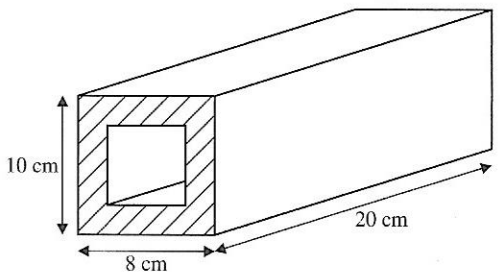
5 The perimeter of a rectangle is 34 cm. Given that the diagonal is of length 13 cm, and that the width is x cm, derive the equation $x^2 - 17x + 60 = 0$. Hence find the dimensions of the rectangle.

6 A garden is in the shape of a rectangle, 20 metres by 8 metres. Around the outside is a border of uniform width, and in the middle is a square pond. The width of the border is the same as the width of the pond. The size of the area which is not occupied by either border or pond is 124 m^2. Letting the width of the border be x m, derive the equation $3x^2 - 56x + 36 = 0$. Solve this equation to find the value of x.

7 A metal sleeve of length 20 cm has rectangular cross-section 10 cm by 8 cm. The metal has uniform thickness, x cm, along the sleeve, and the total volume of metal in the sleeve is 495 cm^3.



Derive the equation $16x^2 - 144x + 99 = 0$, and solve it to find the value of x.

8 A strand of wire of length 32 cm is cut into two pieces. One piece is bent to form a rectangle of width x cm and length $(x + 2)$ cm, and the other piece is bent to form a square.

a) Show that the square has sides of length $(7 - x)$ cm.

b) Given that the total of the areas enclosed by both the rectangle and the square is 31 cm^2, form an equation for x and solve it to find the value of x.

9 A train usually covers a journey of 240 km at a steady speed of v km h^{-1}. One day, due to adverse weather conditions, it reduces its speed by 40 km h^{-1} and the journey takes one hour longer.

Derive the equation $v^2 - 40v - 9600 = 0$, and solve it to find the value of v.

10 As part of his training an athlete usually runs 80 km at a steady speed of v km h^{-1}. One day he decides to reduce his speed by 2.5 km h^{-1} and his run takes him an extra 2 h 40 min.

Derive the equation $\frac{80}{v} + \frac{8}{3} = \frac{160}{2v - 5}$, and solve it to find the value of v.

Exercise 1E

1 Use the method of completing the square to express the solutions to each of the following quadratic equations in the form $a \pm b\sqrt{n}$, where a and b are rational, and n is an integer.

a) $x^2 - 4x - 1 = 0$ **b)** $x^2 + 6x + 2 = 0$ **c)** $x^2 - 2x - 1 = 0$

d) $x^2 - 8x - 3 = 0$ **e)** $x^2 + x - 1 = 0$ **f)** $x^2 + 3x + 1 = 0$

g) $x^2 - 5x - 2 = 0$ **h)** $x^2 - x - 3 = 0$ **i)** $x^2 + 5x + 1 = 0$

j) $x^2 + 12x + 5 = 0$ **k)** $x^2 - 9x + 10 = 0$ **l)** $x^2 - \frac{1}{2}x - \frac{1}{4} = 0$

2 Use the method of completing the square to solve each of the following quadratic equations, expressing your solutions in the form $a \pm b\sqrt{n}$, where a, b and n are rational.

a) $2x^2 - 3x - 3 = 0$ **b)** $3x^2 - 6x + 1 = 0$ **c)** $4x^2 + 4x - 5 = 0$

d) $3x^2 + 5x - 1 = 0$ **e)** $5x^2 + x - 3 = 0$ **f)** $2x^2 - 3x - 1 = 0$

g) $2x^2 - x - 2 = 0$ **h)** $4x^2 + 3x - 2 = 0$ **i)** $7x^2 - 14x + 5 = 0$

j) $6x^2 + 4x - 3 = 0$ **k)** $5x^2 - 20x + 17 = 0$ **l)** $2x^2 + 18x + 21 = 0$

3 Express $x^2 + 4x + 7$ in the form $(x + p)^2 + q$. Hence show that the equation $x^2 + 4x + 7 = 0$ has no real root.

4 Express $5x^2 - 30x + 47$ in the form $a(x + p)^2 + q$. Hence show that the equation $5x^2 - 30x + 47 = 0$ has no real root.

5 Show that the equation $\dfrac{x+2}{x-3} = \dfrac{x+4}{2x+3}$ has no real root.

***6** Given that the equation $x^2 + ax = b$, where a and b are real numbers, has a unique solution, prove that $a^2 + 4b = 0$.

