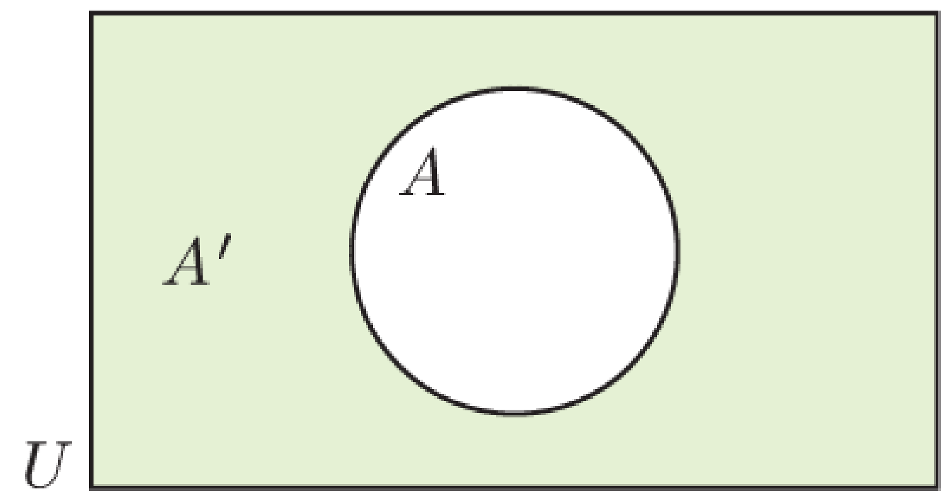
F VENN DIAGRAMS

A **Venn diagram** consists of a universal set U represented by a rectangle, and subsets within it that are generally represented by circles.

The Venn diagram alongside shows set A within the universal set U.

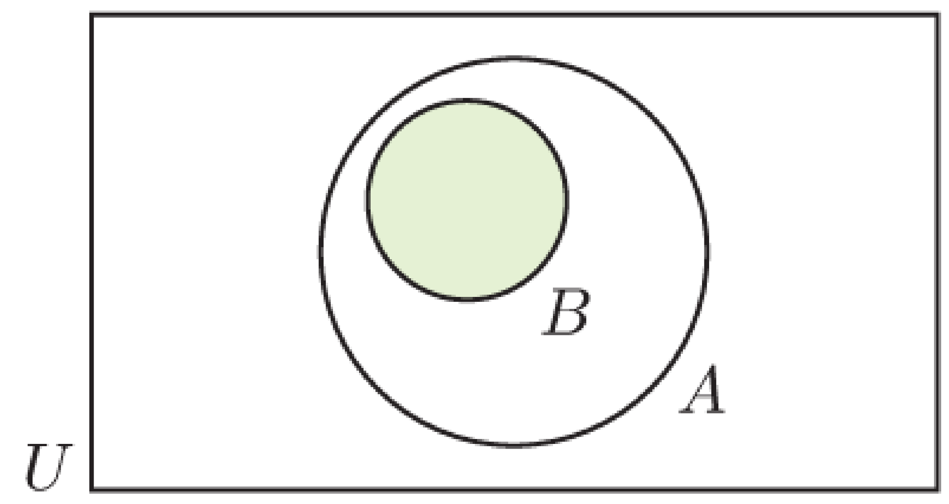
The **complement** of A is the shaded region outside the circle.



SUBSETS

If $B \subseteq A$ then every element of B is also in A.

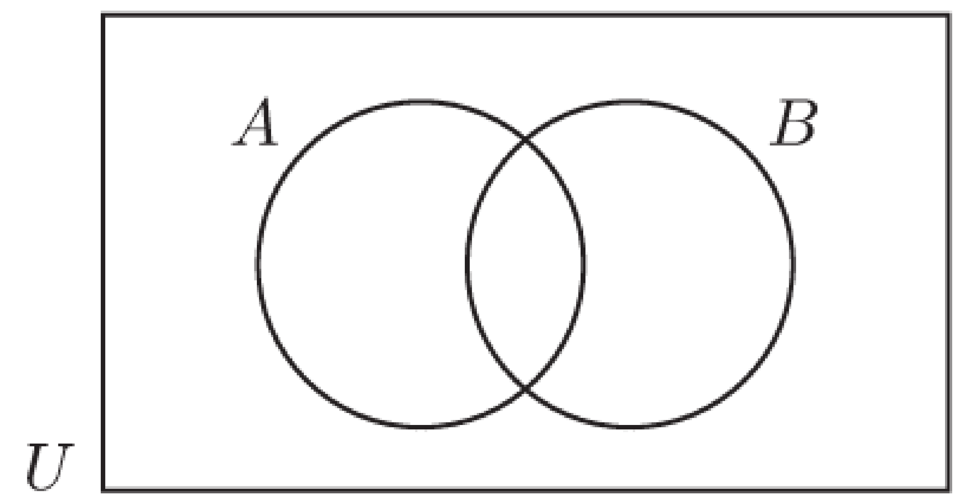
The circle representing B is placed within the circle representing A.



$B \subseteq A$

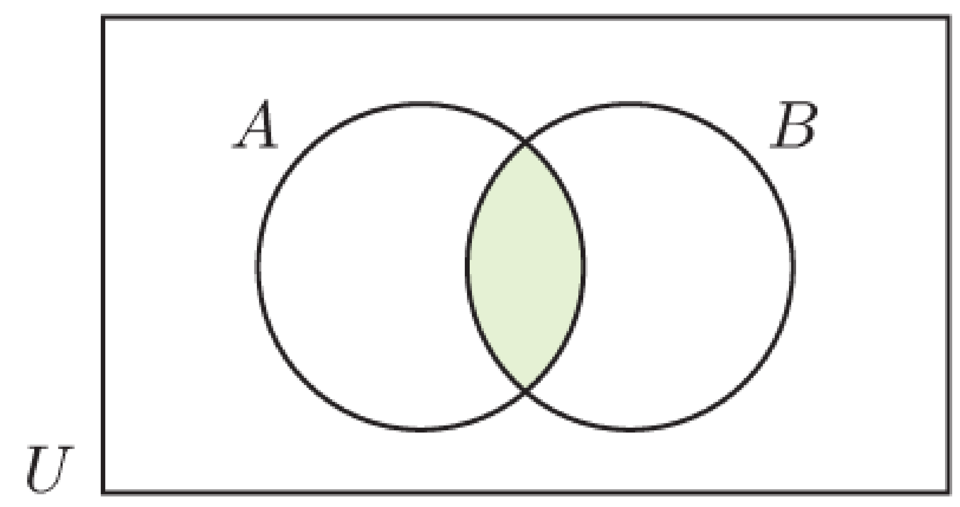
INTERSECTING SETS

For two sets A and B which have some elements in common, but where neither is a subset of the other, we draw the circles overlapping.



