Venn diagrams

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The presentation has 3 parts:

- Applications of Venn diagrams with 2 sets;
- Marking regions on Venn diagrams with 3 sets;
- Applications of Venn diagrams with 2 sets.

We will do a short test on Tuesday on a combination of the above.

Applications of Venn diagrams with 2 sets

Example 1

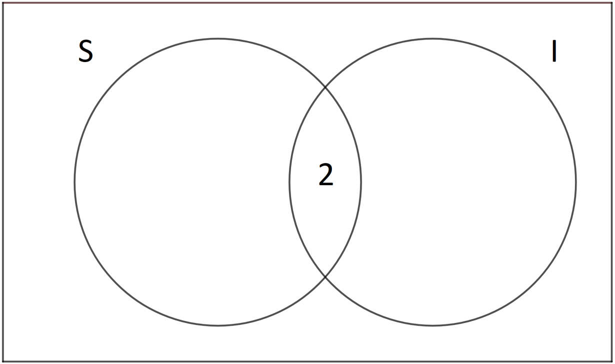
There are 18 students in class. 9 of them speak Spanish, 6 speak Italian, 2 speak both Spanish and Italian.

Represent this information on a Venn diagram and find number of students (i) who do not speak any of the mentioned languages (ii) exactly one of the two languages.

Draw a Venn diagram with two sets.

Draw a Venn diagram with two sets. Start, if possible, by putting numbers that correspond to **one** region. For example 9 (number of students who speak Spanish) corresponds to two regions and we don't know how to divide this number between these two regions. So we start with 2 (number of students who speak both Italian and Spanish):

Draw a Venn diagram with two sets. Start, if possible, by putting numbers that correspond to **one** region. For example 9 (number of students who speak Spanish) corresponds to two regions and we don't know how to divide this number between these two regions. So we start with 2 (number of students who speak both Italian and Spanish):



Now we can figure out how many students speak Spanish only. $9 - 2 = 7$, so we put 7 in the appropriate region.

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Similarly for those who speak Italian only we have $6 - 2 = 4$, so we put 4 into appropriate region.

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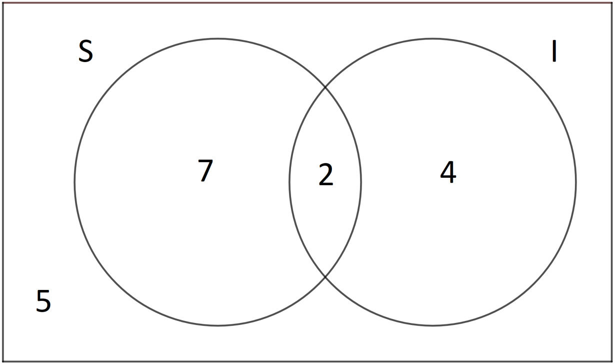
Similarly for those who speak Italian only we have $6 - 2 = 4$, so we put 4 into appropriate region.
Now we have a total of 13 students. We want to have 18, so we put 5 in the appropriate region.

Now we can figure out how many students speak Spanish only. $9 - 2 = 7$, so we put 7 in the appropriate region.
Similarly for those who speak Italian only we have $6 - 2 = 4$, so we put 4 into appropriate region.
Now we have a total of 13 students. We want to have 18, so we put 5 in the appropriate region.



Now to answer the questions:

i. 5 students do not speak any of the mentioned languages,

ii. 11 students speak exactly one language.

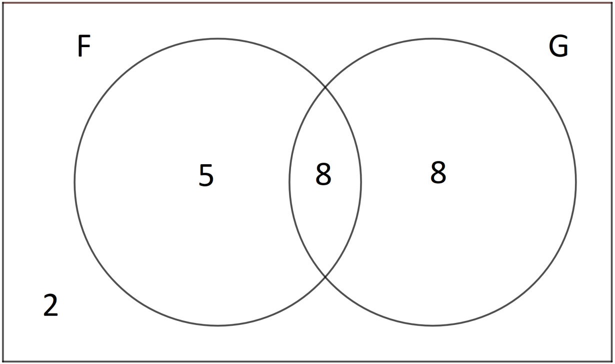
Question 1

There are 23 students in class. 13 of them speak French, 16 speak German, 8 speak both French and German.

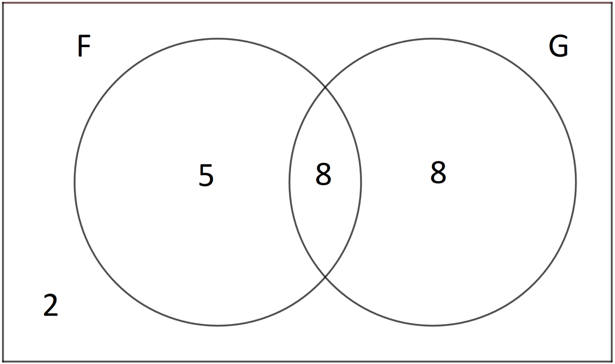
Represent this information on a Venn diagram and find number of students (i) who do not speak any of the mentioned languages (ii) exactly one of the two languages.

