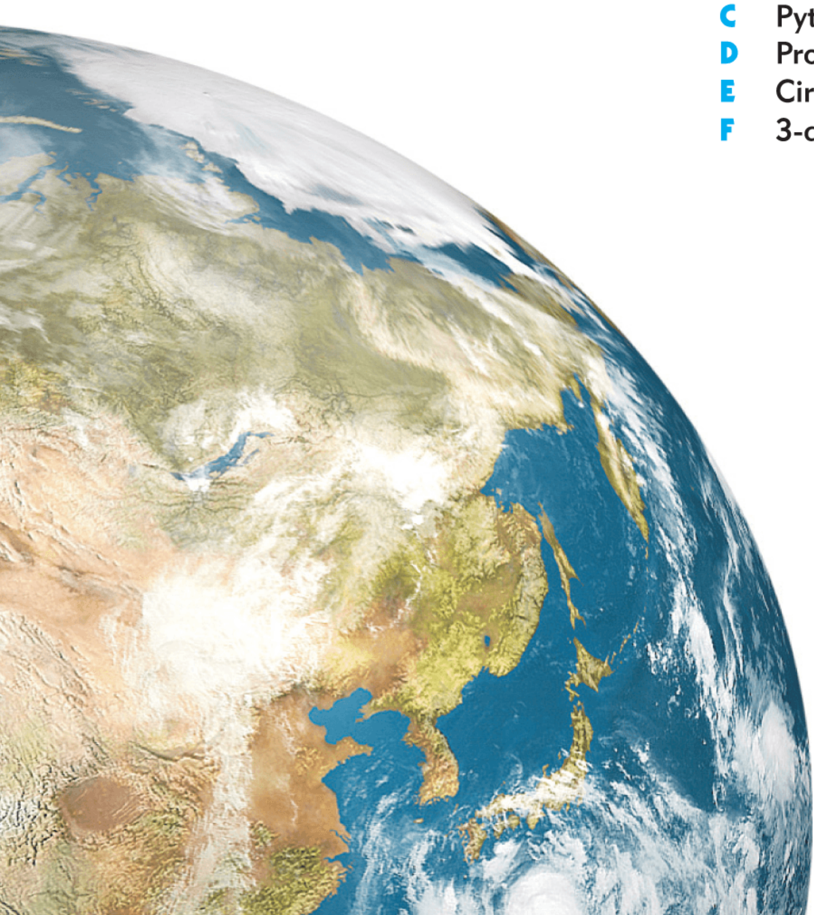
Chapter 5

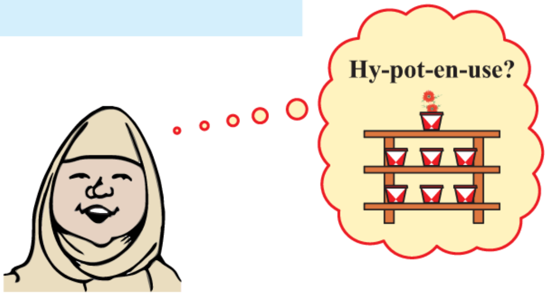
Pythagoras' theorem

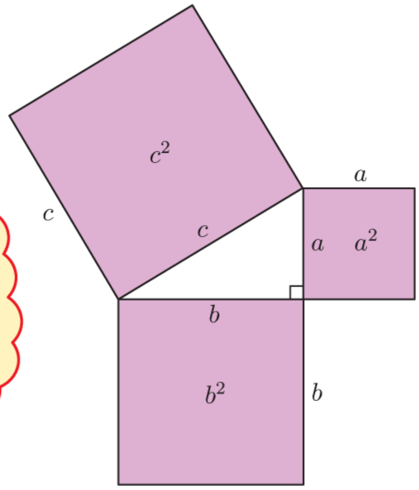
Contents:



In geometric form, **Pythagoras' theorem** states that:

In any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

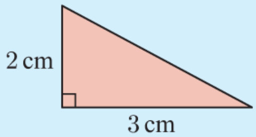




We can use Pythagoras' theorem to find unknown side lengths in right angled triangles.

Example 1 — Self Tutor

Find the length of the hypotenuse in:



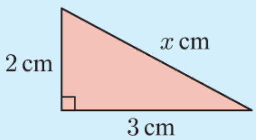
Let the hypotenuse have length x cm.

$\therefore \ x^2 = 3^2 + 2^2$ {Pythagoras}

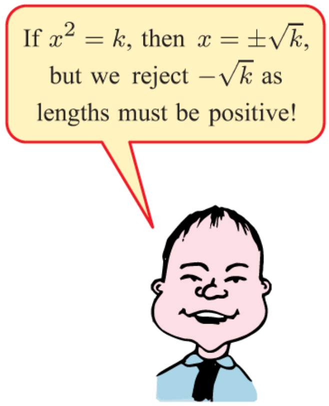
$\therefore \ x^2 = 9 + 4$

$\therefore \ x^2 = 13$

$\therefore \ x = \sqrt{13}$ {as $x > 0$}



The hypotenuse is $\sqrt{13}$ cm long.



EXERCISE 5A

1 Find the length of the hypotenuse of each of the following triangles, leaving your answer in simplest radical form where appropriate:

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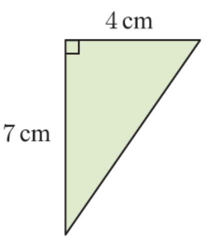

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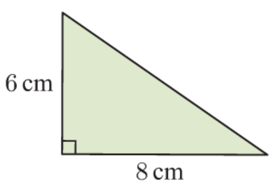

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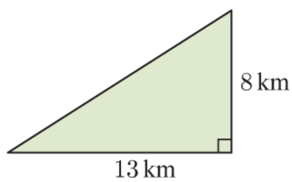

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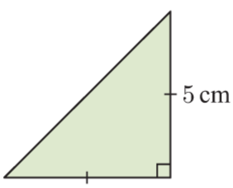

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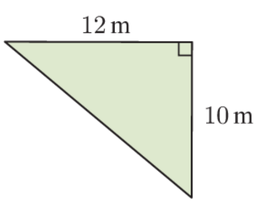

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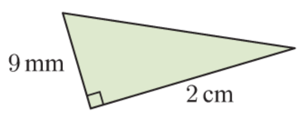
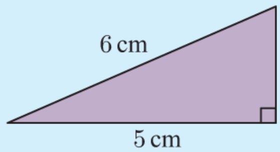


Example 2 — Self Tutor

Find the length of the third side of the triangle.



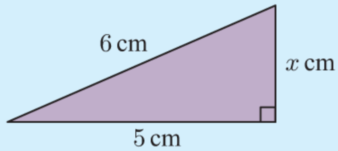
Let the third side have length x cm.

$\therefore\ x^2 + 5^2 = 6^2$ {Pythagoras}

$\therefore\ x^2 + 25 = 36$

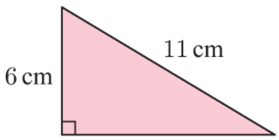
$\therefore\ x^2 = 11$

$\therefore\ x = \sqrt{11}$ {as $x > 0$}



The third side is $\sqrt{11}$ cm long.