The **intersection** $A \cap B$ consists of all elements common to both A and B.

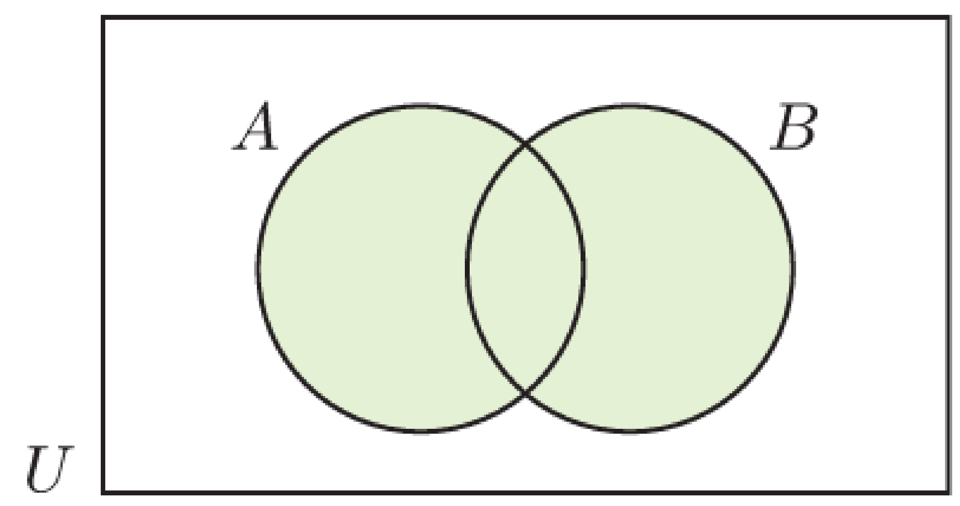
It is the region where the circles overlap.



$A \cap B$

The **union** $A \cup B$ consists of all elements in A or B or both.

It is the region which includes the two circles.

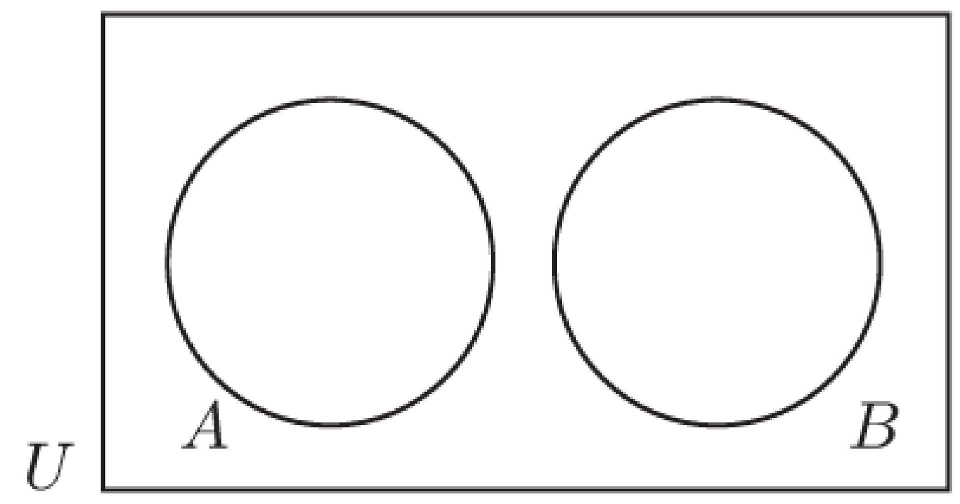


$A \cup B$

DISJOINT OR MUTUALLY EXCLUSIVE SETS

Disjoint sets do not have common elements.

They are represented by non-overlapping circles.



$A \cap B = \varnothing$

Example 6 **Self Tutor**

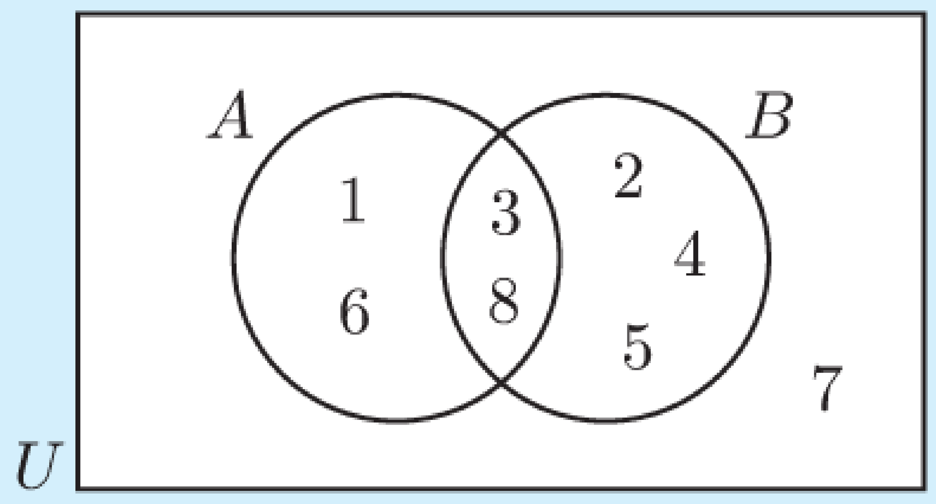
Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Illustrate on a Venn diagram the sets:

a $A = \{1, 3, 6, 8\}$ and $B = \{2, 3, 4, 5, 8\}$

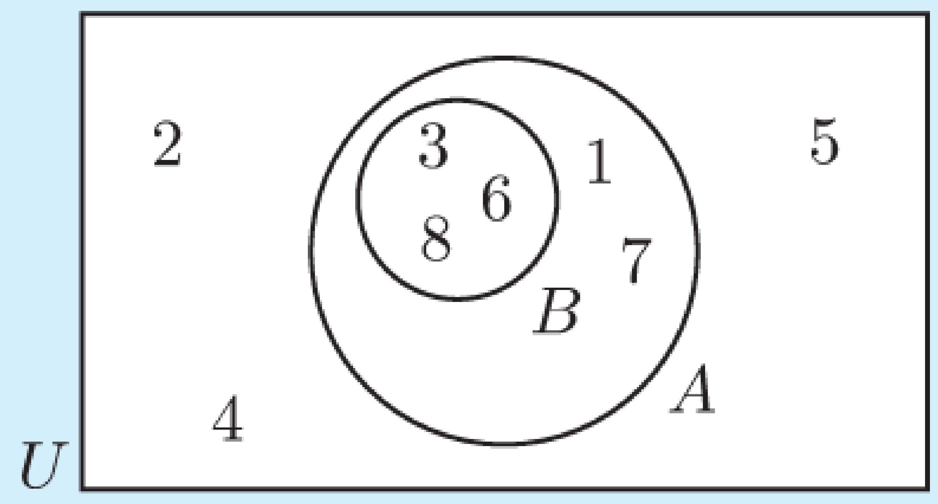
b $A = \{1, 3, 6, 7, 8\}$ and $B = \{3, 6, 8\}$

c $A = \{2, 4, 8\}$ and $B = \{1, 3, 5\}$.