Multiplying throughout by 16 gives

$$x^2 + (100 - x)^2 = 5200$$
$$\therefore \quad x^2 + 10\,000 - 200x + x^2 = 5200$$
$$\therefore \quad 2x^2 - 200x + 4800 = 0$$
$$\therefore \quad 2(x^2 - 100x + 2400) = 0$$
$$\therefore \quad 2(x - 40)(x - 60) = 0$$
$$\therefore \quad x - 40 = 0 \quad \text{or} \quad x - 60 = 0$$
$$\therefore \quad x = 40 \quad \text{or} \quad x = 60$$

If $x = 40$ cm, the square formed from the piece of wire AB has perimeter 40 cm, and the square formed from the piece of wire BC has perimeter 60 cm.

If $x = 60$ cm, the square formed from the piece of wire AB has perimeter 60 cm, and the square formed from the piece of wire BC has perimeter 40 cm.

The perimeters of the squares are 40 cm and 60 cm.

Exercise 1D

In Questions **1** to **4**, solve each of the given quadratic equations for x.

1 **a)** $x^2 - 5x + 6 = 0$ **b)** $x^2 - 3x - 4 = 0$ **c)** $x^2 - 7x + 10 = 0$
d) $x^2 + 5x + 6 = 0$ **e)** $x^2 - 6x + 8 = 0$ **f)** $x^2 - 5x - 6 = 0$
g) $x^2 = 9$ **h)** $x^2 + 2x = 8$ **i)** $x^2 = x + 12$
j) $x^2 + 20 = 9x$ **k)** $x^2 = 4x$ **i)** $x^2 - 8 = 7x$

2 **a)** $2x^2 + 5x + 2 = 0$ **b)** $3x^2 - 7x + 2 = 0$ **c)** $2x^2 - 3x - 5 = 0$
d) $5x^2 + 14x - 3 = 0$ **e)** $4x^2 + 5x + 1 = 0$ **f)** $6x^2 - 5x + 1 = 0$
g) $3x^2 = 10x + 8$ **h)** $2x^2 + x = 15$ **i)** $16x^2 = 9$
j) $3x^2 - x = 10$ **k)** $5x^2 + 13x = 6$ **l)** $8x^2 + 3 = 14x$

3 **a)** $(x + 1)(x + 3) = 8$ **b)** $(x + 2)^2 = 2x + 12$
c) $(2x + 3)(x - 1) = 2(5x + 1)$ **d)** $(x - 3)(x - 4) + 7 = (2x + 5)(x - 1)$
e) $(3x + 5)(x - 1) = 2(x - 1)^2 - 12$ **f)** $(4x + 1)(x + 3) - (x - 3)(x - 1) = 6$
g) $(x - 2)(x + 1) + 3(2x - 1)(x - 3) = 4$ **h)** $(2x + 1)(x - 2) + 3x = x^2 - 1$
i) $x(4 - 3x) + (x + 1)(2x - 3) = 1 - x$ **j)** $(x + 3)(2x + 1) = (5x - 2)(x + 2) + 3x + 8$
k) $(x - 2)^2 = (5x + 3)(2x + 1) + 6x^2 + 1$ **l)** $x(x + 1) + (x + 1)(x + 2) + (x + 2)(x + 3) = 8$

4 **a)** $\dfrac{2}{x+1} = \dfrac{x}{3-2x}$ **b)** $\dfrac{x+2}{4} = \dfrac{x}{4-x}$ **c)** $\dfrac{x^2}{x+3} = \dfrac{2-3x}{2}$

d) $\dfrac{2x-1}{8-x} = \dfrac{5}{x+2}$ **e)** $\dfrac{3x+2}{2x-1} = \dfrac{5x+6}{x+4}$ **f)** $\dfrac{1-x}{x^2} = \dfrac{4}{2x+1}$

g) $1 + \dfrac{4}{x} = \dfrac{3}{x-1}$ **h)** $\dfrac{6}{x-1} + \dfrac{1}{x-4} = 2$ **i)** $\dfrac{2}{3x+1} + \dfrac{3}{1-x} = \dfrac{1}{2}$

j) $\dfrac{3}{x-2} + \dfrac{4}{x+1} = 4$ **k)** $\dfrac{4}{x} + \dfrac{3}{x-1} = 5$ **l)** $\dfrac{5}{x+3} + \dfrac{7}{x-1} = 8$