Solution:

Solution:



Solution:



i. 2 students do not speak any of the mentioned languages,

ii. 13 students speak exactly one language.

Example 2

There are 13 students in class. 8 of them speak Spanish, 4 speak only Italian, 3 speak both Spanish and Italian.

Represent this information on a Venn diagram and find number of students (i) who do not speak any of the mentioned languages (ii) who speak Italian.

We start by drawing a Venn diagram for two sets.

We start by drawing a Venn diagram for two sets. We can put two numbers in. 3 in the middle. And we can also put the 4 in. This is because it says "Italian *only*", so these are the students who speak Italian, but do not speak Spanish.

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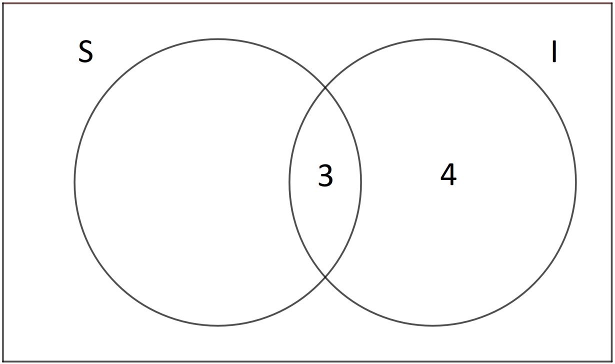
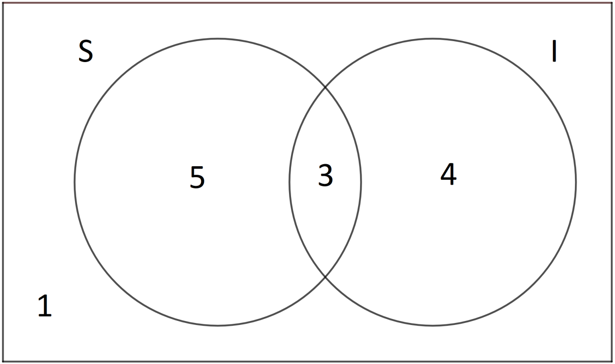


Now we have $8 - 3 = 5$, so 5 students study Spanish only. We can put this information on the diagram.
We have 12 students. We need 13, $13 - 12 = 1$, so we have:

Now we have $8 - 3 = 5$, so 5 students study Spanish only. We can put this information on the diagram.

We have 12 students. We need 13, $13 - 12 = 1$, so we have:



Now to answer the questions:

i. 1 student does not speak any of the mentioned languages,
ii. 7 students speak Italian.

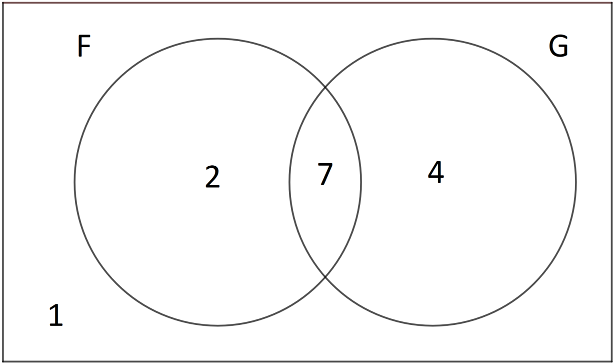
Question 2

There are 14 students in class. 11 of them speak German, 2 speak only French, 7 speak both German and French.

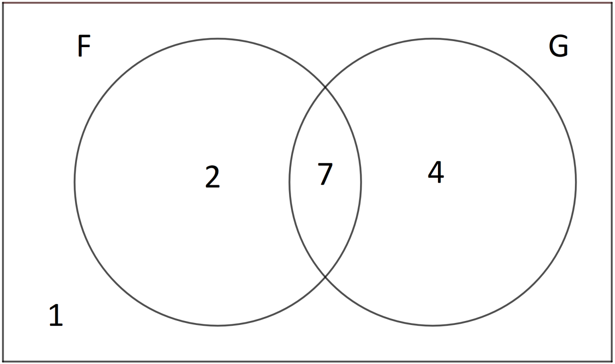
Represent this information on a Venn diagram and find number of students (i) who do not speak any of the mentioned languages (ii) who speak exactly one of the two languages.

Solution:

Solution:



Solution:



i. 1 student does not speak any of the mentioned languages,

ii. 6 students speak exactly one language.