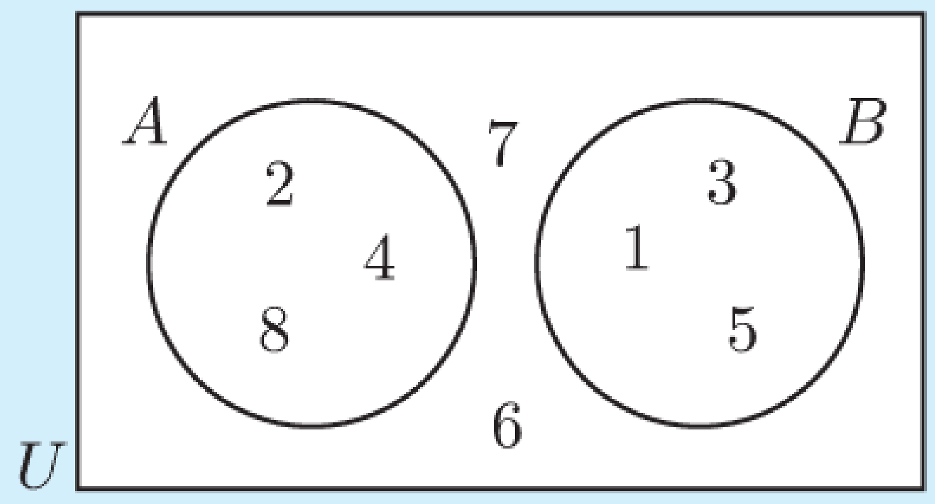
a $A \cap B = \{3, 8\}$



b $A \cap B = \{3, 6, 8\} = B$ and $B \neq A$, so $B \subset A$.



c $A \cap B = \varnothing$



EXERCISE 2F

1 Represent sets A and B on a Venn diagram, given:

a $U = \{2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, and $B = \{5, 7\}$

b $U = \{2, 3, 5, 7, 11, 13\}$, $A = \{2, 3, 7\}$, and $B = \{3, 5, 11\}$

c $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 5, 6\}$, and $B = \{1, 4, 6, 7\}$

d $U = \{1, 3, 4, 5, 7\}$, $A = \{3, 4, 5, 7\}$, and $B = \{3, 5\}$

2 Suppose $U = \{x \in \mathbb{Z} \mid 1 \leqslant x \leqslant 10\}$, $A = \{\text{odd numbers} < 10\}$, and $B = \{\text{primes} < 10\}$.

a List sets A and B.

b Find $A \cap B$ and $A \cup B$.

c Represent the sets A and B on a Venn diagram.

3 Suppose $U = \{x \in \mathbb{Z} \mid 1 \leqslant x \leqslant 9\}$, $A = \{\text{factors of } 6\}$, and $B = \{\text{factors of } 9\}$.

a List sets A and B.

b Find $A \cap B$ and $A \cup B$.

c Represent the sets A and B on a Venn diagram.

4 Suppose $U = \{\text{even numbers between 0 and 30}\}$, $P = \{\text{multiples of 4 less than 30}\}$, and $Q = \{\text{multiples of 6 less than 30}\}$.

a List sets P and Q.

b Find $P \cap Q$ and $P \cup Q$.

c Represent the sets P and Q on a Venn diagram.

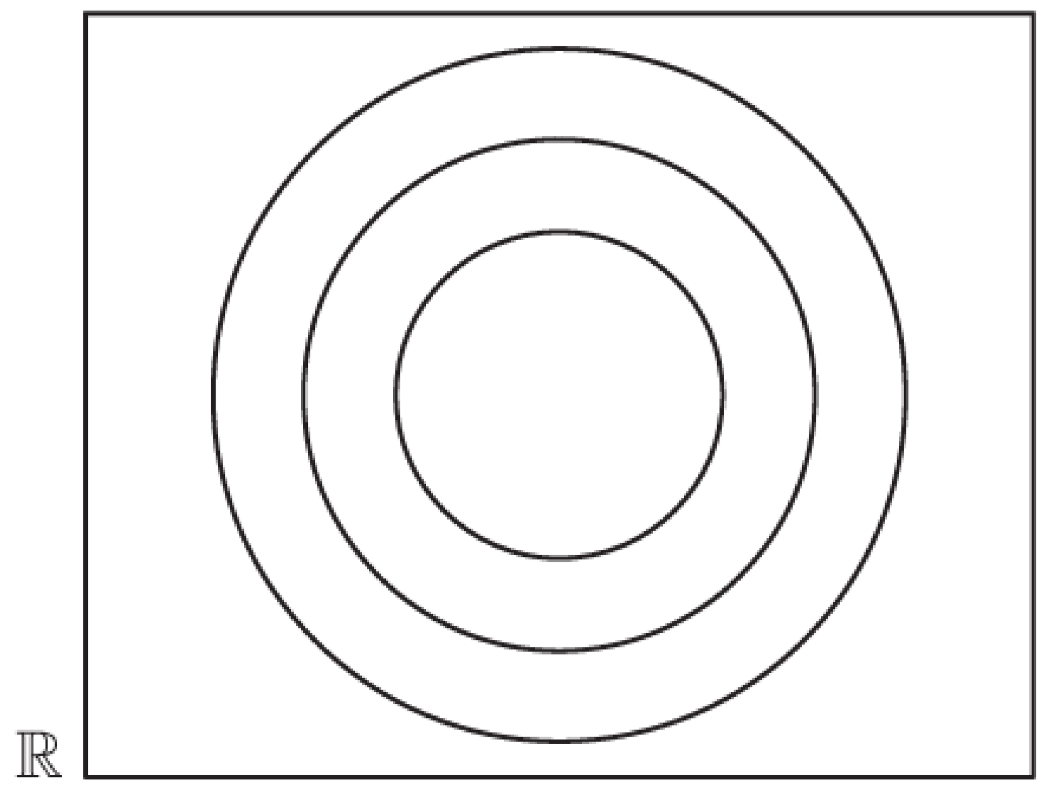
5 Suppose $U = \{x \in \mathbb{Z}^+ \mid x \leqslant 30\}$, $R = \{\text{primes less than 30}\}$, and $S = \{\text{composites less than 30}\}$.

a List sets R and S.

b Find $R \cap S$ and $R \cup S$.

c Represent the sets R and S on a Venn diagram.