2 Find the length of the third side of each of the following right angled triangles. Where appropriate, leave your answer in simplest radical form.

a


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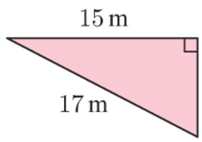

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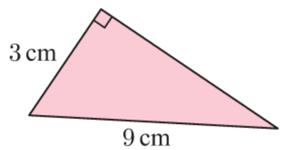

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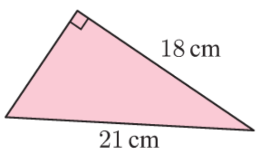

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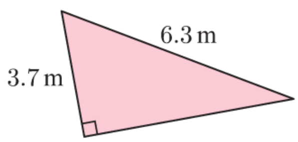

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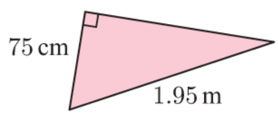
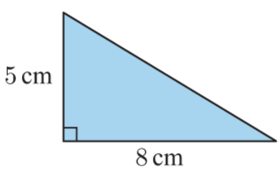


3 Find the length of the unknown side of each of the following right angled triangles. Give your answer to 1 decimal place where appropriate.

a


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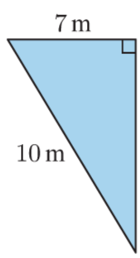

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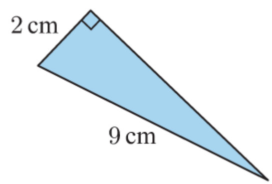

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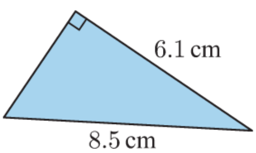

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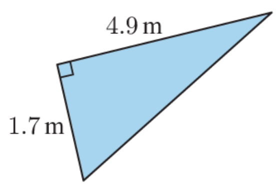

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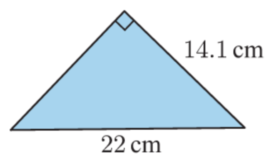
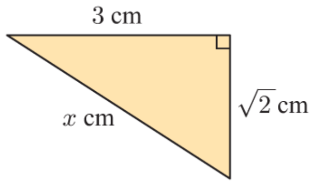


4 Find x in each of the following:

a



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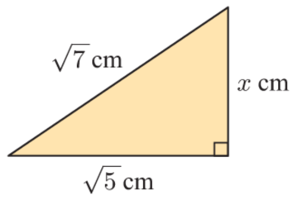

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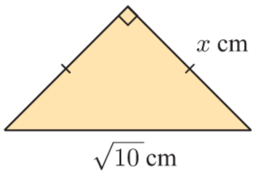

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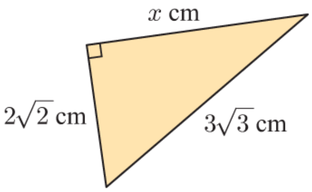

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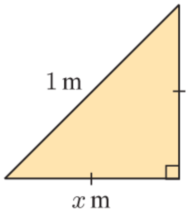

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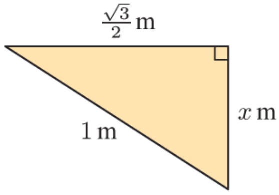
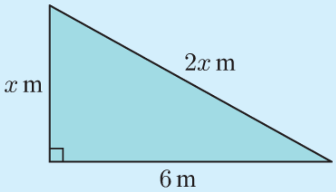


Example 3 **Self Tutor**

Find x:



$(2x)^2 = x^2 + 6^2$ {Pythagoras}

$\therefore\ 4x^2 = x^2 + 36$

$\therefore\ 3x^2 = 36$

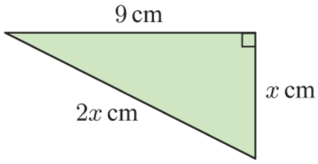
$\therefore\ x^2 = 12$

$\therefore\ x = \sqrt{12}$ {as $x > 0$}

$\therefore\ x = 2\sqrt{3}$

5 Find the value of x:

a



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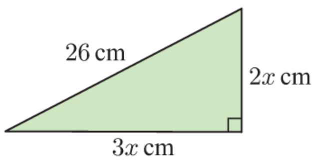

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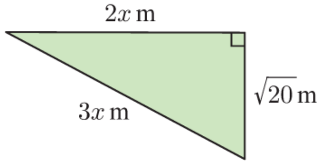
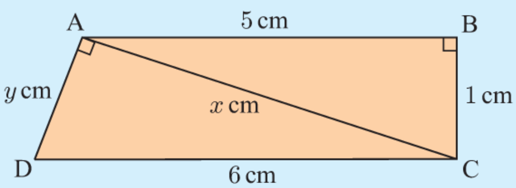


Example 4 **Self Tutor**

Find the values of the unknowns:



In triangle ABC, the hypotenuse is x cm.

$\therefore\ x^2 = 5^2 + 1^2$ {Pythagoras}

$\therefore\ x^2 = 26$

$\therefore\ x = \sqrt{26}$ {as $x > 0$}

In triangle ACD, the hypotenuse is 6 cm.

$\therefore\ y^2 + (\sqrt{26})^2 = 6^2$ {Pythagoras}

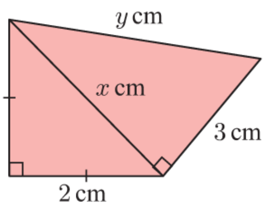
$\therefore\ y^2 + 26 = 36$

$\therefore\ y^2 = 10$

$\therefore\ y = \sqrt{10}$ {as $y > 0$}

6 Find the values of the unknowns:

a



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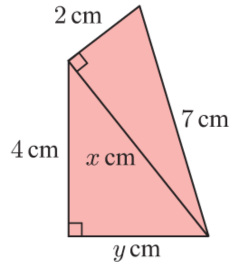

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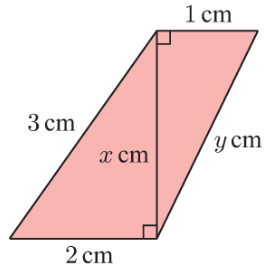
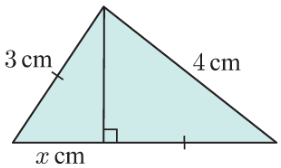


7 Find x:

a



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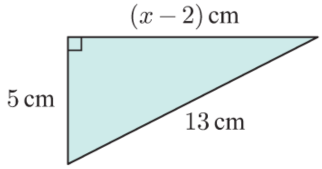

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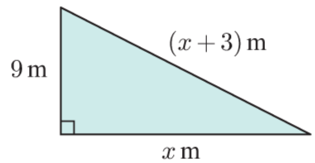
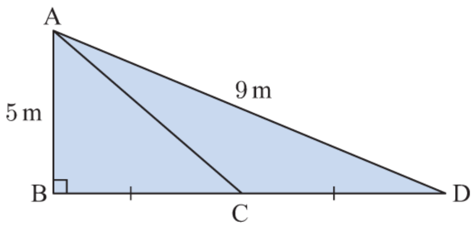
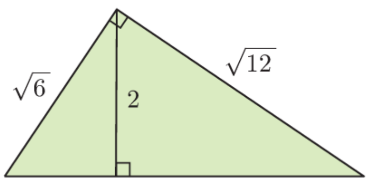


8 Find the length of [AC]:

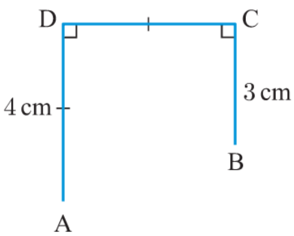


9 Use the figure below to show that $\sqrt{2}+\sqrt{8}=\sqrt{18}$.



10 Find the distance AB in each of the following figures:

a



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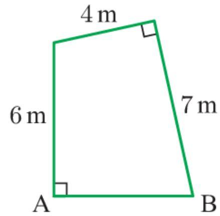

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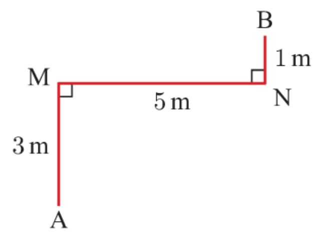
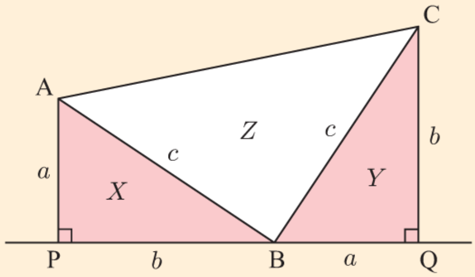


INVESTIGATION PRESIDENT GARFIELD'S PROOF

Prior to being President of the United States, **James Garfield** used the diagram alongside to prove Pythagoras' theorem. When he found this proof he was so pleased he gave cigars out to his many friends.



