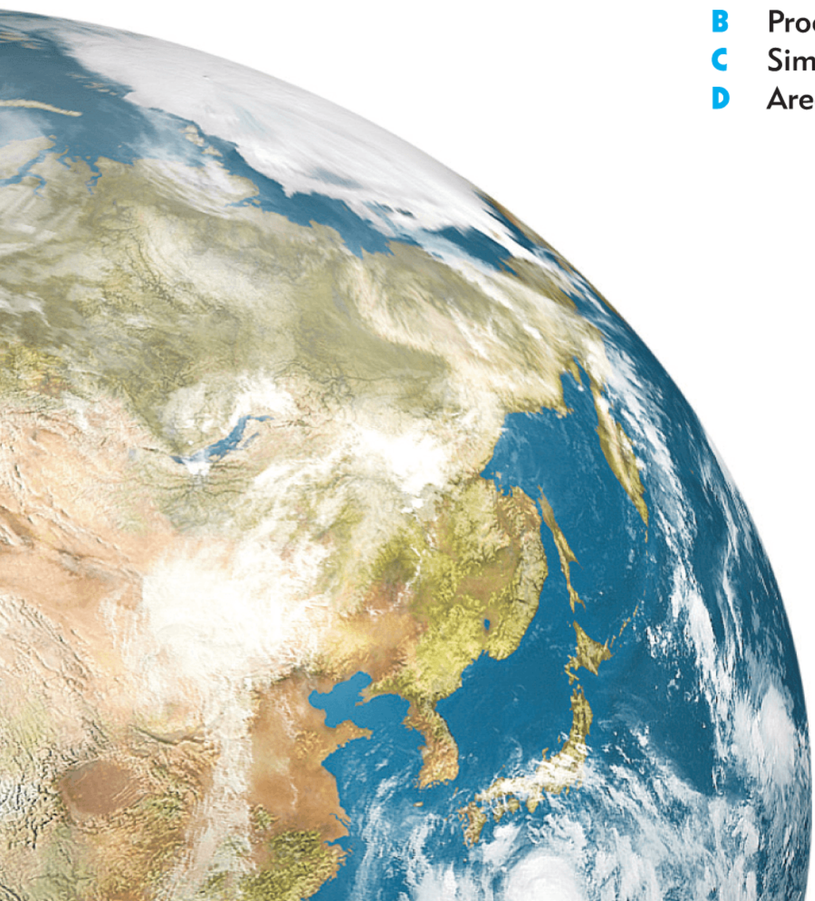
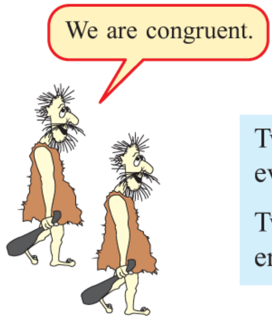
Chapter 7

Congruence and similarity

Contents:



CONGRUENCE AND SIMILARITY



Two figures are **congruent** if they are identical in every respect except for position.

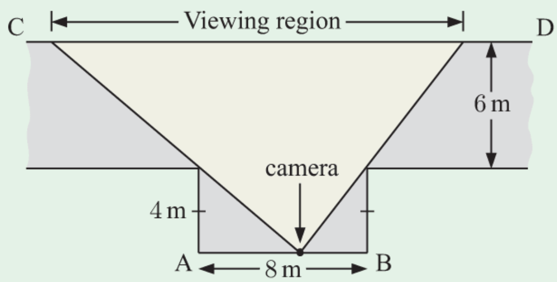
Two figures are **similar** if one figure is an enlargement of the other.

OPENING PROBLEM

In an art gallery, a security camera is being installed along the wall [AB], to view the opposite wall [CD].

Peter is wondering which location for the camera along [AB] maximises the viewing region on the opposite wall.

"It doesn't matter," says Linda, "no matter where the camera is placed, the size of the viewing region will be the same."

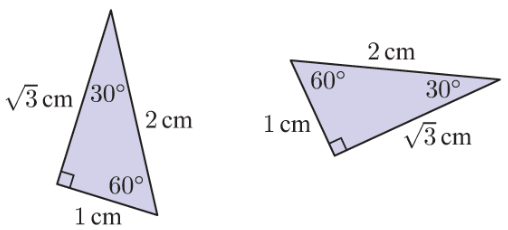


Things to think about:

Can you use similar triangles to determine whether Linda is correct?

A CONGRUENT TRIANGLES

Two triangles are **congruent** if they are identical in every respect except for position.



The triangles alongside are congruent.

They have identical side lengths and angles.

If we are given sufficient information about a triangle, there will be only one way in which it can be drawn. Any two triangles which have this information in common must be **congruent**.

TESTS FOR TRIANGLE CONGRUENCE

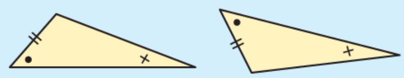
There are four acceptable tests for the congruence of two triangles.

Two triangles are **congruent** if one of the following is true:

- All corresponding sides are equal in length. **(SSS)**

- Two sides and the **included** angle are equal. **(SAS)**

- Two angles and a pair of **corresponding sides** are equal. **(AAcorS)**



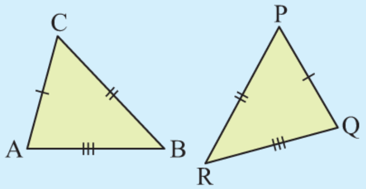
- For right angled triangles, the hypotenuses and one other pair of sides are equal. **(RHS)**

The information we are given will help us decide which congruence test to use. The diagrams in the following Exercise are sketches only and are *not* drawn to scale. However, the information marked on them is correct.