6 Suppose $U = \mathbb{R}$.
Copy the Venn diagram and label the sets $\mathbb{Q}$, $\mathbb{Z}$, and $\mathbb{N}$.
Shade the region representing $\mathbb{Q}'$.
Place these numbers on the Venn diagram: 7, $\frac{1}{5}$, $2.\overline{8}$, $-\pi$, 0, $\sqrt{7}$, -2, -0.35.

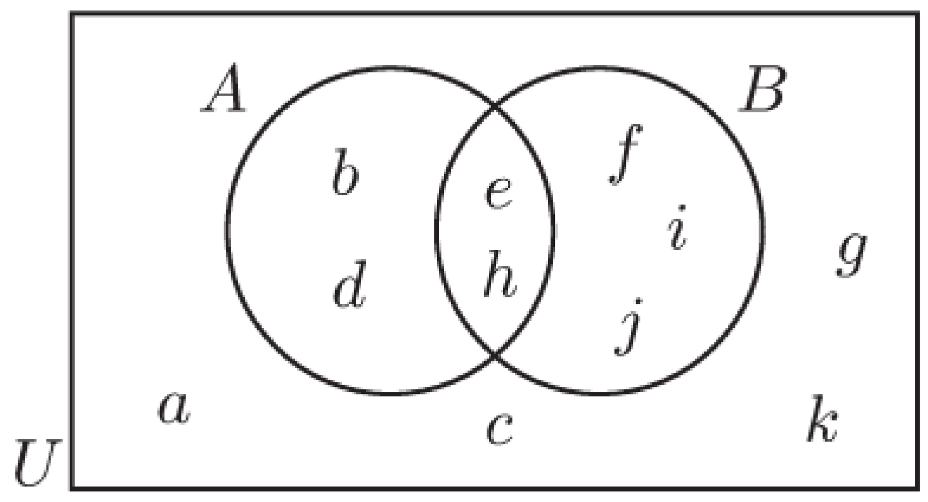


7 Display on a Venn diagram:

a $U = \{\text{parallelograms}\}$, $R = \{\text{rectangles}\}$, $S = \{\text{squares}\}$

b $U = \{\text{polygons}\}$, $Q = \{\text{quadrilaterals}\}$, $T = \{\text{triangles}\}$.

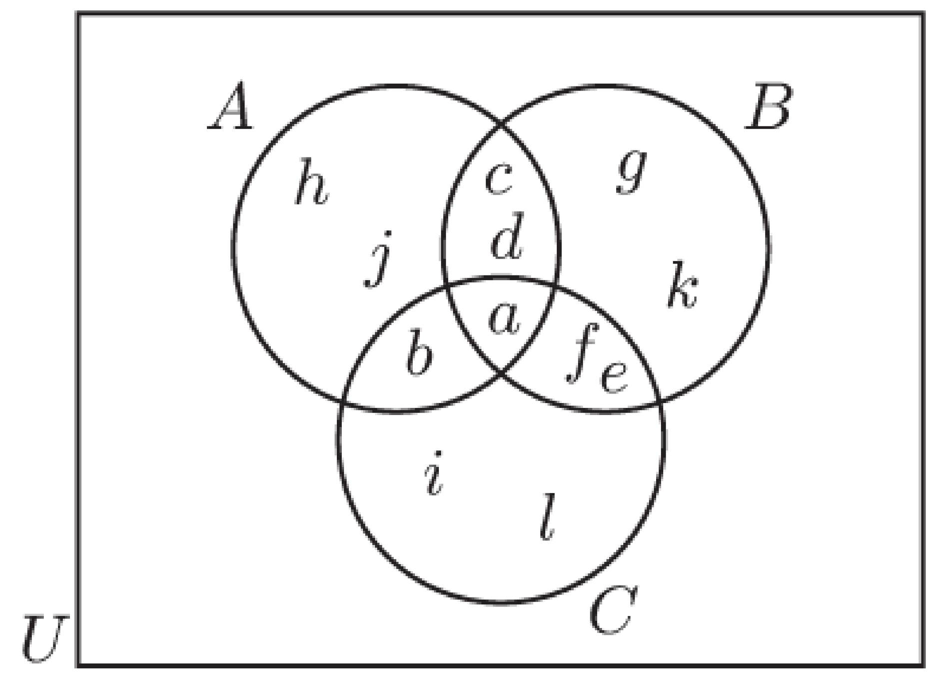
8



List the elements of set:

a A	**b** B	**c** A'
d B'	**e** $A \cap B$	**f** $A \cup B$
g $(A \cup B)'$	**h** $A' \cap B'$	**i** $A' \cup B'$

9 This Venn diagram consists of three overlapping circles A, B, and C.

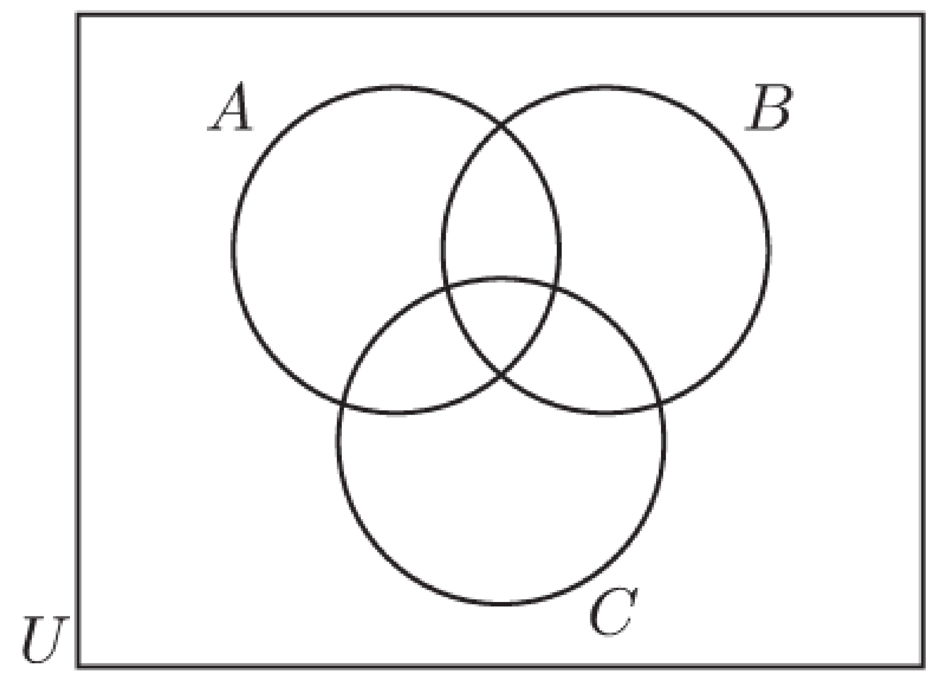


a List the letters in set:

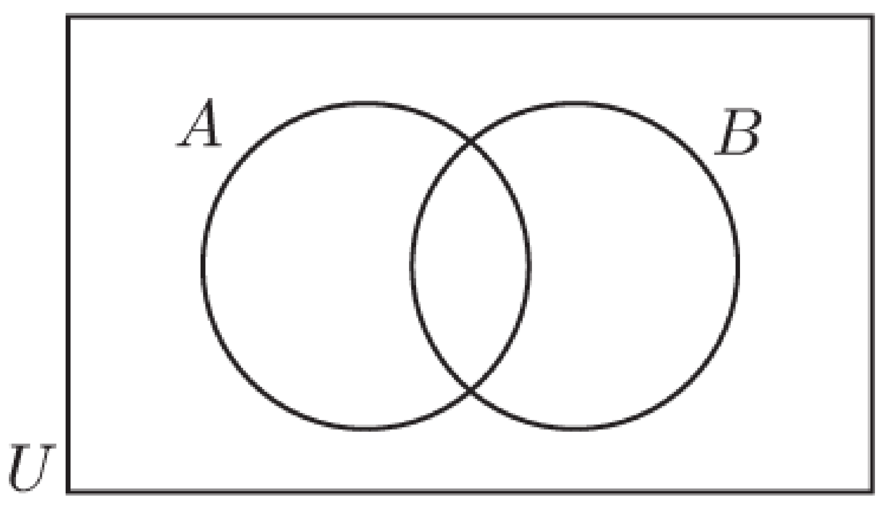
i A	**ii** B	**iii** C
iv $A \cap B$	**v** $A \cup B$	**vi** $B \cap C$
vii $A \cap B \cap C$	**viii** $A \cup B \cup C$	

b Show that
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

10 Suppose $U = \{x \in \mathbb{Z}^+ \mid 40 \leqslant x \leqslant 60\}$,
$A = \{\text{multiples of } 2\}$, $B = \{\text{multiples of } 3\}$, and
$C = \{\text{multiples of } 5\}$.
Display the elements of these sets using a Venn diagram.



11

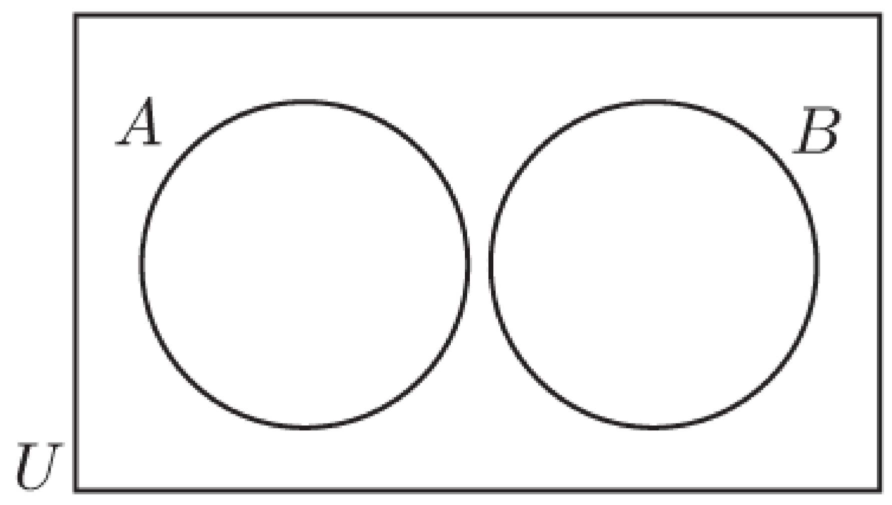


On separate Venn diagrams, shade regions for:

a A	**b** A'
c $A \cap B$	**d** $A \cap B'$
e $A' \cup B$	**f** $A \cup B'$
g $(A \cap B)'$	**h** $(A \cup B)'$

PRINTABLE VENN DIAGRAMS (OVERLAPPING)

12

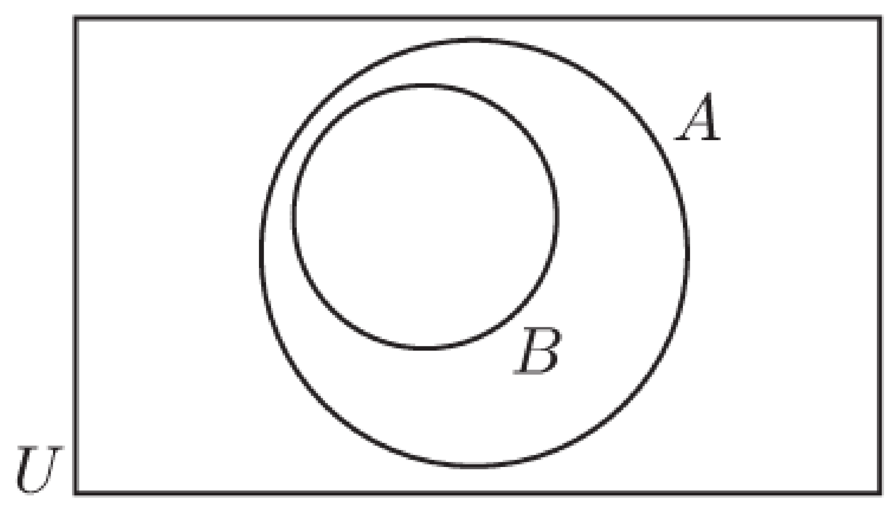


Suppose A and B are two disjoint sets.
Shade on separate Venn diagrams:

a A **b** B **c** A'
d B' **e** $A \cap B$ **f** $A \cup B$
g $A' \cap B$ **h** $A \cup B'$ **i** $(A \cap B)'$

PRINTABLE VENN DIAGRAMS (DISJOINT)

