 6 marks)

4. A small manufacturing company makes and sells  $x$  machines each month. The monthly cost  $C$ , in dollars, of making  $x$  machines is given by

$$C(x) = 2600 + 0.4x^2.$$

The monthly income  $I$ , in dollars, obtained by selling  $x$  machines is given by

$$I(x) = 150x - 0.6x^2.$$

- (a) Show that the company's monthly profit can be calculated using the quadratic function

$$P(x) = -x^2 + 150x - 2600.$$

(2)

- (b) The maximum profit occurs at the vertex of the function  $P(x)$ . How many machines should be made and sold each month for a maximum profit?

(2)

- (c) If the company does maximize profit, what is the selling price of each machine?

(4)

- (d) Given that  $P(x) = (x - 20)(130 - x)$ , find the smallest number of machines the company must make and sell each month in order to make **positive** profit.

(4)

(Total 12 marks)

5. Consider the graphs of the following functions.

(i)  $y = 7x + x^2$ ;

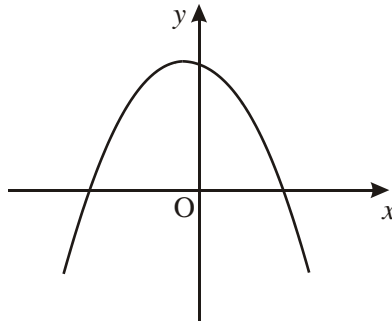
(ii)  $y = (x - 2)(x + 3)$ ;

(iii)  $y = 3x^2 - 2x + 5$ ;

(iv)  $y = 5 - 3x - 2x^2$ .

Which of these graphs

- (a) has a  $y$ -intercept below the  $x$ -axis?
- (b) passes through the origin?
- (c) does not cross the  $x$ -axis?
- (d) could be represented by the following diagram?



**(Total 8 marks)**

6. A rectangle has dimensions  $(5 + 2x)$  metres and  $(7 - 2x)$  metres.

(a) Show that the area,  $A$ , of the rectangle can be written as  $A = 35 + 4x - 4x^2$ .

(1)

(b) The following is the table of values for the function  $A = 35 + 4x - 4x^2$ .

$x$	-3	-2	-1	0	1	2	3	4
$A$	-13	$p$	27	35	$q$	$r$	11	$s$

(i) Calculate the values of  $p$ ,  $q$ ,  $r$  and  $s$ .

(ii) On graph paper, using a scale of 1 cm for 1 unit on the  $x$ -axis and 1 cm for 5 units on the  $A$ -axis, plot the points from your table and join them up to form a smooth curve.

(6)

(c) Answer the following, using your graph or otherwise.

(i) Write down the equation of the axis of symmetry of the curve,

(ii) Find one value of  $x$  for a rectangle whose area is  $27 \text{ m}^2$ .

(iii) Using this value of  $x$ , write down the dimensions of the rectangle.

(4)

(d) (i) On the same graph, draw the line with equation  $A = 5x + 30$ .

(ii) Hence or otherwise, solve the equation  $4x^2 + x - 5 = 0$ .

(3)

**(Total 14 marks)**

7. The perimeter of a rectangle is 24 metres.

- (a) The table shows some of the possible dimensions of the rectangle. Find the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$ .

Length (m)	Width (m)	Area (m <sup>2</sup> )
1	11	11
$a$	10	$b$
3	$c$	27
4	$d$	$e$

(2)

- (b) If the length of the rectangle is  $x$  m, and the area is  $A$  m<sup>2</sup>, express  $A$  in terms of  $x$  only.

(1)

- (c) What are the length and width of the rectangle if the area is to be a maximum?

(3)

(Total 6 marks)

8. The profit ( $P$ ) in Swiss Francs made by three students selling homemade lemonade is modelled by the function

$$P = -\frac{1}{20}x^2 + 5x - 30$$

where  $x$  is the number of glasses of lemonade sold.

- (a) **Copy** and complete the table below

$x$	0	10	20	30	40	50	60	70	80	90
$P$		15			90			75	50	

(3)

- (b) On graph paper draw axes for  $x$  and  $P$ , placing  $x$  on the horizontal axis and  $P$  on the vertical axis. Use suitable scales. Draw the graph of  $P$  against  $x$  by plotting the points. Label your graph.

(5)

- (c) **Use your graph** to find

(i) the maximum possible profit;

(1)

(ii) the number of glasses that need to be sold to make the maximum profit;

(1)

(iii) the number of glasses that need to be sold to make a profit of 80 Swiss Francs;

(2)

(iv) the amount of money initially invested by the three students.

(1)

- (d) The three students Baljeet, Jane and Fiona share the profits in the ratio of 1:2:3 respectively. If they sold 40 glasses of lemonade, calculate Fiona's share of the profits.

(2)

**(Total 15 marks)**