Example 1 — Self Tutor

Are the following pairs of triangles congruent? If so, state the congruence relationship and give a brief reason.

a



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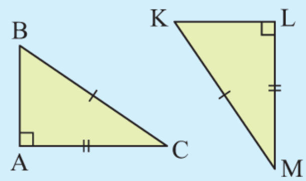

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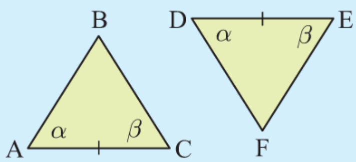

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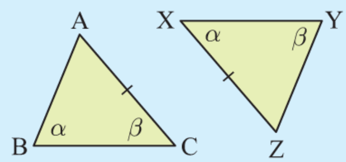
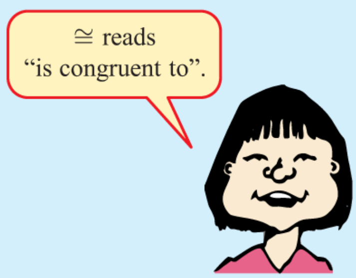


a $\triangle ABC \cong \triangle QRP$ {SSS}

b $\triangle ABC \cong \triangle LKM$ {RHS}

c $\triangle ABC \cong \triangle DFE$ {AAcorS}

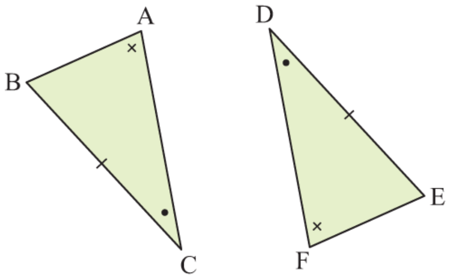
d The two angles α and β are common, but although AC equals XZ, these sides are *not* corresponding. [AC] is opposite α whereas [XZ] is opposite β.
So, the triangles are not congruent.



When we describe congruent triangles, we label the vertices in corresponding positions in the same order. In **Example 1** part **a** above, A and Q are opposite two tick marks, B and R are opposite one tick mark, and C and P are opposite three tick marks. So we write $\triangle ABC \cong \triangle QRP$, not $\triangle ABC \cong \triangle PQR$.

EXERCISE 7A

1 Are the following pairs of triangles congruent? If so, state the congruence relationship and give a brief reason.

a


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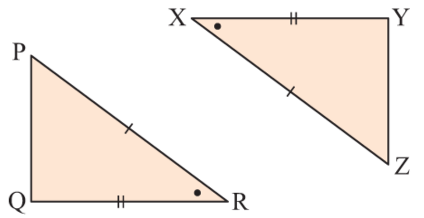

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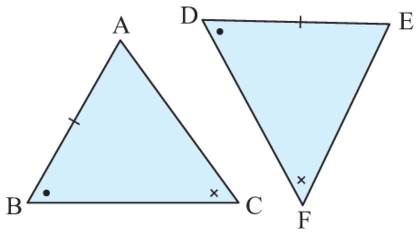

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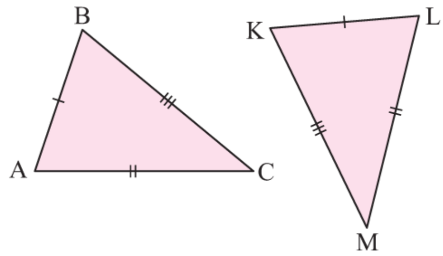

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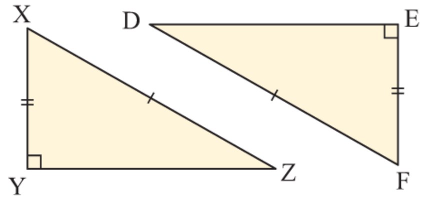

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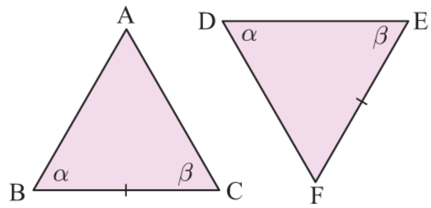

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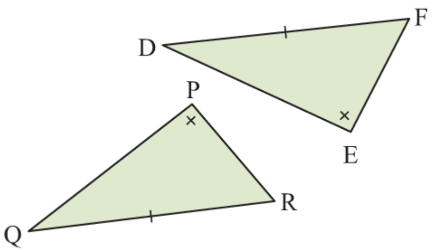

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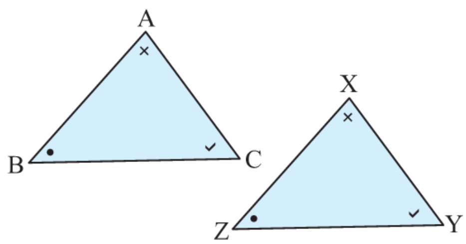

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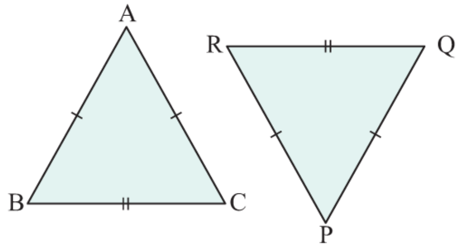

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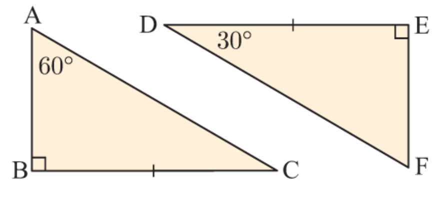
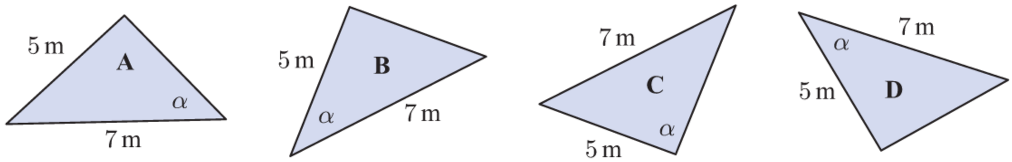


2 For the following groups of triangles, determine which two triangles are congruent. Give reasons for your answers. The triangles are not drawn to scale, but contain correct information.

a


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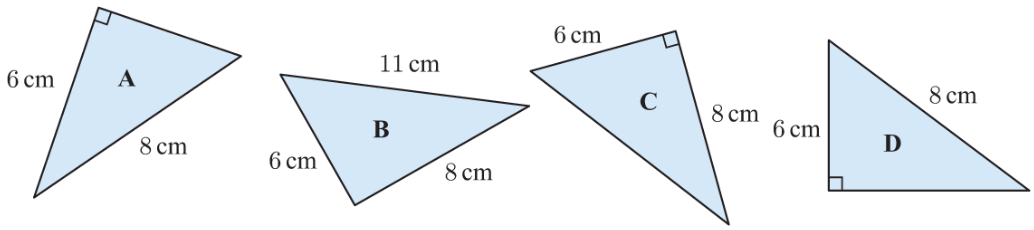