What to do:

1 Two identical right angled triangles, ABP and BCQ, are placed on a line. What can you deduce about $A\widehat{B}C$? Explain your answer.

2 Find the areas of triangles X, Y, and Z.
Hence, express area X + area Y + area Z in simplest form.

3 The combined regions X, Y, and Z form a trapezium. Find:

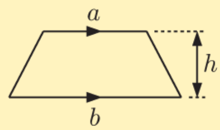
a the average length of the parallel sides

b the distance between the parallel sides

c the area of the trapezium in terms of a and b.

4 Use your results from **2** and **3 c** to find a relationship between a, b, and c.

Area of trapezium
$= \left(\frac{a+b}{2}\right) \times h$



B THE CONVERSE OF PYTHAGORAS' THEOREM

If we know all of the side lengths of a triangle, we can use the **converse of Pythagoras' theorem** to test whether the triangle is right angled.

GEOMETRY PACKAGE

If a triangle has sides of length a, b, and c units where $a^2 + b^2 = c^2$, then the triangle is right angled.

Example 5 — Self Tutor

Is a triangle with side lengths 6 cm, 8 cm, and 5 cm right angled?

The two shorter sides have lengths 5 cm and 6 cm.

Now $5^2 + 6^2 = 25 + 36 = 61$, but $8^2 = 64$.

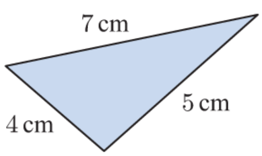
$\therefore$ $5^2 + 6^2 \neq 8^2$, and hence the triangle is not right angled.

The hypotenuse would be the longest side!

EXERCISE 5B

1 The following figures are not drawn to scale. Which of the triangles are right angled?

a



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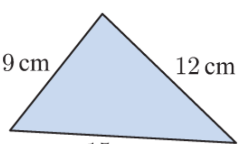

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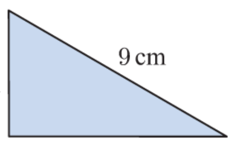

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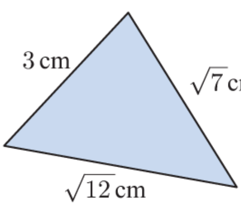

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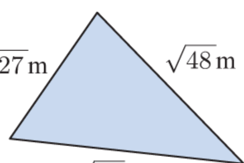

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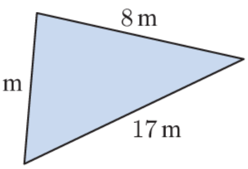
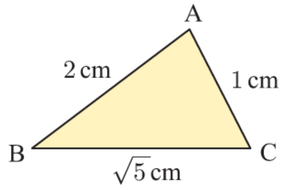


2 The following triangles are not drawn to scale. If any of them is right angled, identify the right angle.

a


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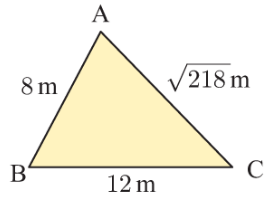

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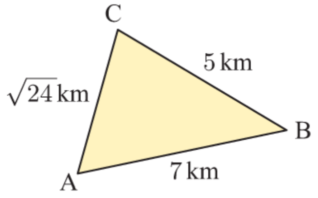

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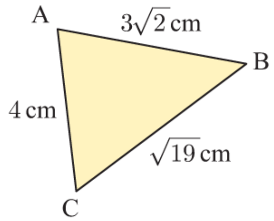

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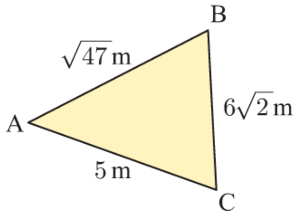

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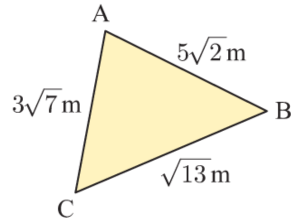
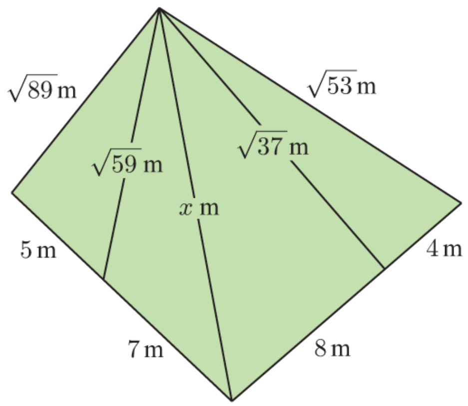


3 Find x:

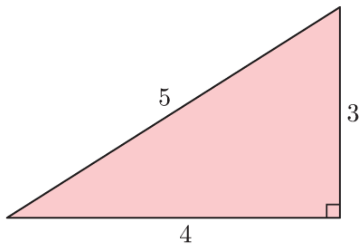


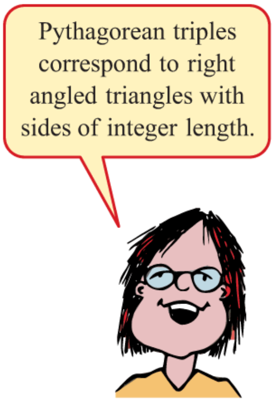
C PYTHAGOREAN TRIPLES

> The set of positive integers $\{a, b, c\}$ where $a < b < c$ is a **Pythagorean triple** if it obeys the rule $a^2 + b^2 = c^2$.

For example, $\{3, 4, 5\}$ is a Pythagorean triple because $3^2 + 4^2 = 5^2$.

Other examples of Pythagorean triples include $\{5, 12, 13\}$ and $\{8, 15, 17\}$.





Example 6 **Self Tutor**

Determine whether the following sets of numbers are Pythagorean triples:

a $\{5, 8, 9\}$ **b** $\{6, 8, 10\}$ **c** $\{2, 3, \sqrt{13}\}$

a $5^2 + 8^2 = 25 + 64 = 89$
and $9^2 = 81$
Since $5^2 + 8^2 \neq 9^2$, $\{5, 8, 9\}$ is not a Pythagorean triple.

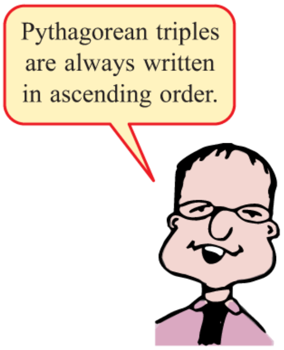
b $6^2 + 8^2 = 36 + 64 = 100$
and $10^2 = 100$
Since $6^2 + 8^2 = 10^2$, $\{6, 8, 10\}$ is a Pythagorean triple.

c $\{2, 3, \sqrt{13}\}$ is not a Pythagorean triple, as these numbers are not all positive integers.