13

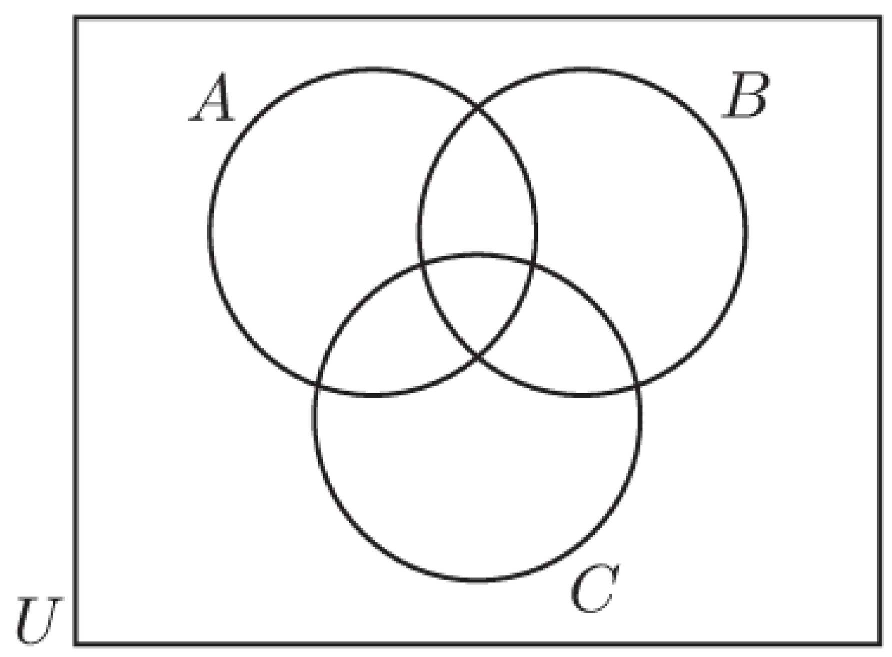


Suppose $B \subseteq A$. Shade on separate Venn diagrams:

a A **b** B **c** A'
d B' **e** $A \cap B$ **f** $A \cup B$
g $A' \cap B$ **h** $A \cup B'$ **i** $(A \cap B)'$

PRINTABLE VENN DIAGRAMS (SUBSET)

14



This Venn diagram consists of three intersecting sets. Shade on separate Venn diagrams:

a A **b** B'
c $B \cap C$ **d** $A \cup B$
e $A \cap B \cap C$ **f** $A \cup B \cup C$
g $(A \cap B \cap C)'$ **h** $(A \cup B) \cap C$
i $(B \cap C) \cup A$ **j** $A' \cap (B \cup C)$

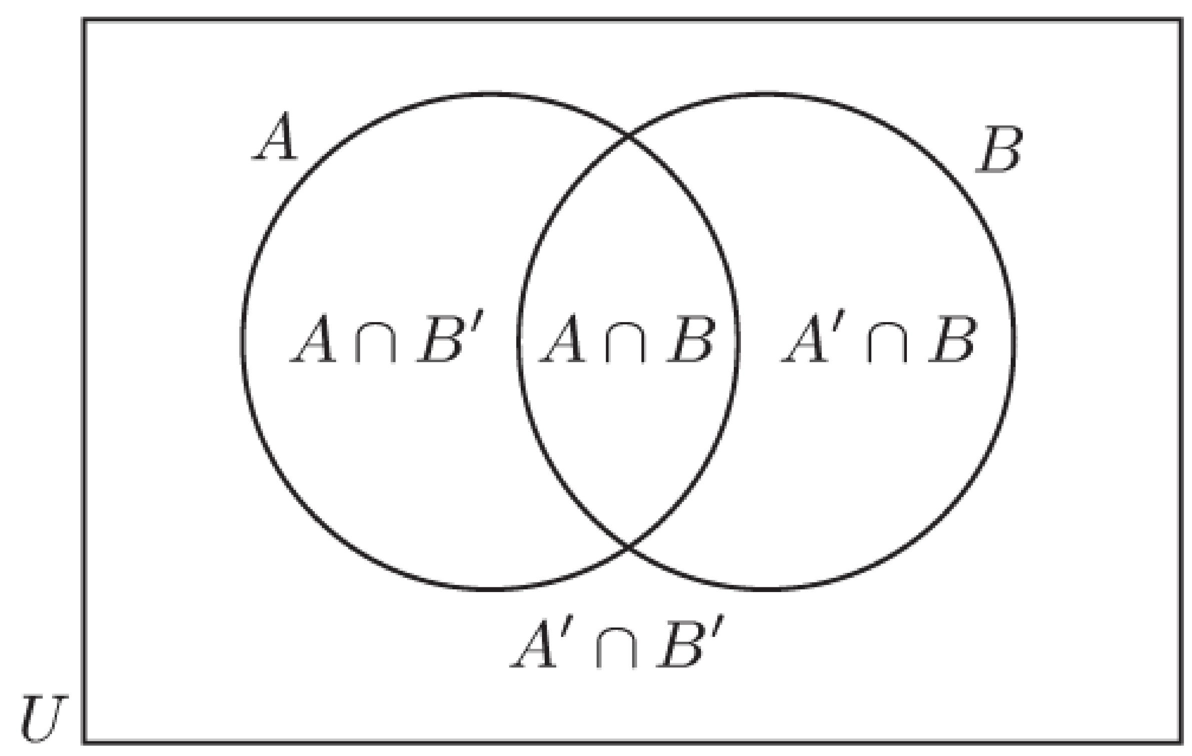
PRINTABLE VENN DIAGRAMS (3 SETS)

Click on the icon to practise shading regions representing various subsets. You can practise with both two and three intersecting sets.