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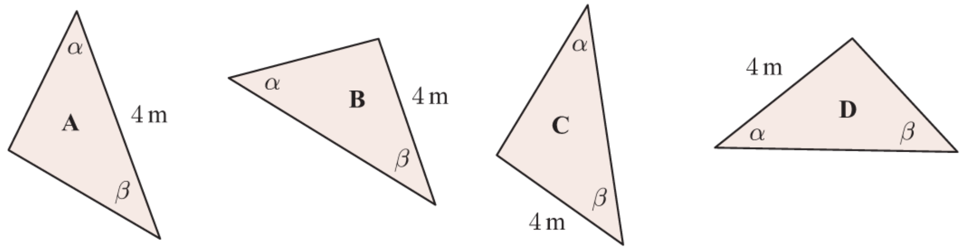

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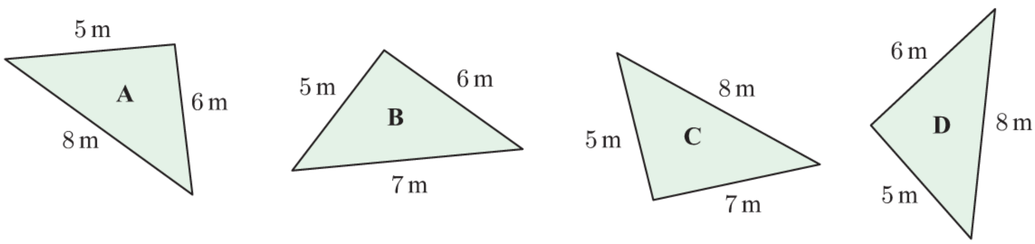
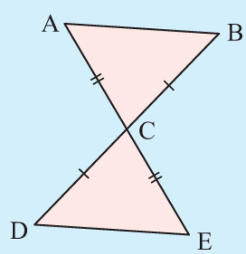


B PROOF USING CONGRUENCE

Once we have proven that two triangles are congruent, we can deduce that the remaining corresponding sides and angles of the triangles are equal. We can therefore use congruence to prove facts about geometric figures.

Example 2 Self Tutor

In the figure alongside, show that [AB] is parallel to [DE].



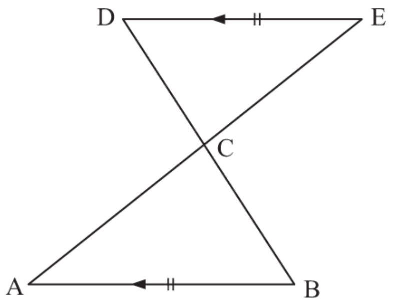
In triangles ABC and EDC:

- BC = DC {given}
- AC = EC {given}
- $A\widehat{C}B = E\widehat{C}D$ {vertically opposite angles}

So, $\triangle ABC \cong \triangle EDC$ {SAS}

Therefore, $A\widehat{B}C = E\widehat{D}C$ {equating angles}
and so [AB] || [DE] {equal alternate angles}

EXERCISE 7B

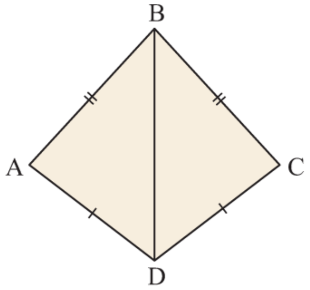
1 

In the given figure, [DE] is parallel to [AB] and DE = AB.
Show that the triangles are congruent.

2 **a** Show that triangles ABD and CBD are congruent.

b Given that $A\widehat{B}D = 47°$ and $B\widehat{A}D = 82°$, find the size of:

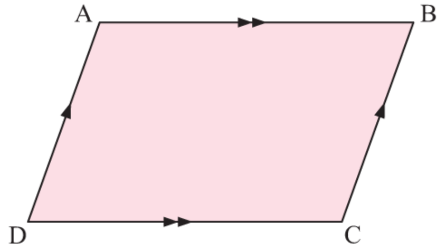
   i $C\widehat{B}D$ **ii** $C\widehat{D}B$.

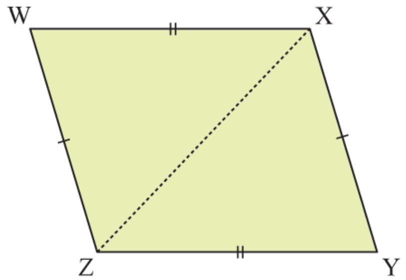


3 Consider the quadrilateral ABCD alongside. [AB] is parallel to [DC], and [AD] is parallel to [BC].

a Use congruence to show that the opposite sides are equal in length.

b Hence, show that the diagonals of a parallelogram bisect each other.



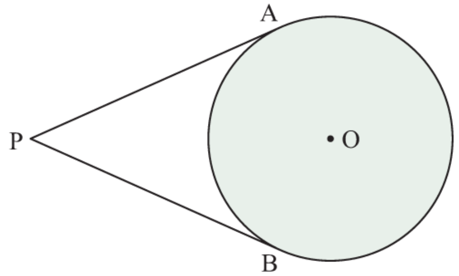
4 

WXYZ is a quadrilateral with opposite sides equal. [XZ] is added to the figure.

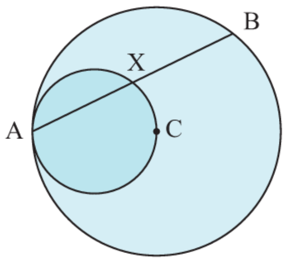
a Show that the two triangles created are congruent.

b Hence deduce that WXYZ is a parallelogram.

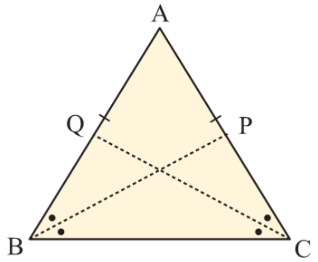
5 The tangents to a circle at A and B intersect at P.
Show that AP = BP.