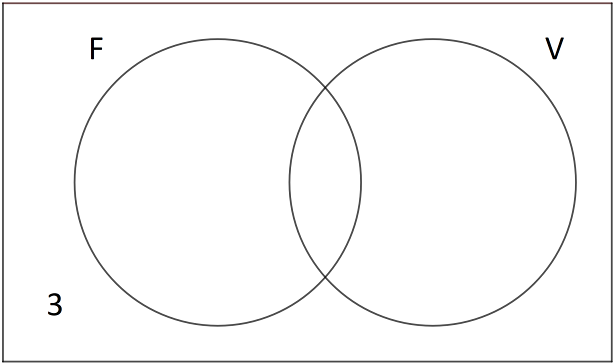
Example 3

There are 20 students in class. 11 of them like football, 12 like volleyball, 17 like at least one of the two sports. Represent this information on a

Venn diagram and find number of students (i) who like both sports (ii) who like football only.

We start by drawing a Venn diagram for two sets.

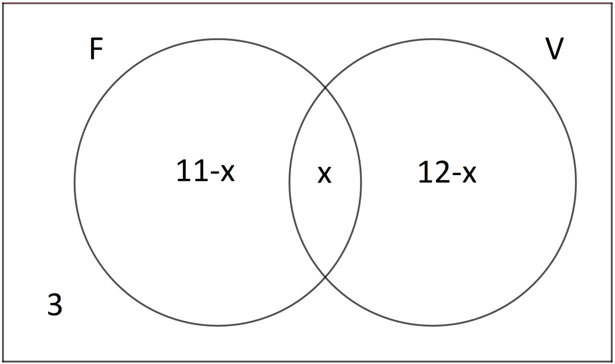
We start by drawing a Venn diagram for two sets. Now we can start with those who don't like any of the two sports. There are 20 students, 17 like at least one, so $20 - 17 = 3$, 3 students don't like any. Let's represent this on the diagram.



Now we are stuck. So let's put x in the middle. x will represent the number of students who like both sports.

Now we are stuck. So let's put x in the middle. x will represent the number of students who like both sports. Now the number of students who like football only is $11 - x$ and the number of students who like volleyball only is $12 - x$. So we get the following diagram.

Now we are stuck. So let's put x in the middle. x will represent the number of students who like both sports. Now the number of students who like football only is $11 - x$ and the number of students who like volleyball only is $12 - x$. So we get the following diagram.



We can form an equation

$$(11 - x) + x + (12 - x) = 17$$

Because 17 students like at least one of the sports.

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$$(11 - x) + x + (12 - x) = 17$$

Because 17 students like at least one of the sports. Solving this equation gives:

$$23 - x = 17$$
$$x = 6$$

So there are 6 students who like both volleyball and football.

We can form an equation

$$(11 - x) + x + (12 - x) = 17$$

Because 17 students like at least one of the sports. Solving this equation gives:

$$23 - x = 17$$
$$x = 6$$

So there are 6 students who like both volleyball and football. Now the number of students who like football only is $11 - x = 11 - 6 = 5$.

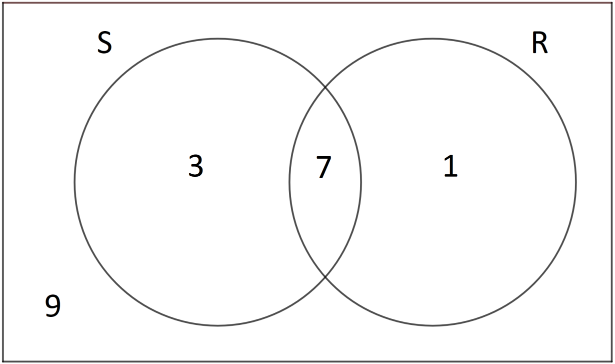
Question 3

There are 20 students in class. 10 of them like swimming, 8 like running, 11 like at least one of the two activities.

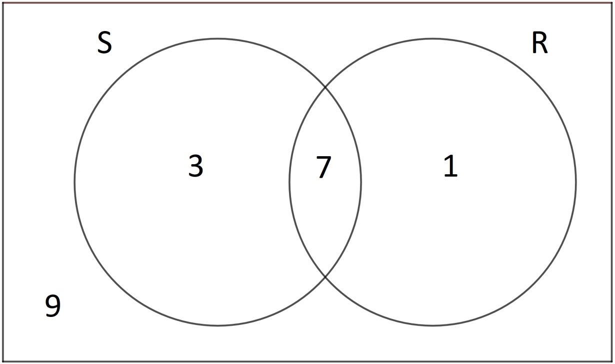
Represent this information on a Venn diagram and find number of students (i) who like both activities (ii) who like running, but don't like swimming.