Example 7 **Self Tutor**

Find k given that $\{9, k, 15\}$ is a Pythagorean triple.

$$9^2 + k^2 = 15^2$$
$$\therefore 81 + k^2 = 225$$
$$\therefore k^2 = 144$$
$$\therefore k = 12 \quad \{\text{as } k > 0\}$$



EXERCISE 5C

1 Determine whether the following are Pythagorean triples:

a $\{15, 20, 25\}$ **b** $\{5, 6, 7\}$ **c** $\{14, 48, 50\}$
d $\{1, 6, \sqrt{37}\}$ **e** $\{20, 48, 52\}$ **f** $\{-15, 8, 17\}$

2 Find k given that the following are Pythagorean triples:

a $\{12, 16, k\}$ **b** $\{k, 24, 26\}$ **c** $\{14, k, 50\}$
d $\{8, k, k+2\}$ **e** $\{20, k, k+8\}$ **f** $\{k, 60, k+50\}$

3 **a** Given that $\{a, b, c\}$ is a Pythagorean triple and k is a positive integer, show that $\{ka, kb, kc\}$ is also a Pythagorean triple.

b Use the multiples $k = 2, 3, 4,$ and 5 to construct new Pythagorean triples from these Pythagorean triples:

i $\{3, 4, 5\}$ **ii** $\{5, 12, 13\}$

4 **a** Given that $\{a, b, c\}$ and $\{d, e, f\}$ are Pythagorean triples, show that $\{be - ad, bd + ae, cf\}$ is also a Pythagorean triple.

b Given that $\{3, 4, 5\}$ and $\{8, 15, 17\}$ are Pythagorean triples, use **a** to construct a new Pythagorean triple. Use technology to check your answer.

5 **a** For each of these values of n, show that $\{2n+1,\ 2n^2+2n,\ 2n^2+2n+1\}$ is a Pythagorean triple:

i $n=1$ **ii** $n=2$ **iii** $n=3$ **iv** $n=4$

b Prove that $\{2n+1,\ 2n^2+2n,\ 2n^2+2n+1\}$ is a Pythagorean triple for all $n \in \mathbb{Z}^+$.

Hint: Let $a = 2n+1$, $b = 2n^2+2n$, and $c = 2n^2+2n+1$, then simplify $c^2 - b^2 = (2n^2+2n+1)^2 - (2n^2+2n)^2$ using the *difference of two squares* factorisation.

PUZZLE — PYTHAGOREAN TRIPLE SEQUENCES

Consider a sequence of numbers such that any two consecutive numbers are members of a Pythagorean triple.

For example, one such sequence is 6, 10, 24, 18.

{6, 8, 10} {10, 24, 26} {18, 24, 30}

What to do:

1 Create a sequence of 6 numbers with this property, which starts with 3 and ends with:

a 6 **b** 50 **c** 75.

2 Find the shortest such sequence you can, that starts with 3 and ends with 1000.

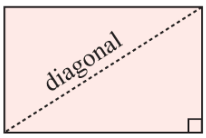
3, ... ? ...., 1000

D PROBLEM SOLVING USING PYTHAGORAS

Right angled triangles occur in many practical problems. In these situations we can apply Pythagoras' theorem to help find unknown side lengths. The problem solving method involves the following steps:

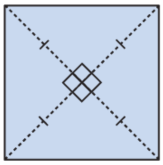
Step 1: Draw a neat, clear diagram of the situation.
Step 2: Mark known lengths and right angles on the diagram.
Step 3: Use a symbol such as x to represent the unknown length.
Step 4: Apply Pythagoras' theorem to the right angled triangle.
Step 5: Solve the equation.
Step 6: Where necessary, write your answer in sentence form.

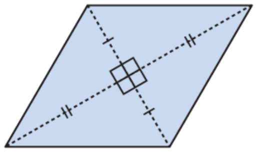
The following special figures contain right angled triangles:



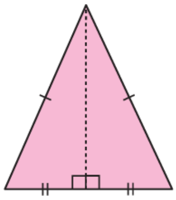
rectangle

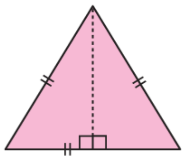
In a **rectangle**, right angles exist between adjacent sides. We can construct a diagonal to form a right angled triangle.


square


rhombus

In a **square** and a **rhombus**, the diagonals bisect each other at right angles.

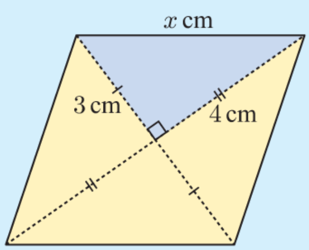

isosceles triangle


equilateral triangle

In an **isosceles triangle** and an **equilateral triangle**, the altitude bisects the base at right angles.

Example 8 **Self Tutor**

A rhombus has diagonals of length 6 cm and 8 cm.
Find the length of its sides.



The diagonals of a rhombus bisect at right angles.

Let each side of the rhombus have length x cm.

$\therefore\ x^2 = 3^2 + 4^2$ {Pythagoras}

$\therefore\ x^2 = 25$

$\therefore\ x = 5$ {as $x > 0$}

The sides are 5 cm long.

EXERCISE 5D

1 A rectangle has sides of length 8 cm and 3 cm. Find the length of its diagonals.

2 The longer side of a rectangle is three times the length of the shorter side. The length of the diagonal is 10 cm. Find the dimensions of the rectangle.

3 A rectangle with diagonals of length 20 cm has sides in the ratio 2 : 1. Find the:

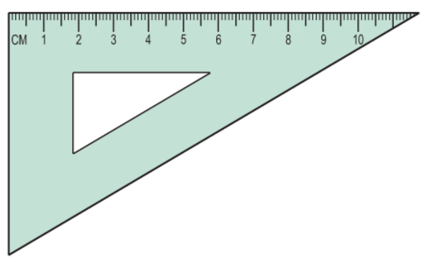
a perimeter **b** area of the rectangle.

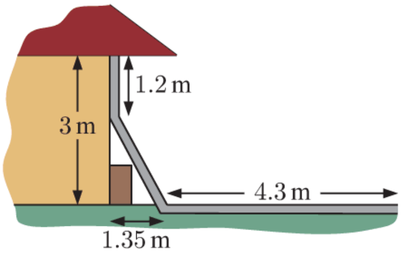
4 A rhombus has sides of length 6 cm. One of its diagonals is 10 cm long. Find the length of the other diagonal.

5 A square has diagonals of length 10 cm. Find the length of its sides.

6 A rhombus has diagonals of length 8 cm and 10 cm. Find its perimeter.

7 To check that his set square was right angled, Roger measured its sides. The two shorter sides were 8 cm and 11.55 cm long, and the longest side was 14.05 cm long. Is the set square right angled?

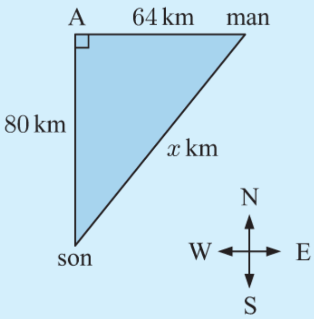


8 

A drain pipe runs down the wall of a house, then out to the road as shown. Find the length of the pipe.

Example 9 Self Tutor

A man and his son both leave point A at the same time. The man rides his bicycle due east at 16 km h^{-1}. The son rides his bicycle due south at 20 km h^{-1}. How far apart are they after 4 hours?



After 4 hours, the man has travelled $4 \times 16 = 64$ km, and his son has travelled $4 \times 20 = 80$ km.

Let the distance between them be x km.

Thus $x^2 = 64^2 + 80^2$ {Pythagoras}

$\therefore\ x^2 = 10\,496$

$\therefore\ x = \sqrt{10\,496}$ {as $x > 0$}

$\therefore\ x \approx 102$

$\therefore$ they are about 102 km apart after 4 hours.