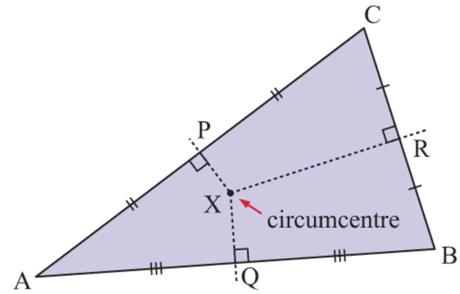
6 [AC] is a radius of the large circle, and a diameter of the small circle. A line through A cuts the small circle at X and the large circle at B.
Show that X is the midpoint of [AB].



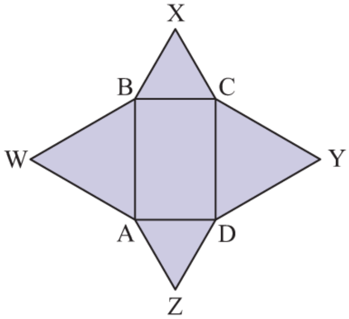
7 Triangle ABC is isosceles, with AB = AC. The angle bisectors of B and C are drawn, meeting the triangle at P and Q respectively. Show that AP = AQ.



8 The perpendicular bisectors of a triangle's edges meet at a point called the **circumcentre** of the triangle.
Prove that the circumcentre is equidistant from each vertex of the triangle.



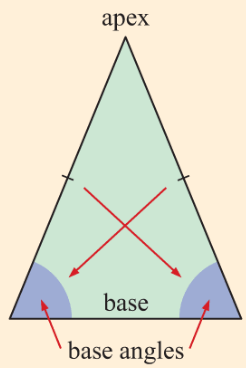
9 ABCD is a rectangle. Equilateral triangles are drawn from each side of the rectangle, with apexes W, X, Y, and Z.
Show that WXYZ is a rhombus.



INVESTIGATION 1 — THE ISOSCELES TRIANGLE THEOREM AND ITS CONVERSES

THE ISOSCELES TRIANGLE THEOREM

In an isosceles triangle:
- the base angles are equal
- the line joining the apex to the midpoint of the base bisects the vertical angle and meets the base at right angles.



GEOMETRY PACKAGE

CONVERSES OF THE ISOSCELES TRIANGLE THEOREM

With many theorems there are converses which we can use in problem solving. We have already seen one example in the converse to Pythagoras' theorem.

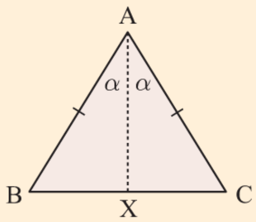
The isosceles triangle theorem has these converses:

Converse 1: If a triangle has two equal angles then it is isosceles.

Converse 2: The angle bisector of the apex of an isosceles triangle bisects the base at right angles.

Converse 3: The perpendicular bisector of the base of an isosceles triangle passes through its apex.

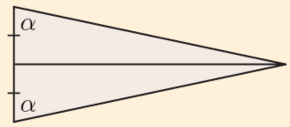
What to do:

1 

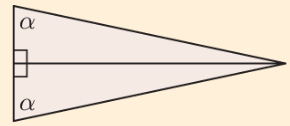
Triangle ABC is isosceles. The angle bisector at A meets [BC] at X. Prove *Converse 2* by using congruence to show that [AX] is perpendicular to [BC].

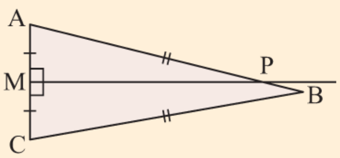
2 Sam wants to prove *Converse 1*.

a Suppose Sam draws a line from the apex to the midpoint of the base. Can he use congruence to prove *Converse 1*?



b Sam now decides to begin by drawing the perpendicular from the apex to the base. Can he now use congruence to prove *Converse 1*?



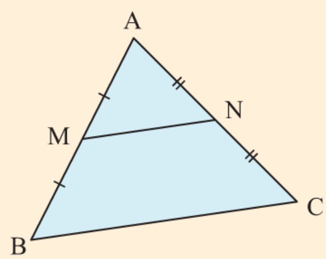
3 

Mustafa is trying to prove *Converse 3*. He draws the perpendicular bisector of the base so that it does not pass through vertex B, but instead meets [AB] at some other point P. By joining [CP], help Mustafa complete his proof.

INVESTIGATION 2 THE MIDPOINT THEOREM

In triangle ABC, M is the midpoint of [AB], and N is the midpoint of [AC].

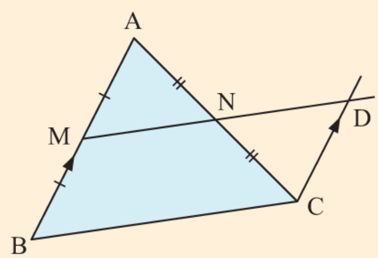
The **midpoint theorem** states that the line [MN] is parallel to [BC], and half its length.



Proving the midpoint theorem

What to do:

Suppose we extend [MN], and draw a line through C parallel to [AB]. We let these lines meet at D.



1 Show that triangles AMN and CDN are congruent.

2 Hence show that:

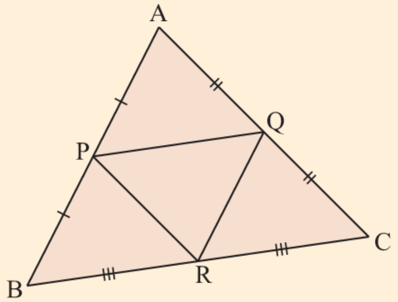
a MN = DN **b** BM = CD.