Solution:

Solution:



Solution:



i. 7 students like both activities,

ii. 1 student likes swimming but not running.

Example 4

There are 25 students in class. 13 of them have dark hair, 6 have blue eyes, 9 have neither dark hair nor blue eyes.

Represent this information on a Venn diagram and find number of students (i) who have dark hair and blue eyes (ii) who have dark hair but do not have blue eyes.

We start by drawing a Venn diagram for two sets.

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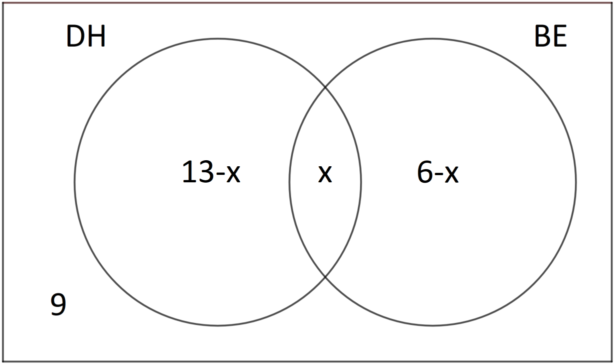
We start by drawing a Venn diagram for two sets. Let x represent the number of students with dark hair and blue eyes. Now the number of students who have dark hair but don't have blue eyes is $13 - x$ and the number of students who have blue eyes but don't have dark hair is $6 - x$. So we get the following diagram.

We start by drawing a Venn diagram for two sets. Let x represent the number of students with dark hair and blue eyes. Now the number of students who have dark hair but don't have blue eyes is $13 - x$ and the number of students who have blue eyes but don't have dark hair is $6 - x$. So we get the following diagram.



We can form an equation

$$(13 - x) + x + (6 - x) + 9 = 25$$

We've counted all of the students and there are 25 of them. We solve the equation:

$$28 - x = 25$$
$$x = 3$$

So there are 3 students with dark hair and blue eyes.

We can form an equation

$$(13 - x) + x + (6 - x) + 9 = 25$$

We've counted all of the students and there are 25 of them. We solve the equation:

$$28 - x = 25$$
$$x = 3$$

So there are 3 students with dark hair and blue eyes.
So the number of dark haired students who who do not have blue eyes is $13 - x = 13 - 3 = 10$.

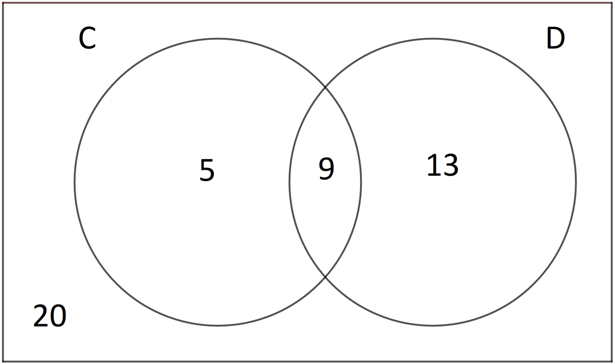
Question 4

There are 47 students in class. 14 of them have a cat, 22 have a dog, 20 have no pets.

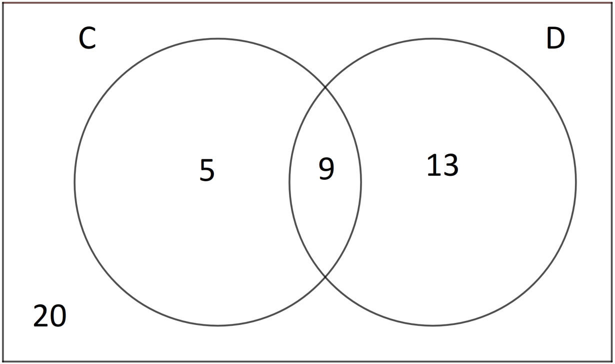
Represent this information on a Venn diagram and find number of students (i) who have a cat and a dog (ii) who have a cat, but no dog.

Solution:

Solution:



Solution:



i. 9 students have a cat and a dog,

ii. 5 students have a cat only.

Marking regions on Venn diagrams with 3 sets

The presentation will introduce two ways of representing appropriate regions on Venn diagrams with 3 sets. The fist one is done by shading regions step by step (it may require some erasing as well). In the second method we do most of the work in our heads.

Example 1

Represent the set $(A \cap B') \cup C$ on a Venn diagram.