9 A yacht sails 5 km due west and then 8 km due south. How far is it from its starting point?

10 Pirate Captain William Hawk left his hat on Treasure Island. He sailed 18 km northeast through the Forbidden Strait, then 11 km southeast to his home before realising it was missing. He sent his parrot to fetch the hat and return it to the boat. How far did the parrot need to fly?

11 Two runners set off from town A at the same time. One runs due east to town B, and the other runs due south to town C at twice the speed of the first. They both arrive at their destinations two hours later. Given that B and C are 50 km apart, find the average speed of each runner.

12 Answer the **Opening Problem** on page **80**.

13 A highway runs east-west between two towns B and C that are 25 km apart. Town A lies 15 km directly north of B. A straight road is built from A to meet the highway at D. Given that D is equidistant from A and C, find the position of D on the highway.

Example 10 — Self Tutor

An equilateral triangle has sides of length 6 cm. Find its area.

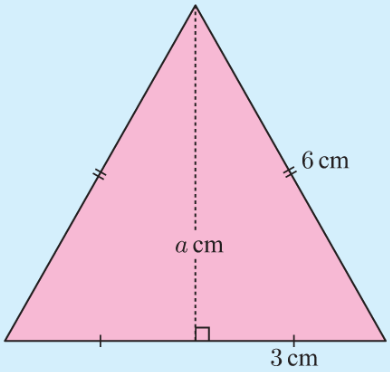
The altitude bisects the base at right angles.

$\therefore\ a^2 + 3^2 = 6^2$ {Pythagoras}

$\therefore\ a^2 + 9 = 36$

$\therefore\ a^2 = 27$

$\therefore\ a = \sqrt{27}$ {as $a > 0$}



Area $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6 \times \sqrt{27}$

$= 3\sqrt{27}\ \text{cm}^2$

$\approx 15.6\ \text{cm}^2$

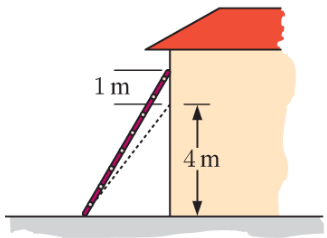
So, the area is about $15.6\ \text{cm}^2$.

14 An equilateral triangle has sides of length 12 cm. Find the length of one of its altitudes.

15 An isosceles triangle has equal sides of length 8 cm and a base of length 6 cm. Find the area of the triangle.

16 An equilateral triangle has area $16\sqrt{3}\ \text{cm}^2$. Find the length of its sides.

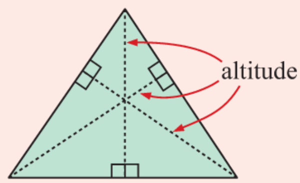
17 An extension ladder rests 4 m up a wall. If the ladder is extended a further 0.8 m without moving its feet, then it will now rest 1 m further up the wall. How long is the extended ladder?



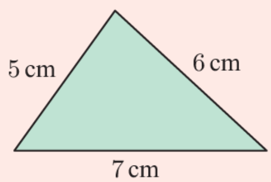
18 A rectangular piece of paper, 10 cm by 25 cm, is folded so that a pair of diagonally opposite corners coincide. Find the length of the crease.

ACTIVITY — ALTITUDES

An **altitude** of a triangle is a line which is perpendicular to one side of the triangle, and which passes through the opposite vertex. Every triangle has three altitudes.



Can you use Pythagoras' theorem to find the lengths of the three altitudes of this triangle?



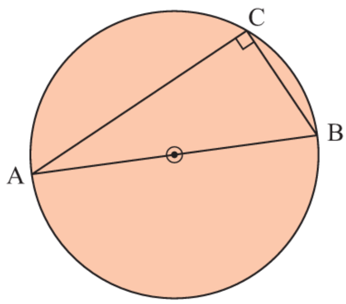
E CIRCLE PROBLEMS

There are also certain properties of circles which involve right angles. They are described in the following theorems:

ANGLE IN A SEMI-CIRCLE

The angle in a semi-circle is a right angle.

No matter where C is placed on the circle, $A\widehat{C}B$ is always a right angle.



Example 11 Self Tutor

A circle has diameter [XY] of length 13 cm. Z is a point on the circle such that XZ is 5 cm. Find the length YZ.

From the angle in a semi-circle theorem, $X\widehat{Z}Y$ is a right angle.

Let the length YZ be x cm.

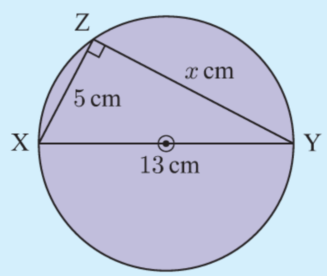
$\therefore\ 5^2 + x^2 = 13^2$ {Pythagoras}

$\therefore\ x^2 = 169 - 25 = 144$

$\therefore\ x = \sqrt{144}$ {as $x > 0$}

$\therefore\ x = 12$

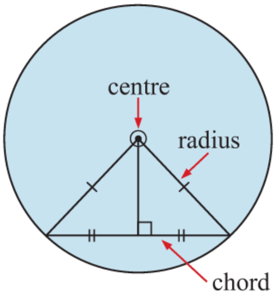
So, YZ has length 12 cm.



A CHORD OF A CIRCLE

The line drawn from the centre of a circle at right angles to a chord, bisects the chord.

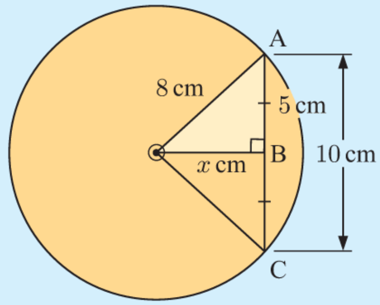
The construction of radii from the centre of the circle to the end points of the chord produces an isosceles triangle. The above property then follows from the **isosceles triangle theorem**.



Example 12 **Self Tutor**

A circle with radius 8 cm has a chord of length 10 cm.
Find the shortest distance from the centre of the circle to the chord.

The shortest distance is the 'perpendicular distance'. The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.



$\therefore$ AB = BC = 5 cm

In $\triangle$AOB, $5^2 + x^2 = 8^2$ {Pythagoras}

$\therefore$ $x^2 = 64 - 25 = 39$

$\therefore$ $x = \sqrt{39}$ {as $x > 0$}

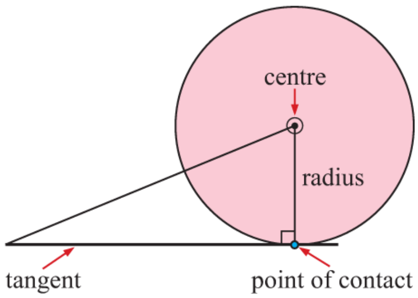
$\therefore$ $x \approx 6.24$

So, the shortest distance is about 6.24 cm.

TANGENT-RADIUS PROPERTY

A tangent to a circle and a radius at the point of contact meet at right angles.

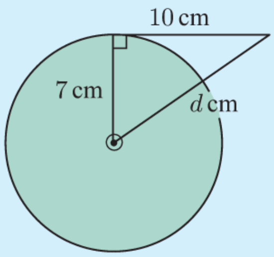
Notice that we can now form a right angled triangle.



Example 13 **Self Tutor**

A tangent of length 10 cm is drawn to a circle with radius 7 cm. How far is the centre of the circle from the end point of the tangent?

Let the distance be d cm.



$\therefore$ $d^2 = 7^2 + 10^2$ {Pythagoras}

$\therefore$ $d^2 = 149$

$\therefore$ $d = \sqrt{149}$ {as $d > 0$}

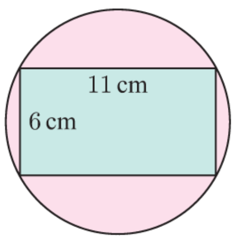
$\therefore$ $d \approx 12.2$

The centre is about 12.2 cm from the end point of the tangent.