3 Show that BCDM is a parallelogram.

4 Hence, show that: **a** [MN] is parallel to [BC] **b** MN = $\frac{1}{2}$BC.

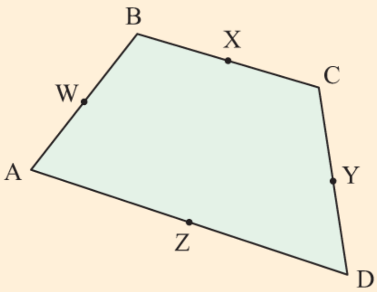
Using the midpoint theorem

What to do:

1 

In the diagram alongside, P, Q, and R are the midpoints of [AB], [AC], and [BC] respectively. Use the midpoint theorem to show that the four small triangles are all congruent.

2 For any quadrilateral ABCD, let W, X, Y, and Z be the midpoints of [AB], [BC], [CD], and [DA] respectively. Use the midpoint theorem to show that WXYZ is a parallelogram.

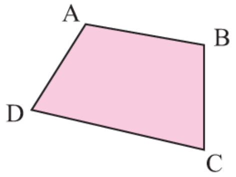


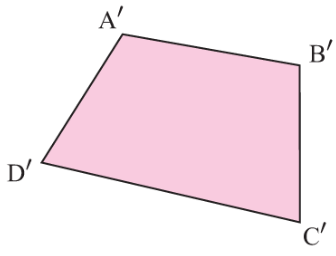
C SIMILARITY

Two figures are **similar** if one is an enlargement of the other, regardless of orientation.

If two figures are similar then their corresponding sides are **in proportion**. The lengths of their sides will be increased (or decreased) by the **same ratio** from one figure to the next. This ratio is called the **enlargement factor**.

Consider the enlargement below for which the enlargement factor k is 1.5.





Since $k = 1.5$, $\dfrac{A'B'}{AB} = \dfrac{B'C'}{BC} = \dfrac{C'D'}{CD} = \dfrac{D'A'}{DA} = \dfrac{B'D'}{BD} = ... = 1.5$.

When a figure is enlarged or reduced, the size of its angles does not change. The figures are therefore **equiangular**.

Two figures are **similar** if:

- the figures are **equiangular** *and*
- the corresponding sides are in the **same ratio**.

SIMILAR TRIANGLES

When we are dealing with triangles, if either of the above conditions is true, then the other condition must also be true. Therefore, when testing for similar triangles, we only need to check that *one* of the conditions is true.

> Two triangles are similar if *either*:
> - they are equiangular *or* • their side lengths are in the same ratio.

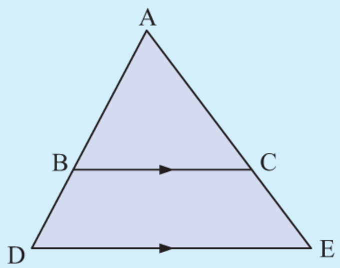
If we can show that two of the angles in one triangle are equal in size to two of the angles in another triangle, then the remaining angles must also be equal, since the angles in each triangle sum to 180°.

Once we have established that two triangles are similar, we can use the fact that corresponding sides are in the same ratio to find unknown lengths.

Example 3 Self Tutor

Show that the following figures possess similar triangles:

a



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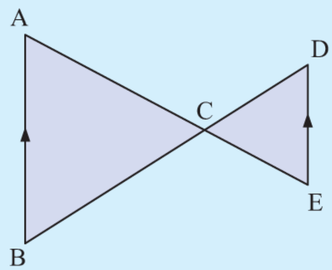
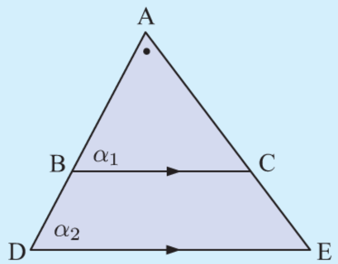


a

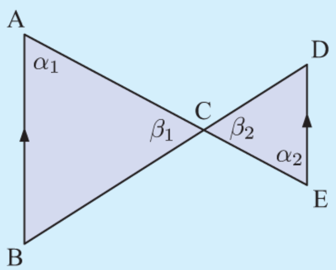


$\triangle$s ABC and ADE are equiangular as:

- $\alpha_1 = \alpha_2$ {equal corresponding angles}
- $\widehat{A}$ is common to both triangles

$\therefore$ the triangles are similar.

b



$\triangle$s ABC and EDC are equiangular as:

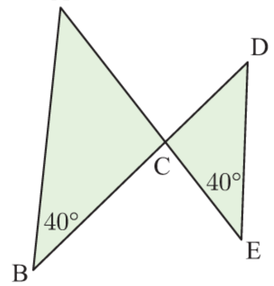
- $\alpha_1 = \alpha_2$ {equal alternate angles}
- $\beta_1 = \beta_2$ {vertically opposite angles}

$\therefore$ the triangles are similar.

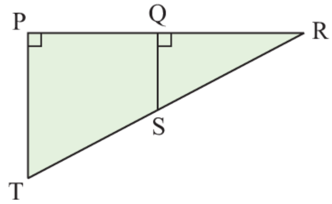
EXERCISE 7C

1 Show that the following figures possess similar triangles:

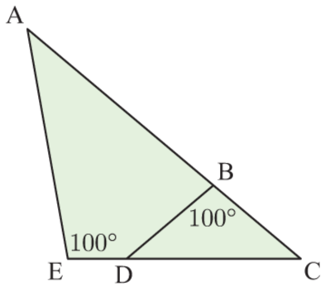
a



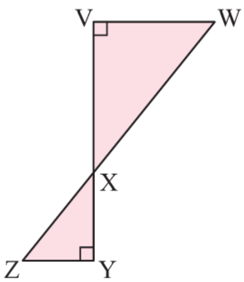
b



c



d



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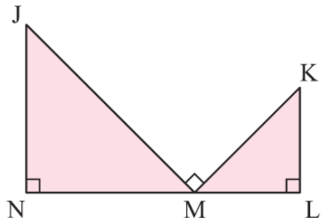