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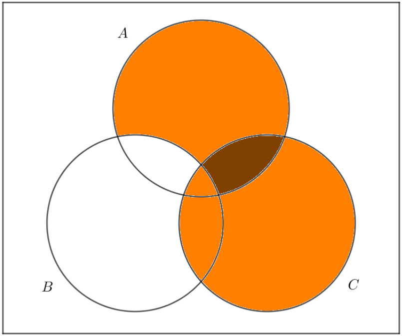
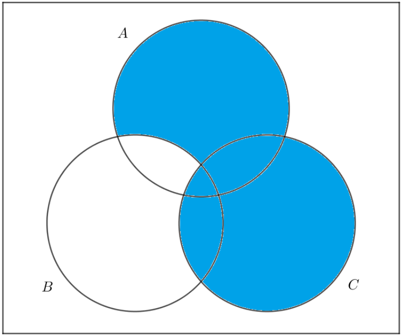
Represent the set $(A \cap B') \cup C$ on a Venn diagram.
We can start by shading $A \cap B'$ and C. We get the following diagram:



The darker colour means that this region has been shaded twice.

Example 1

Now we want the union ∪ of these two sets, this means that we take everything that has been shaded at least once, so the answer will be:



Example 2

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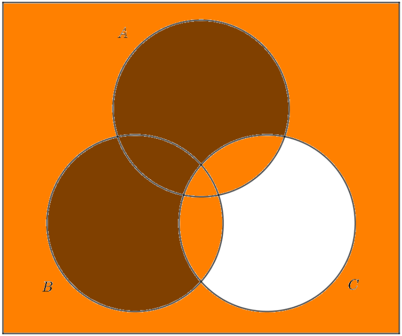
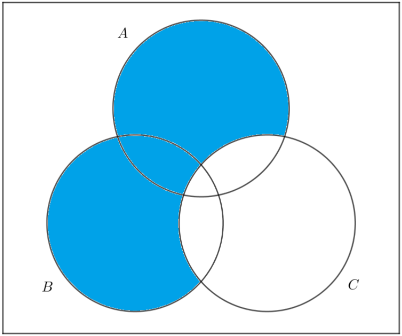


Example 2

Now we want the intersection ∩ of these two sets, so we take everything that has been shaded twice, so the answer will be:



Example 3

Represent the set $B \cap (A \cap C')$ on a Venn diagram.

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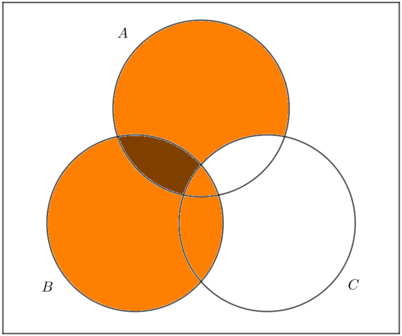
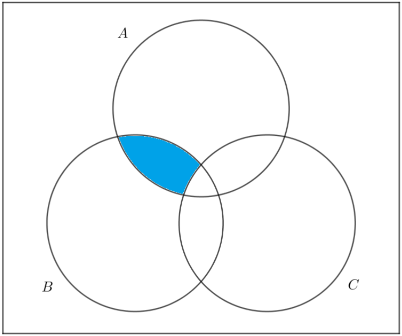
Represent the set $B \cap (A \cap C')$ on a Venn diagram.

We can start by shading B and $A \cap C'$. We get the following diagram:



Example 3

Now we want the intersection ∩ of these two sets, so the answer will be:



Example 4

Represent the set $B \cup (A' \cap C')$ on a Venn diagram.

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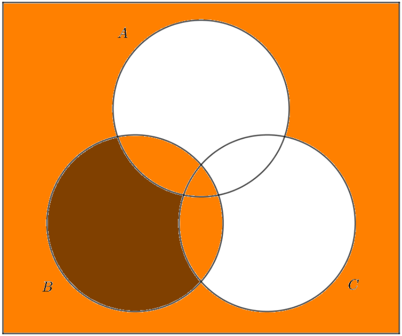
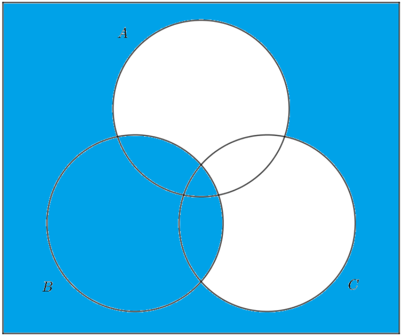
Represent the set $B \cup (A' \cap C')$ on a Venn diagram.

We can start by shading B and $A' \cap C'$. We get the following diagram:



Example 4

Now we want the union ∪ of these two sets, so the answer will be:



Next slides will show a more direct approach.

Example 5

Mark on the diagram the set corresponding to $(A \cap B') \cup C$.

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Let's make some observations:

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Let's make some observations:

- $(A \cap B')$ is everything in A and not in B.

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- C is of course everything in C.