EXERCISE 5E

1 A circle has diameter [AB] of length 10 cm. C is a point on the circle such that AC is 8 cm. Find the length BC.

2 A rectangle with side lengths 11 cm and 6 cm is inscribed in a circle. Find the radius of the circle.



3 A circle with radius 4 cm has a chord of length 3 cm. Find the shortest distance from the centre of the circle to the chord.

4 A chord of length 6 cm is 3 cm from the centre of a circle. Find the radius of the circle.

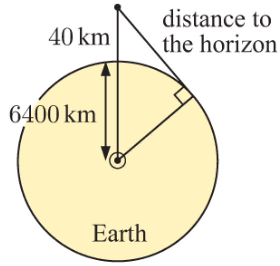
5 A chord is 5 cm from the centre of a circle of radius 8 cm. Find the length of the chord.

6 A circle has radius 3 cm. A tangent is drawn to the circle from point P, which is 9 cm from the circle's centre. How long is the tangent?

7 A tangent of length 12 cm has end point 16 cm from the circle's centre. Find the radius of the circle.

8 A circular table of diameter 2 m is placed in the corner of a room so that its edges touch two perpendicular walls. Find the shortest distance from the corner of the room to the edge of the table.

9 The radius of the Earth is about 6400 km. Determine the distance to the horizon from a rocket which is 40 km above the Earth's surface.



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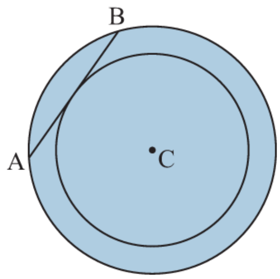
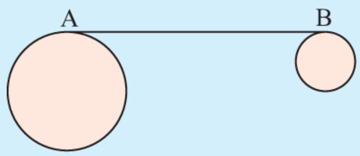


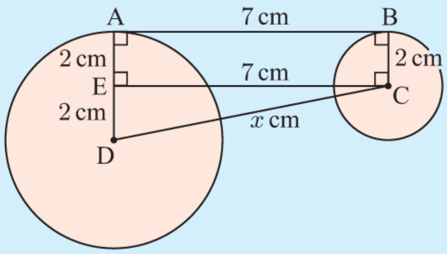
C is the centre of two circles with radii 7 cm and 5 cm. [AB] is a chord of the larger circle, and a tangent of the smaller circle.

Find the length of [AB].

Example 14 Self Tutor

Two circles have a common tangent with points of contact A and B which are 7 cm apart. The radii of the circles are 4 cm and 2 cm respectively. Find the distance between the centres.





For centres C and D, we draw [BC], [AD], [CD], and [CE] ∥ [AB].

$\therefore$ ABCE is a rectangle.

$\therefore$ CE = 7 cm {as CE = AB}

and DE = 4 − 2 = 2 cm

Let the distance between the centres be x cm.

$\therefore$ $x^2 = 2^2 + 7^2$ {Pythagoras in △DEC}

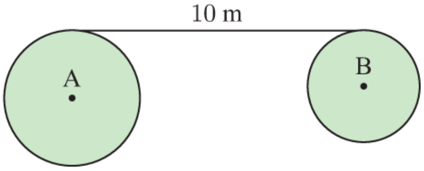
$\therefore$ $x^2 = 53$

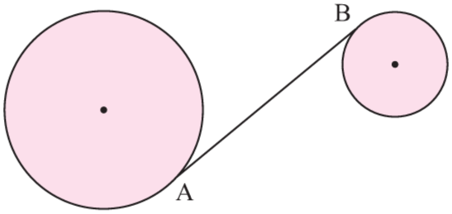
$\therefore$ $x = \sqrt{53}$ {as $x > 0$}

$\therefore$ $x \approx 7.28$

The distance between the centres is about 7.28 cm.

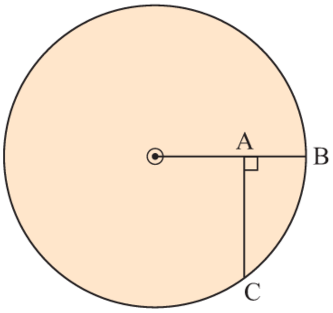
11 A and B are the centres of two circles with radii 4 m and 3 m respectively. The illustrated common tangent has length 10 m.
Find the distance between the centres to the nearest millimetre.



12 

The illustration shows two circles of radii 4 cm and 2 cm respectively. The distance between the two centres is 8 cm. Find the length of the common tangent [AB].

13 In the given figure, AB = 1 cm and AC = 3 cm.
Find the radius of the circle.



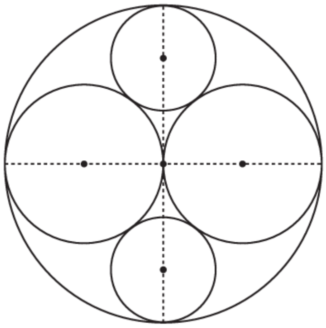
14 In the given figure, the largest circle has radius 12 cm.

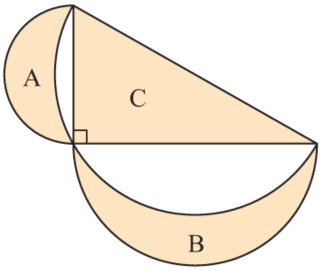
a Find the radius of:

i the medium circles

ii the smallest circles.

b What fraction of the largest circle is occupied by the four inner circles?



15 

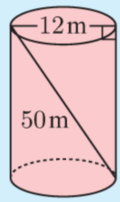
Show that Area A + Area B = Area C.

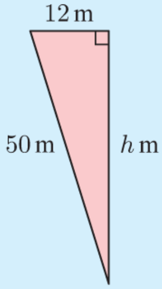
F 3-DIMENSIONAL PROBLEMS

Pythagoras' theorem is often used to find lengths in **3-dimensional** problems.

Example 15 Self Tutor

A 50 m rope is attached inside an empty cylindrical wheat silo of diameter 12 m as shown. How high is the silo?





Let the height be h m.

$\therefore\ h^2 + 12^2 = 50^2$ {Pythagoras}

$\therefore\ h^2 + 144 = 2500$

$\therefore\ h^2 = 2356$

$\therefore\ h \approx 48.5$ {as $h > 0$}

The wheat silo is about 48.5 m high.

Sometimes we need to apply Pythagoras' theorem *twice*.

Example 16 Self Tutor

A room is 6 m by 4 m, and has a height of 3 m. Find the distance from a corner point on the floor to the opposite corner point on the ceiling.

The required distance is AD. We join [BD].

In $\triangle$BCD, $x^2 = 4^2 + 6^2$ {Pythagoras}

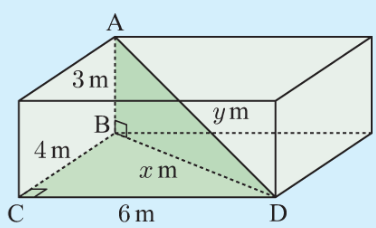
In $\triangle$ABD, $y^2 = x^2 + 3^2$ {Pythagoras}

$\therefore\ y^2 = 4^2 + 6^2 + 3^2$

$\therefore\ y^2 = 61$

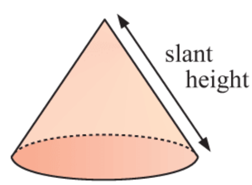
$\therefore\ y \approx 7.81$ {as $y > 0$}

$\therefore$ the distance is about 7.81 m.