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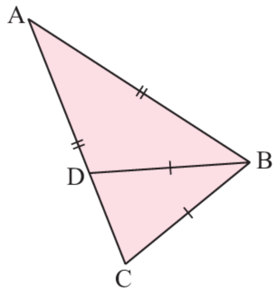
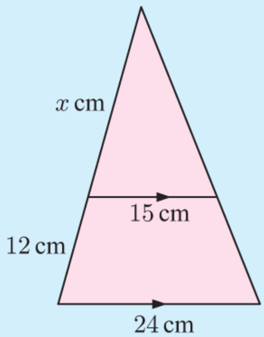
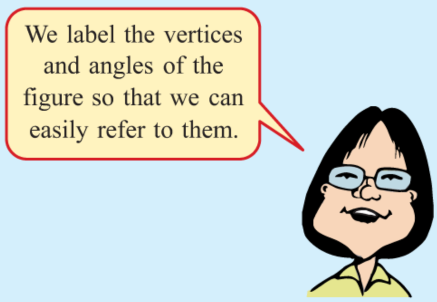
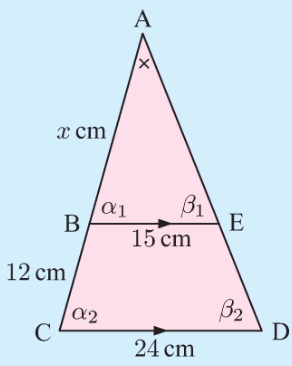


Example 4 — Self Tutor

Establish that a pair of triangles is similar, and hence find x:







$\triangle$s ABE and ACD are equiangular since

$\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ {corresponding angles}

$\therefore$ $\triangle$s ABE and ACD are similar.

Corresponding sides must be in the same ratio.

$$\therefore \frac{AC}{AB} = \frac{CD}{BE}$$

$$\therefore \frac{x+12}{x} = \frac{24}{15}$$

$$\therefore 1 + \frac{12}{x} = \frac{8}{5}$$

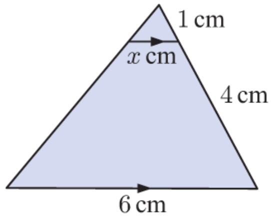
$$\therefore \frac{12}{x} = \frac{3}{5}$$

$$\therefore \frac{x}{12} = \frac{5}{3}$$

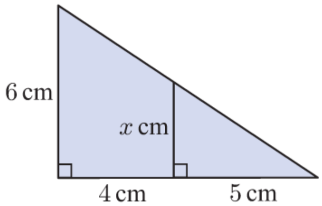
$$\therefore x = 20$$

2 For the following figures, establish that a pair of triangles is similar, and hence find x:

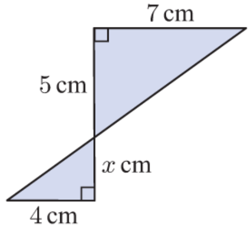
a



b



c



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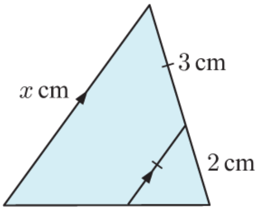
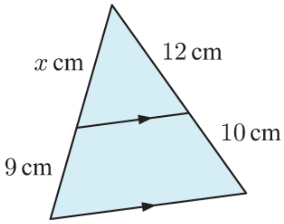


e



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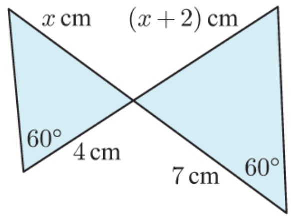

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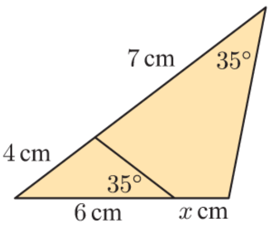

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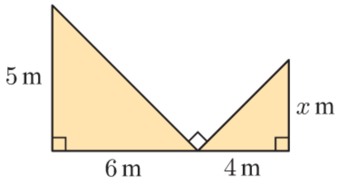
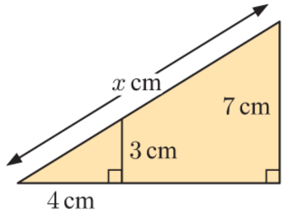


i

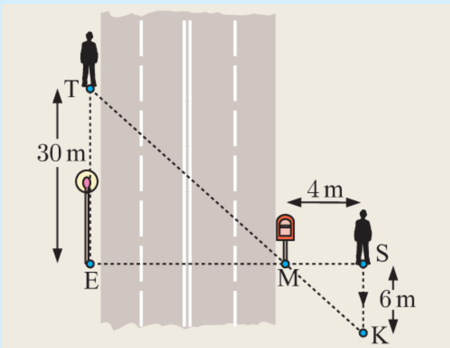


Example 5 **Self Tutor**

An electric light post E is directly opposite a mail box M on the other side of a straight road. Taj walks 30 metres along the road away from E to point T.

Kanvar is 4 metres away from M at point S, so that E, M, and S are in a straight line. Kanvar walks 6 metres parallel to the road in the opposite direction to Taj, to K. Now T, M, and K are in a straight line.

Find the width of the road.



Let the width of the road be x m.

$\triangle$s TEM and KSM are equiangular as:

- $T\widehat{E}M = K\widehat{S}M = 90°$
- $E\widehat{M}T = S\widehat{M}K$ {vertically opposite angles}

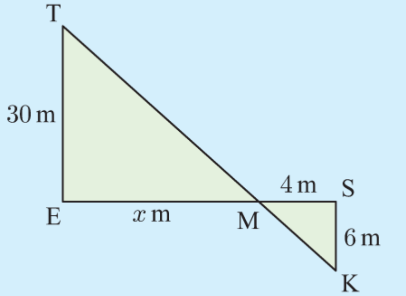
$\therefore$ $\triangle$s TEM and KSM are similar.

Corresponding sides must be in the same ratio.

$$\therefore \frac{EM}{SM} = \frac{TE}{KS}$$

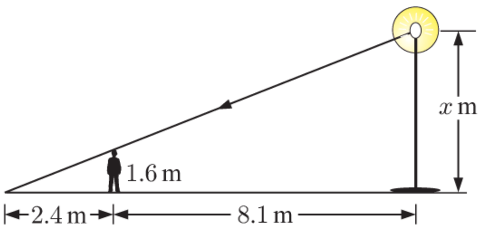
$$\therefore \frac{x}{4} = \frac{30}{6}$$

$$\therefore x = 5 \times 4 = 20$$

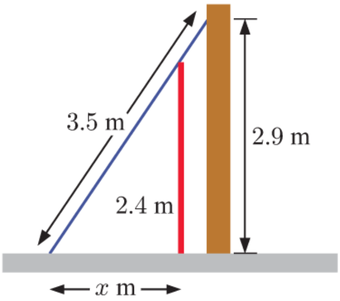


So, the road is 20 metres wide.