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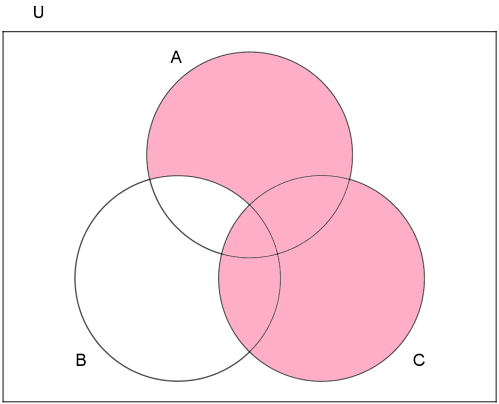
Let's make some observations:

- $(A \cap B')$ is everything in A and not in B.
- C is of course everything in C.
- Finally we have $\cup$ between these, so we want elements that are in at least one of the two sets.

Example 5

Mark on the diagram the set corresponding to $(A \cap B') \cup C$. Answer:



Example 6

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Mark on the diagram the set corresponding to $(A \cup B)' \cap C'$.

Let's make some observations:

- $(A \cup B)'$ is everything outside of A and B. Using symbolic logic we could read this as: *it is not true that it is in A or in B*.

Example 6

Mark on the diagram the set corresponding to $(A \cup B)' \cap C'$.

Let's make some observations:

- $(A \cup B)'$ is everything outside of A and B. Using symbolic logic we could read this as: *it is not true that it is in A or in B.*
- C' is everything outside of C. In logic this is *not in C.*

Example 6

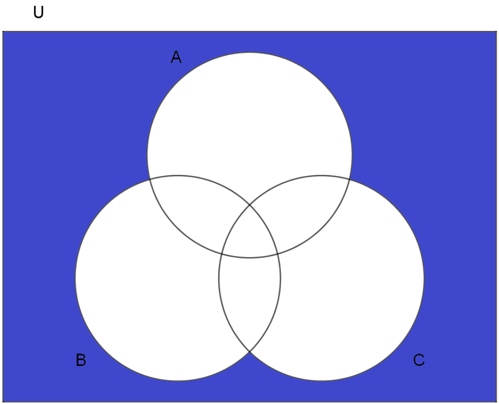
Mark on the diagram the set corresponding to $(A \cup B)' \cap C'$.

Let's make some observations:

- $(A \cup B)'$ is everything outside of A and B. Using symbolic logic we could read this as: *it is not true that it is in A or in B*.
- C' is everything outside of C. In logic this is *not in C*.
- Finally we have $\cap$ between these, so we want elements that are in both sets. Using symbolic logic we have *it is not true that it is in A or in B* ***and*** *it is not in C*.

Example 6

Mark on the diagram the set corresponding to $(A \cup B)' \cap C'$. Answer:



Example 7

Mark on the diagram the set corresponding to $(A \cap B) \cup C'$.

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Observations:

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Observations:

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Mark on the diagram the set corresponding to $(A \cap B) \cup C'$.

Observations:

- $(A \cap B)$ is everything that is both in A and in B.
- C' is again everything outside of C.

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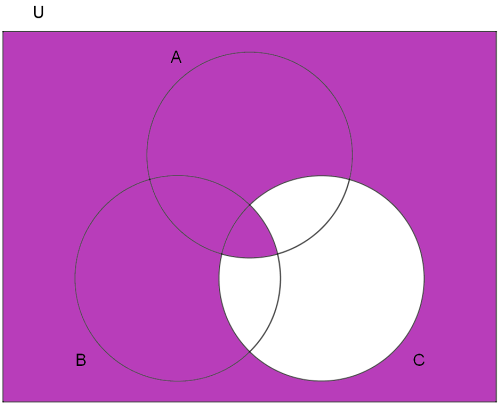
Mark on the diagram the set corresponding to $(A \cap B) \cup C'$.

Observations:

- $(A \cap B)$ is everything that is both in A and in B.
- C' is again everything outside of C.
- Finally we have $\cup$ between these, so we want elements that are in at least one of the two sets. Using logic we have *it is both in A and B* ***or*** *it is not in C*.

Example 7

Mark on the diagram the set corresponding to $(A \cap B) \cup C'$ Answer:



Example 8

Mark on the diagram the set corresponding to $(A \cup B) \cap (C \cap A)$.

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Let's make some observations:

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Mark on the diagram the set corresponding to $(A \cup B) \cap (C \cap A)$.

Let's make some observations:

- $(A \cup B)$ is everything in A or in B.
- $(C \cap A)$ is everything in C and in A.

