1. A closed box has a square base of side x and height h.

(a) Write down an expression for the volume, V, of the box.

(1)

(b) Write down an expression for the total surface area, A, of the box.

(1)

The volume of the box is 1000 cm^3

(c) Express h in terms of x.

(2)

(d) Hence show that $A = 4000x^{-1} + 2x^2$.

(2)

(e) Find $\frac{dA}{dx}$.

(2)

(f) Calculate the value of x that gives a minimum surface area.

(4)

(g) Find the surface area for this value of x.

(3)
(Total 15 marks)

2. A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, *h* represents the height of the can in cm, and *x* represents the radius of the base of the can in cm.

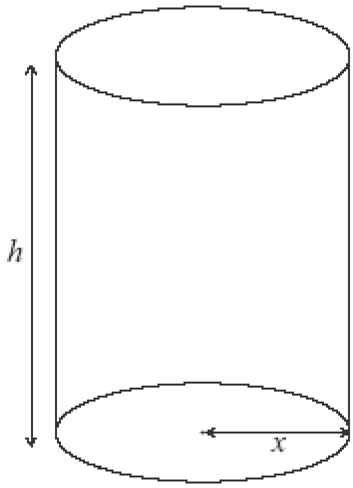


diagram not to scale

The volume of the dog food cans is 600 cm^3.

(a) Show that $h = \frac{600}{\pi x^2}$.

(2)

(b) (i) Find an expression for the curved surface area of the can, in terms of *x*. Simplify your answer.

(ii) Hence write down an expression for *A*, the total surface area of the can, in terms of *x*.

(4)

(c) Differentiate *A* in terms of *x*.

(3)

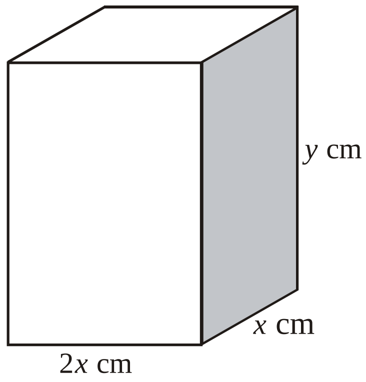
(d) Find the value of *x* that makes *A* a minimum.

(3)

(e) Calculate the minimum total surface area of the dog food can.

(2)
(Total 14 marks)

3. A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2.



(a) Show that $4x^2 + 6xy = 300$.

(2)

(b) Find an expression for y in terms of x.

(2)

(c) Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$.

(2)

(d) Find $\frac{dV}{dx}$.

(2)

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(ii) Calculate the maximum volume.

(5)
(Total 13 marks)

4. A farmer has a rectangular enclosure with a straight hedge running down one side. The area of the enclosure is 162 m^2. He encloses this area using x metres of the hedge on one side as shown on the diagram below.

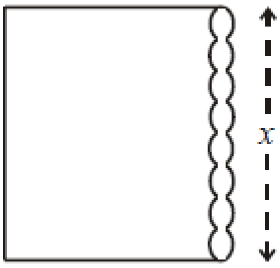


diagram not to scale

(a) If he uses y metres of fencing to complete the enclosure,
show that $y = x + \dfrac{324}{x}$.

(3)

The farmer wishes to use the least amount of fencing.

(b) Find $\dfrac{dy}{dx}$.

(3)

(c) Find the value of x which makes y a minimum.

(3)

(d) Calculate this minimum value of y.

(2)

(e) Using $y = x + \dfrac{324}{x}$ find the values of a and b in the following table.

x	6	9	12	18	24	27	36
y	60	45	39	a	37.5	b	45

(2)

(f) Draw an accurate graph of this function using a horizontal scale starting at 0 and taking 2 cm to represent 10 metres, and a vertical scale **starting at 30** with 4 cm to represent 10 metres.

(5)

(g) Write down the values of x for which y increases.

(2)
(Total 20 marks)