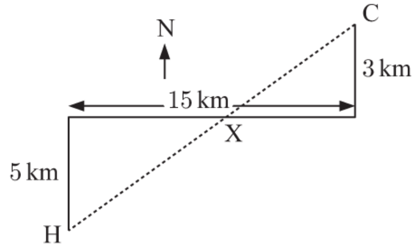
3 A boy who is 1.6 m tall stands 8.1 m from the base of an electric light pole. He casts a shadow 2.4 m long. How high above the ground is the light globe?



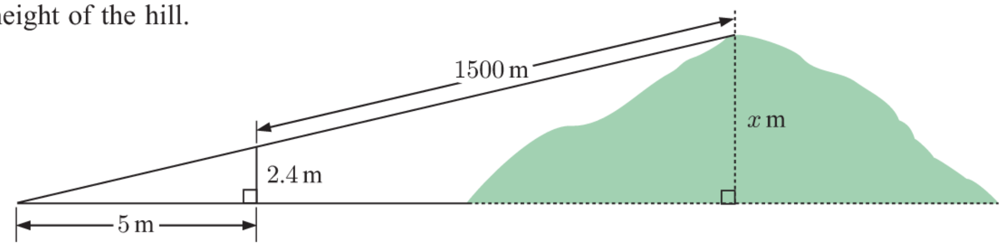
4 A 3.5 m ladder leans on a 2.4 m high fence. One end is on the ground and the other end touches a vertical wall 2.9 m from the ground.
How far is the bottom of the ladder from the fence?



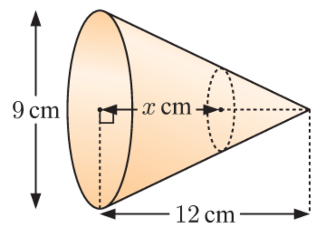
5 A hospital H receives a report about a serious road accident at C. An ambulance reaches the scene by travelling 5 km north, 15 km east, then 3 km north.
A helicopter travels directly from H to C. Their paths intersect at X.
Find the distance from the hospital to X.



6 Two surveyors estimate the height of a nearby hill. One stands 5 m away from the other on horizontal ground holding a measuring stick vertically. The other surveyor finds a "line of sight" to the top of the hill, and observes that this line passes the vertical stick at a height of 2.4 m. They measure the distance from the stick to the top of the hill to be 1500 m using laser equipment.
Find, correct to the nearest metre, their estimate for the height of the hill.

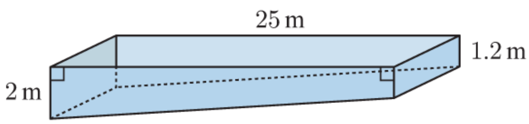


7 Mitchell pushes a coin of diameter 3 cm into a cone with diameter 9 cm and height 12 cm. How far into the cone can Mitchell push the coin before it gets stuck?



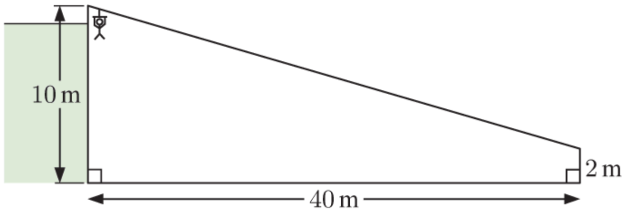
8 Answer the **Opening Problem** on page **132**.

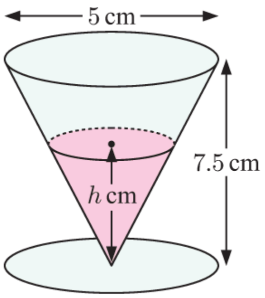
9



A swimming pool is 1.2 m deep at one end, and 2 m deep at the other end. The pool is 25 m long. Isaac jumps into the pool 10 m from the shallow end. How deep is the pool at this point?

10 It is safe to let go of the flying fox shown alongside when you are 3 m above the ground. How far can you travel along the flying fox before letting go?



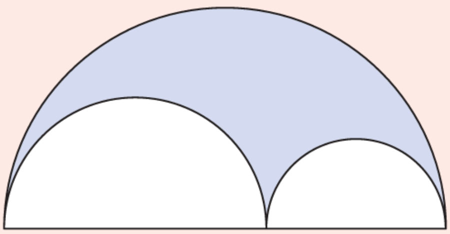
11 

The conical medicine glass alongside is filled with 20 mL of medicine.

To what height does the medicine level rise?

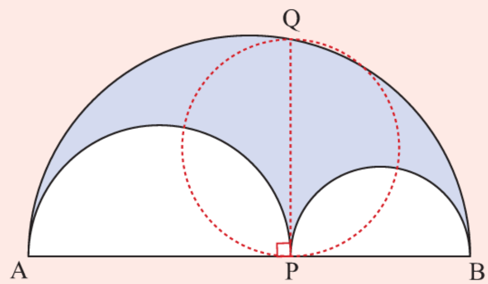
ACTIVITY — THE SHOEMAKER'S KNIFE

The shaded area alongside is called an **arbelos**, a Greek word meaning "shoemaker's knife". It is formed by drawing two smaller semi-circles inside a large semi-circle.



Archimedes showed that if a line is drawn from P perpendicular to [AB], meeting the arbelos again at Q, then the area of the arbelos is equal to the area of the circle with diameter [PQ].

Can you use similarity to prove this fact?



D AREAS AND VOLUMES

Triangle **A** has base b cm and height h cm.