Example 8

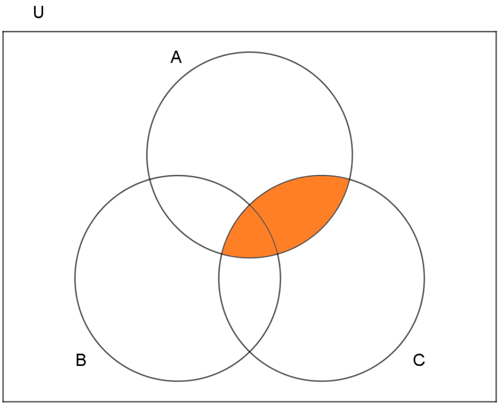
Mark on the diagram the set corresponding to $(A \cup B) \cap (C \cap A)$.

Let's make some observations:

- $(A \cup B)$ is everything in A or in B.
- $(C \cap A)$ is everything in C and in A.
- $(A \cup B) \cap (C \cap A)$ is everything in both of the above so *in A or in B* ***and*** *in C and in A*.

Example 8

Mark on the diagram the set corresponding to $(A \cup B) \cap (C \cap A)$. Answer:



Example 9

Mark on the diagram the set corresponding to $(A' \cap B') \cap (B \cup C)$.

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Mark on the diagram the set corresponding to $(A' \cap B') \cap (B \cup C)$.

Let's make some observations:

- $(A' \cap B')$ is everything that is both outside of A and outside of B.

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Mark on the diagram the set corresponding to $(A' \cap B') \cap (B \cup C)$.

Let's make some observations:

- $(A' \cap B')$ is everything that is both outside of A and outside of B.
- $(B \cup C)$ is everything in B or in C.

Example 9

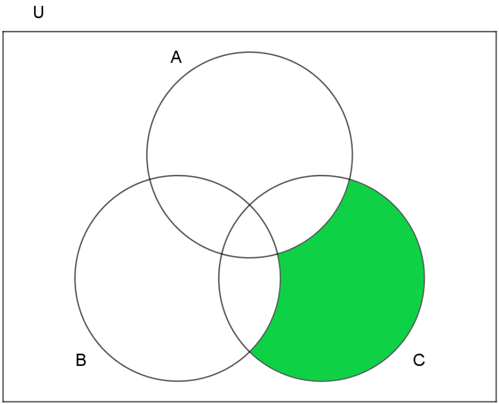
Mark on the diagram the set corresponding to $(A' \cap B') \cap (B \cup C)$.

Let's make some observations:

- $(A' \cap B')$ is everything that is both outside of A and outside of B.
- $(B \cup C)$ is everything in B or in C.
- $(A' \cap B') \cap (B \cup C)$ is everything in both of the above so *not in A and not in B* ***and*** *in B or in C*.

Example 9

Mark on the diagram the set corresponding to $(A' \cap B') \cap (B \cup C)$. Answer:



Applications of Venn diagrams with 3 sets.

Example 1

50 people were asked what they had for breakfast this morning.
27 people had eggs
24 had cheese
14 had bacon
13 had bacon and eggs
7 had eggs and cheese
3 had bacon and cheese
5 had none of the three products.

Example 1

50 people were asked what they had for breakfast this morning.
27 people had eggs
24 had cheese
14 had bacon
13 had bacon and eggs
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5 had none of the three products.

Find the number of people who had (i) all three products (ii) exactly one of the three for breakfast.

Example 1

We start by putting 5 outside of the regions and x in the intersection of all three sets.

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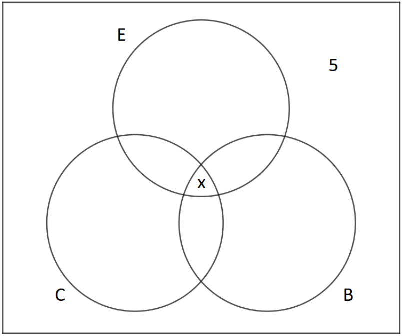

Example 1

Now we can put $13 - x$, $7 - x$ and $3 - x$ in appropriate regions:

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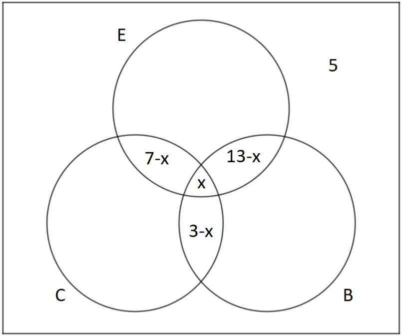


Example 1

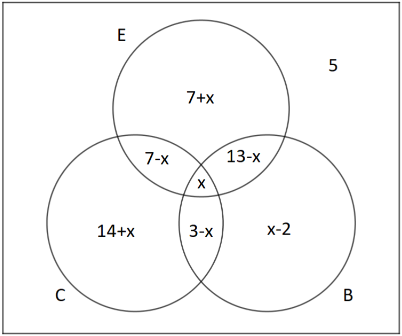
Now the number of those who ate eggs only is
$27 - (13 - x) - x - (7 - x) = 7 + x$.