VENN DIAGRAMS

G VENN DIAGRAM REGIONS

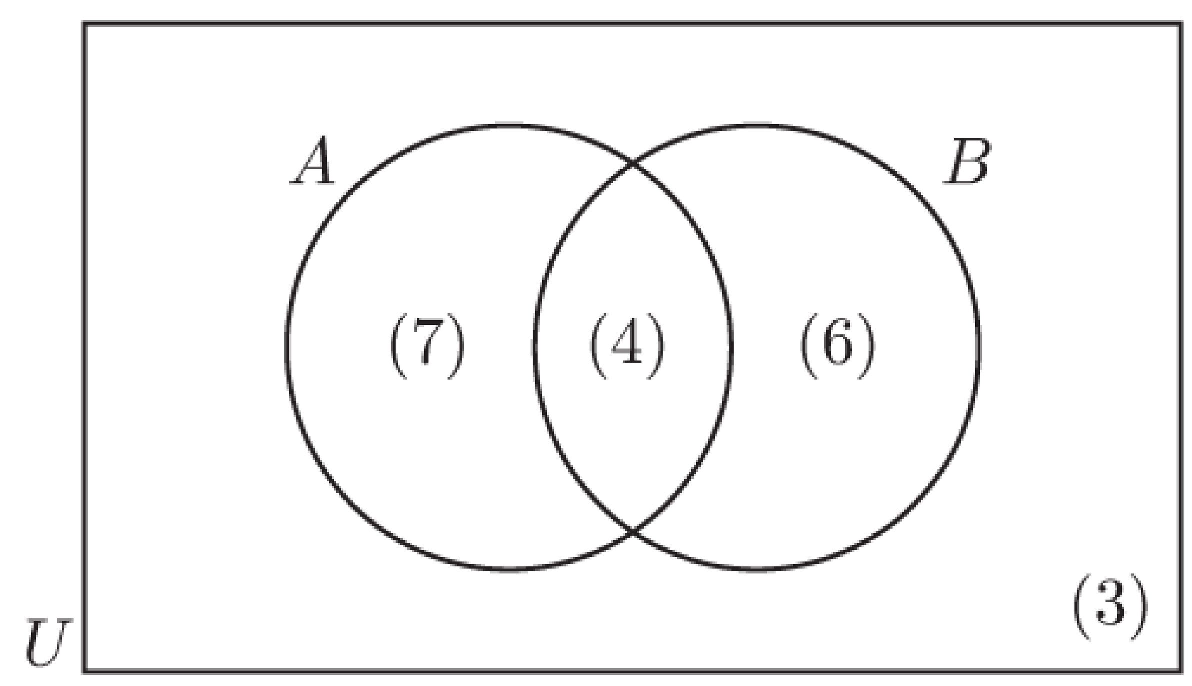
We have seen that there are four distinct regions on a Venn diagram which contains two intersecting sets A and B.



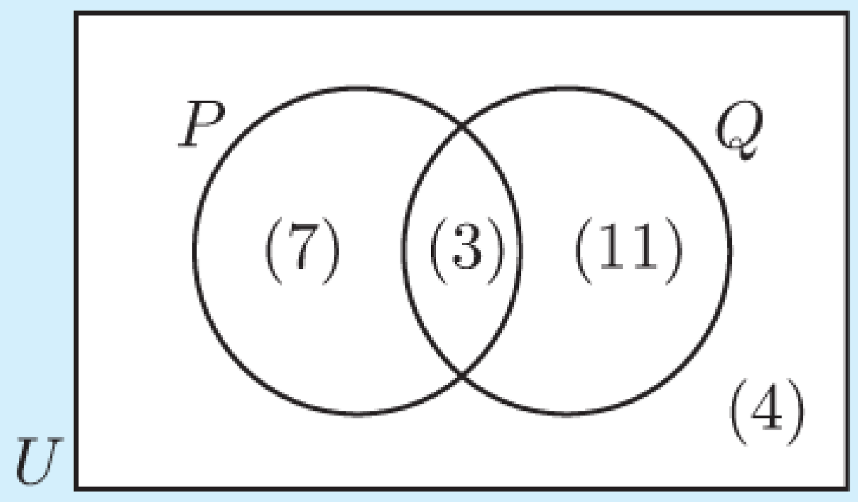
There are many situations where we are only interested in the **number of elements** of U that are in each region. We do not need to show all the elements on the diagram, so instead we write the number of elements in each region in brackets.

For example, the Venn diagram opposite shows there are 4 elements in both sets A and B, and 3 elements in neither set A nor B.

Every element in U belongs in only one region of the Venn diagram. So, in total there are $7+4+6+3=20$ elements in the universal set U.



Example 7 **Self Tutor**



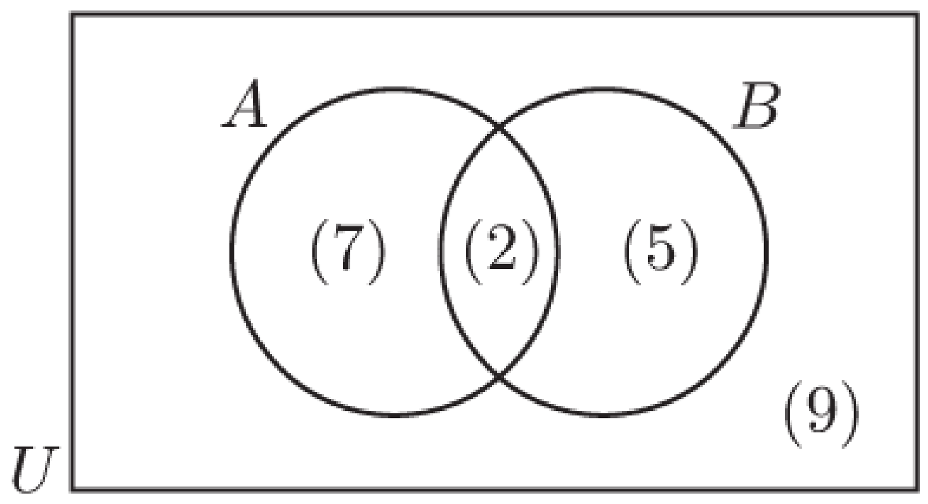
Use the Venn diagram to find the number of elements in:

a P **b** Q'
c $P \cup Q$ **d** P, but not Q
e Q, but not P **f** neither P nor Q.

a $n(P) = 7 + 3 = 10$
b $n(Q') = 7 + 4 = 11$
c $n(P \cup Q) = 7 + 3 + 11 = 21$
d $n(P$, but not $Q) = 7$
e $n(Q$, but not $P) = 11$
f $n($neither P nor $Q) = 4$

EXERCISE 2G

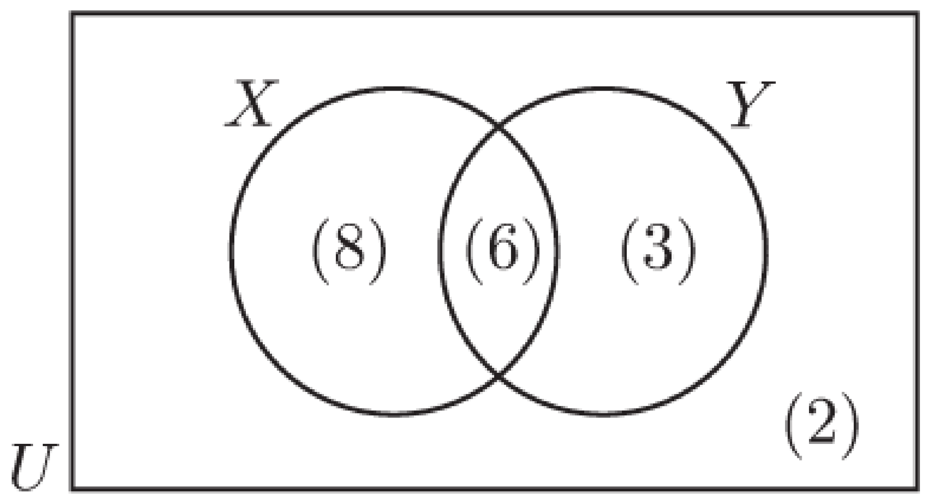
1



Use the Venn diagram to find the number of elements in:

a B **b** A'
c $A \cup B$ **d** A, but not B
e B, but not A **f** neither A nor B.