EXERCISE 5F

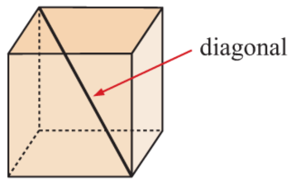
1 A cone has a slant height of 17 cm, and a base radius of 8 cm. How high is the cone?



2 Find the length of the longest nail that could fit entirely within a cylindrical can with radius 3 cm and height 8 cm.

3 A 20 cm nail just fits inside a cylindrical can. Three identical spherical balls need to fit entirely within the can. What is the maximum radius each ball could have?

4 A cube has sides of length 3 cm. Find the length of a diagonal of the cube.



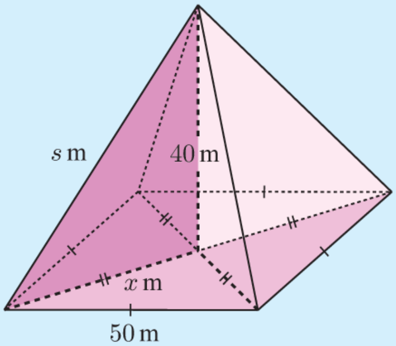
5 A room is 5 m by 3 m, and has a height of 3.5 m. Find the distance from a corner point on the floor to the opposite corner of the ceiling.

6 A rectangular box has internal dimensions 2 cm by 3 cm by 2 cm. Find the length of the longest toothpick that can be placed within the box.

7 Can an 8.5 m long piece of timber be stored in a rectangular shed which is 6 m by 5 m by 2 m high?

Example 17 **Self Tutor**

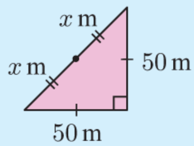
A pyramid of height 40 m has a square base with edges of length 50 m.
Determine the length of the slant edges.



Let a slant edge have length s m.

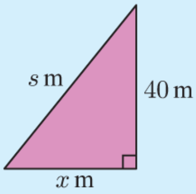
Let half a diagonal have length x m.

Using



$$(2x)^2 = 50^2 + 50^2 \quad \{\text{Pythagoras}\}$$
$$\therefore\ 4x^2 = 5000$$
$$\therefore\ x^2 = 1250$$

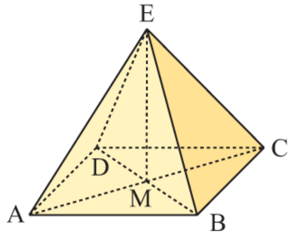
Using



$$s^2 = x^2 + 40^2 \quad \{\text{Pythagoras}\}$$
$$\therefore\ s^2 = 1250 + 1600$$
$$\therefore\ s^2 = 2850$$
$$\therefore\ s \approx 53.4 \quad \{\text{as } s > 0\}$$

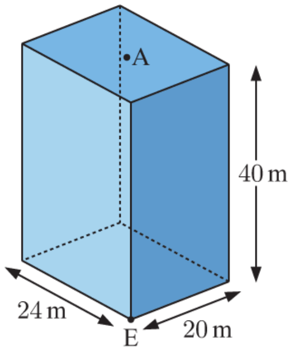
Each slant edge is about 53.4 m long.

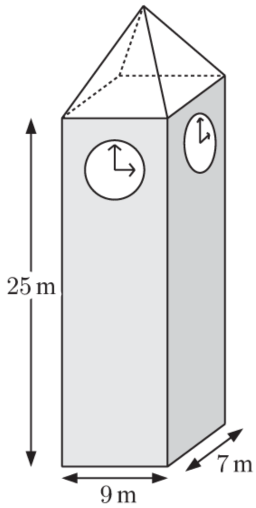
8 An Egyptian Pharaoh wishes to build a square-based pyramid with all edges of length 100 m. Its apex will be directly above the centre of its base. How high, to the nearest metre, will the pyramid reach above the desert sands?



9 A symmetrical square-based pyramid has height 10 cm and slant edges of length 15 cm. Find the dimensions of its square base.

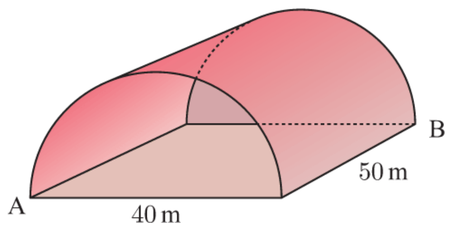
10 In the office building alongside, the entrance is at E. A radio antenna at A, the centre of the roof, receives security signals from the door E.
Find the direct distance between the entrance and the antenna.





11 A clock tower has the dimensions shown. The slant edges of the pyramid are 7.5 m long. Find the height of the tower, to the nearest cm.

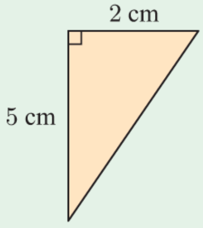
12 An aircraft hangar is semi-cylindrical with diameter 40 m and length 50 m. A helicopter places a cable across the top of the hangar, and one end is pinned to the corner at A. The cable is then pulled tight and pinned at the opposite corner B. Determine the length of the cable, to the nearest cm.



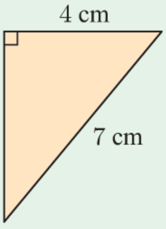
REVIEW SET 5A

1 Find the length of the unknown side in each of the following triangles:

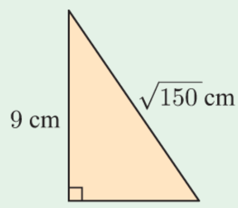
a



b

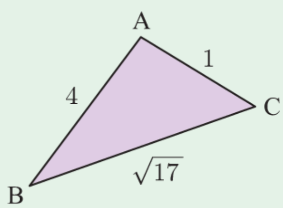


c



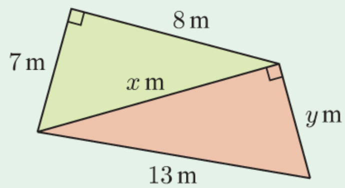
2 Determine whether $\{5, 11, 13\}$ is a Pythagorean triple.

3 Is this triangle right angled?
Explain your answer.

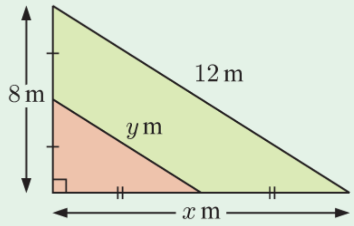


4 Find the values of the unknowns:

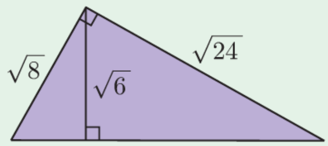
a



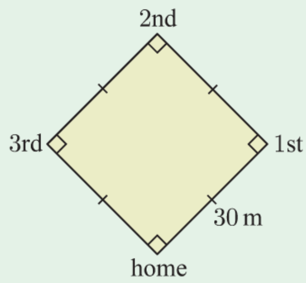
b



5 Use the figure alongside to show that $\sqrt{2}+\sqrt{18}=\sqrt{32}$.



6 A softball diamond has sides of length 30 m. Determine the distance a fielder must throw the ball from second base to reach home base.



7 **a** Find the value of k such that $\{15,\ k,\ k+3\}$ is a Pythagorean triple.

b **i** For any integer $k>2$, show that $\{4k,\ k^2-4,\ k^2+4\}$ is a Pythagorean triple.

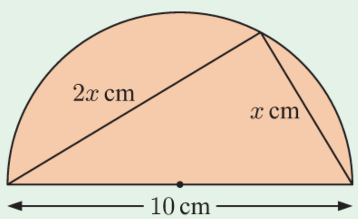
ii Find the Pythagorean triple which results when $k=5$.