Example 1

Now the number of those who ate eggs only is $27 - (13 - x) - x - (7 - x) = 7 + x$. Similarly for those who are cheese only we have $24 - (7 - x) - x - (3 - x) = 14 + x$, and for bacon $14 - (13 - x) - x - (3 - x) = x - 2$.

Example 1

Now the number of those who ate eggs only is $27-(13-x)-x-(7-x)=7+x$. Similarly for those who are cheese only we have $24-(7-x)-x-(3-x)=14+x$, and for bacon $14-(13-x)-x-(3-x)=x-2$. So we can represent this on the diagram:



Example 1

We can now form an equation, since the total number of people surveyed was 50, we have:

$$50 = x + (7 - x) + (3 - x) + (13 - x) + (7 + x) + (14 + x) + (x - 2) + 5$$

Example 1

We can now form an equation, since the total number of people surveyed was 50, we have:

$$50 = x + (7 - x) + (3 - x) + (13 - x) + (7 + x) + (14 + x) + (x - 2) + 5$$

which gives:

$$50 = x + 47$$

Example 1

We can now form an equation, since the total number of people surveyed was 50, we have:

$$50 = x + (7 - x) + (3 - x) + (13 - x) + (7 + x) + (14 + x) + (x - 2) + 5$$

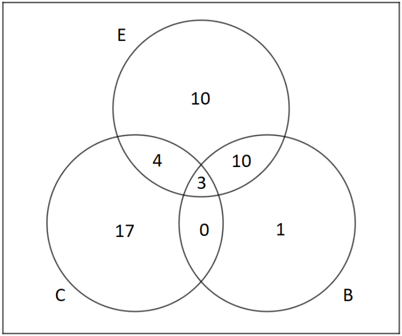
which gives:

$$50 = x + 47$$

So $x = 3$

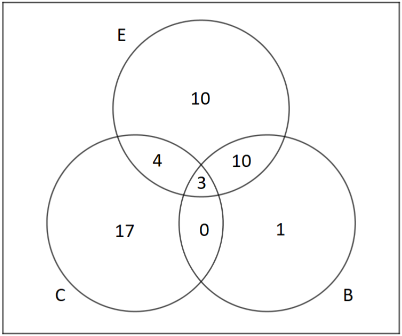
Example 1

We can now update the diagram:



Example 1

We can now update the diagram:



So 3 people had all three products and $17 + 10 + 1 = 28$ had exactly one of the 3 products.

Example 2

100 people were asked what they had for breakfast this morning.
60 people had eggs
51 had cheese
32 had bacon
31 had bacon and eggs
19 had eggs and cheese
11 had bacon and cheese
8 had none of the three products.

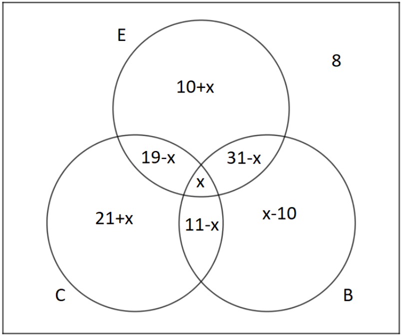
Example 2

100 people were asked what they had for breakfast this morning.
60 people had eggs
51 had cheese
32 had bacon
31 had bacon and eggs
19 had eggs and cheese
11 had bacon and cheese
8 had none of the three products.

Find the number of people who had (i) all three products (ii) exactly one of the three for breakfast.

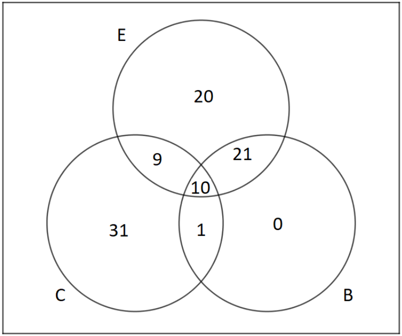
Example 2

The diagram with x as the variable representing the number of people who had all three products:



Example 2

We solve for x and find out the $x = 10$, so the diagram becomes:



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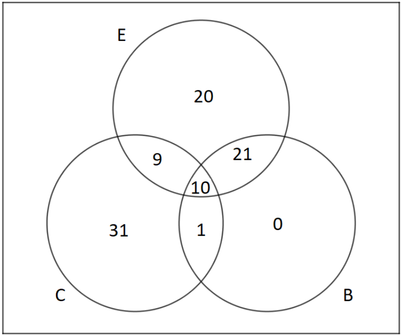

10 people had all three products and 51 had exactly one of the 3 products.

The short test will include an example similar to the ones above.