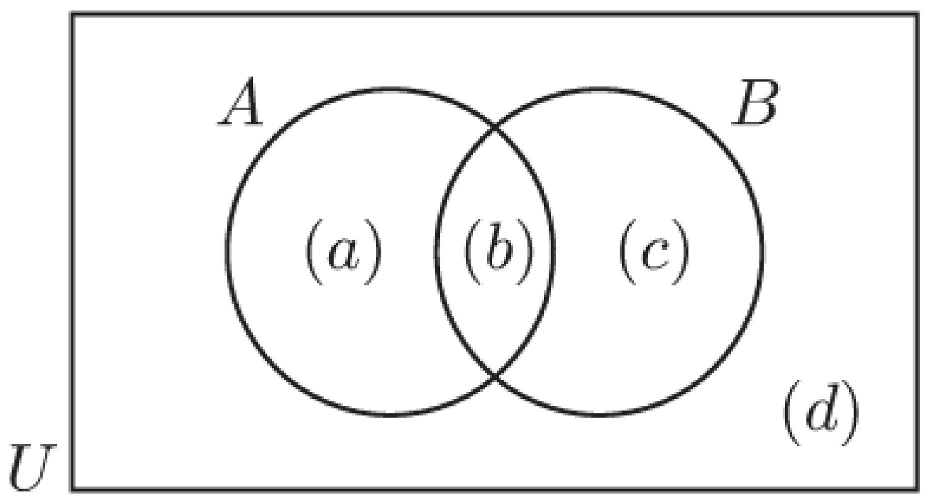
2



Find the number of elements in:

a X' **b** $X \cap Y$
c $X \cup Y$ **d** X, but not Y
e Y, but not X **f** neither X nor Y.

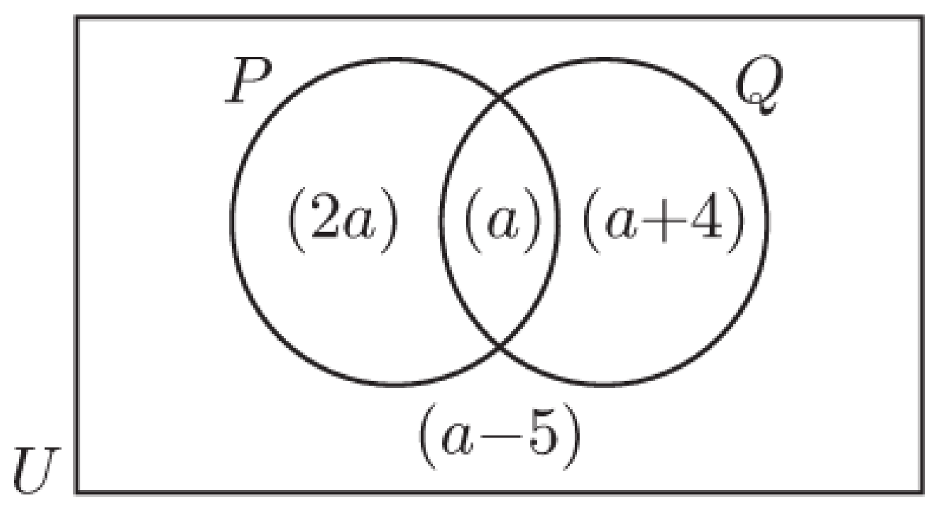
3



Use the Venn diagram to find:

a $n(B)$ **b** $n(A')$ **c** $n(A \cap B)$
d $n(A \cup B)$ **e** $n((A \cap B)')$ **f** $n((A \cup B)')$

4

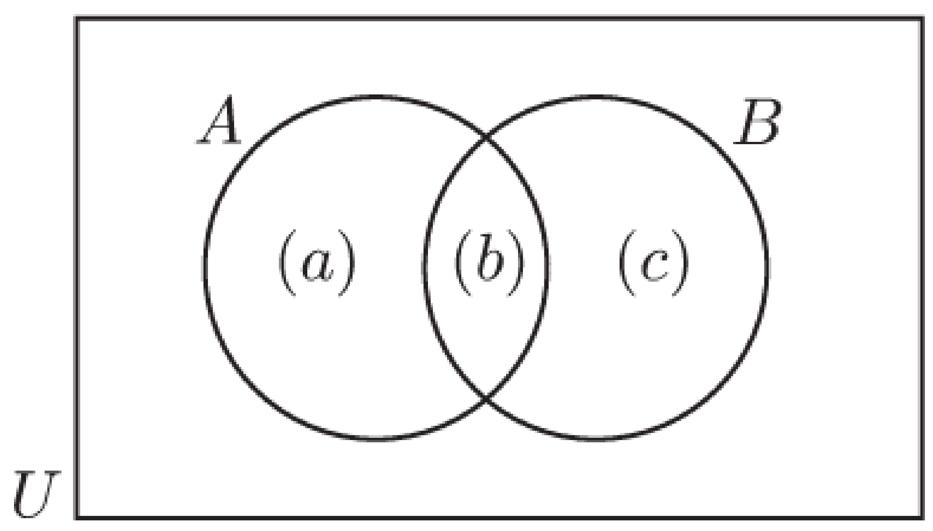


a Use the Venn diagram to find:
  i $n(P \cap Q)$ **ii** $n(P)$
  iii $n(Q)$ **iv** $n(P \cup Q)$
  v $n(Q')$ **vi** $n(U)$

b Find the value of a if:
  i $n(U) = 29$ **ii** $n(U) = 31$

  Comment on your results.

5



Use the Venn diagram to show that:

a $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
b $n(A \cap B') = n(A) - n(A \cap B)$

6 Suppose A and B are disjoint sets. Use a Venn diagram to show that $n(A \cup B) = n(A) + n(B)$.