8 A rectangle has diagonals 15 cm long, and one side is 8 cm long. Find the perimeter of the rectangle.

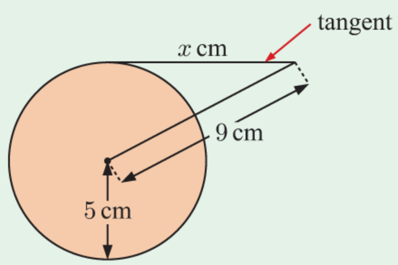
9 A circle has a chord of length 10 cm. The shortest distance from the circle's centre to the chord is 5 cm. Find the radius of the circle.

10 Find x:

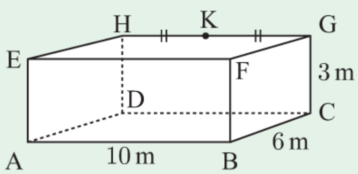
a



b



11

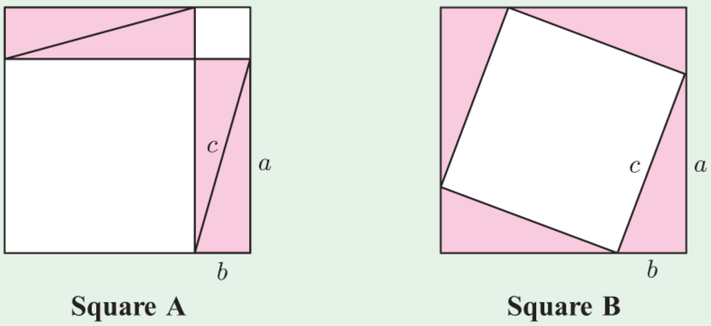


A room is 10 m by 6 m by 3 m. Find the shortest distance from:

a E to K

b A to K.

12 Consider the two squares below.



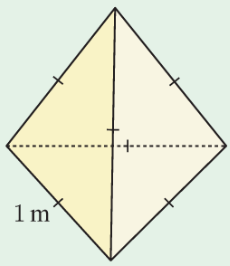
In each square, the 4 shaded right angled triangles have legs a and b, and hypotenuse c.

a Show that the squares have the same area.

b Find an expression for the unshaded area in: **i** square A **ii** square B.

c Hence, prove Pythagoras' theorem.

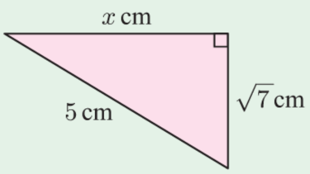
13 Find the height of an equilateral triangular pyramid in which every edge has length 1 m.



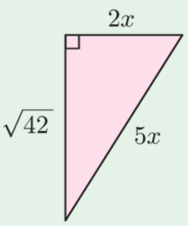
REVIEW SET 5B

1 Find the value of x in the following:

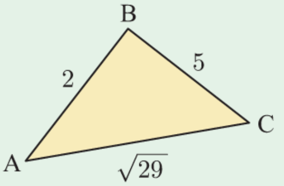
a



b



2 Show that this triangle is right angled, and identify which is the right angle.

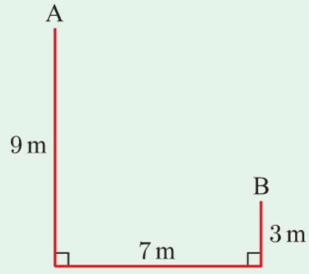


3 Find k given that $\{12, k, 37\}$ is a Pythagorean triple.

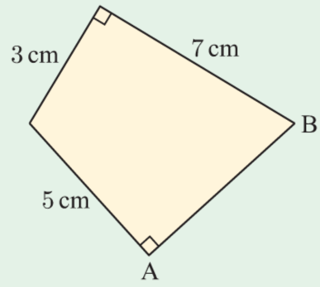
4 The diameter of a circle is 20 cm. Find the shortest distance from a chord of length 16 cm to the centre of the circle.

5 Find the distance AB in the following figures:

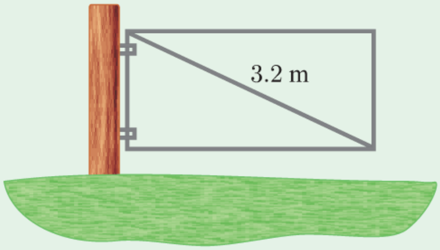
a



b



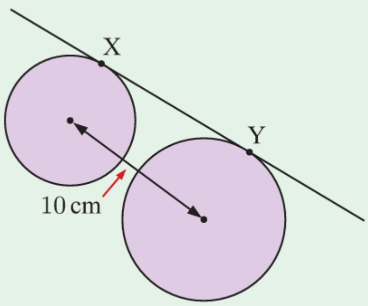
6 A rectangular gate is twice as wide as it is high. It is held in shape by a diagonal strut 3.2 m long. Find the height of the gate to the nearest millimetre.



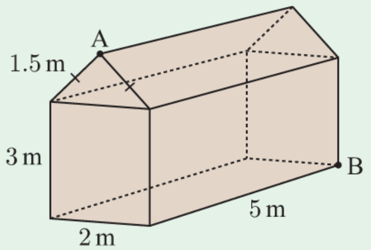
7 A 15 m ladder reaches twice as far up a vertical wall as the base is out from the wall. How far up the wall does the ladder reach?

8 Can a wooden beam 10.5 m long be placed in a rectangular shed 8 m by 7 m by 3 m?

9 Two circles have a common tangent with points of contact X and Y.
The radii of the circles are 4 cm and 5 cm respectively, and the distance between the centres is 10 cm.
Find the length of the common tangent [XY].



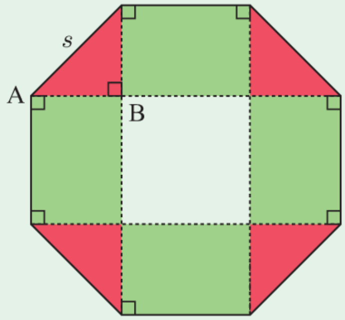
10



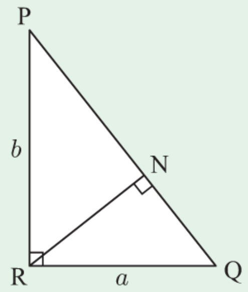
A barn has the dimensions given.
Find the shortest distance from A to B.

11 Consider a regular octagon with side length s.
The interior angles of a regular octagon are $135°$.

- **a** Find AB in terms of s.
- **b** Find the area of:
  - **i** a red triangle
  - **ii** a green rectangle.
- **c** Hence, show that the area of a regular octagon with side length s is $(2+2\sqrt{2})s^2$.



12 **a**

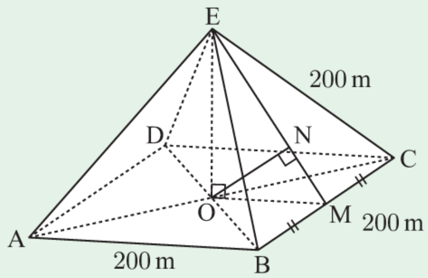


i Find the length of [PQ].

ii Use triangle areas to explain why

$$\text{RN} = \frac{ab}{\sqrt{a^2+b^2}}.$$

b All edges of a square-based pyramid are 200 m long. O is the centre of base ABCD and M is the midpoint of [BC]. [ON] is a small shaft from face BCE to the King's chamber at O. How long is this shaft?



13 Two circles are inscribed inside a semi-circle with radius r as shown.

a Find the radius of the red circle in terms of r.

b Let x be the radius of the green circle.

i Show that

$$\left(\frac{r}{2}+x\right)^2 - \left(\frac{r}{2}-x\right)^2 = (r-x)^2 - x^2.$$

ii Hence, find an expression for the radius of the green circle in terms of r.