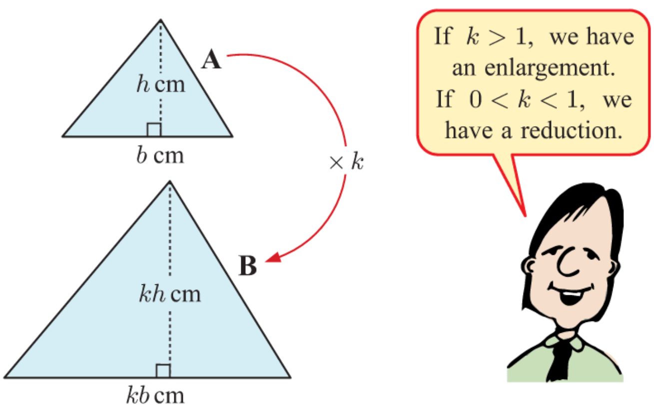
Suppose it is enlarged with scale factor k to produce a *similar* triangle **B**.

Area of triangle **B**

$= \frac{1}{2}(kb)(kh)$

$= k^2(\frac{1}{2}bh)$

$= k^2 \times$ area of triangle **A**.



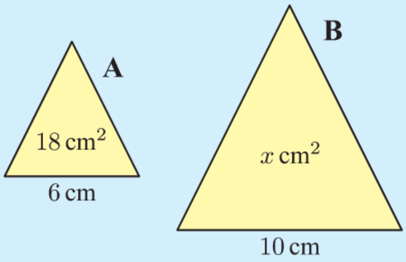
This suggests that:

> If a figure is enlarged with scale factor k to produce a similar figure, then the new area $= k^2 \times$ the old area.

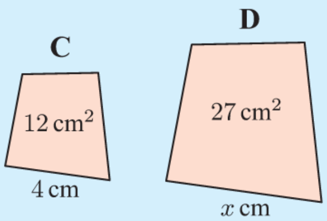
Example 6 — Self Tutor

For the following similar figures, find x:

a



b



a **A** is enlarged with scale factor k to give **B**.

$\therefore \quad k = \frac{10}{6} = \frac{5}{3}$

Area of **B** $= k^2 \times$ area of **A**

$\therefore \quad x = (\frac{5}{3})^2 \times 18$

$\therefore \quad x = 50$

b **C** is enlarged with scale factor k to give **D**.

Area of **D** $= k^2 \times$ area of **C**

$\therefore \quad 27 = k^2 \times 12$

$\therefore \quad \frac{9}{4} = k^2$

$\therefore \quad k = \frac{3}{2} \quad \{\text{as } k > 0\}$

Since the sides are in the same ratio,

$x = \frac{3}{2} \times 4$

$\therefore \quad x = 6$

VOLUME

The cylinder **A** has radius r cm and height h cm. Suppose it is enlarged with scale factor k to produce a *similar* cylinder **B**.

The radius of cylinder **B** will be kr, and its height will be kh.

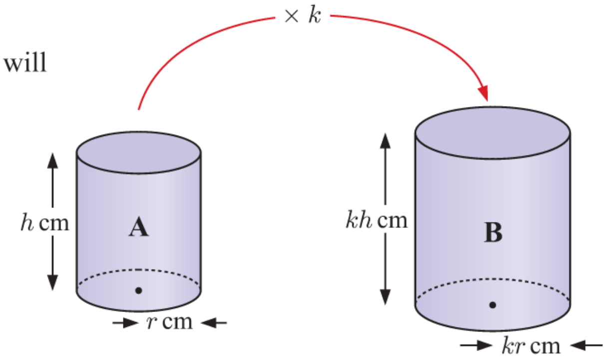
Volume of cylinder **B**

$= \pi(kr)^2(kh)$

$= \pi(k^2r^2)(kh)$

$= k^3(\pi r^2 h)$

$= k^3 \times$ volume of cylinder **A**.



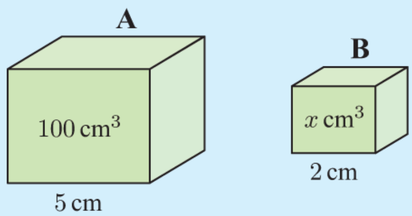
This suggests that:

> If a 3-dimensional figure is enlarged with scale factor k to produce a similar figure, then the new volume $= k^3 \times$ the old volume.

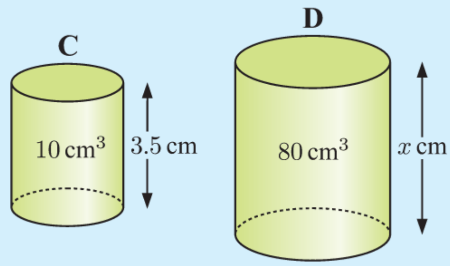
Example 7 **Self Tutor**

For the following similar figures, find x:

a



b



a **A** is reduced with scale factor k to give **B**.

$\therefore \quad k = \frac{2}{5}$

Volume of **B** $= k^3 \times$ volume of **A**

$\therefore \quad x = (\frac{2}{5})^3 \times 100$

$\therefore \quad x = 6.4$

b **C** is enlarged with scale factor k to give **D**.

Volume of **D** $= k^3 \times$ volume of **C**

$\therefore \quad 80 = k^3 \times 10$

$\therefore \quad 8 = k^3$

$\therefore \quad k = 2$

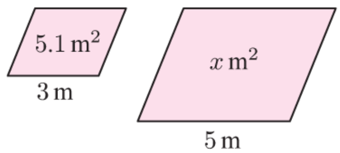
So, $\quad x = 2 \times 3.5$

$\therefore \quad x = 7$

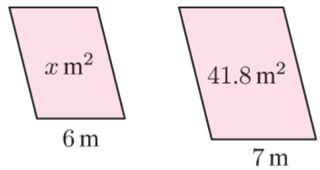
EXERCISE 7D

1 For each pair of similar figures, find x:

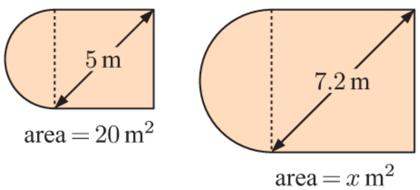
a



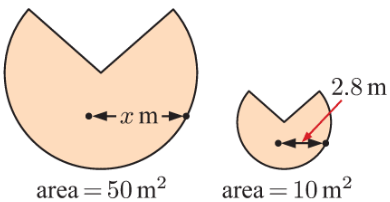
b



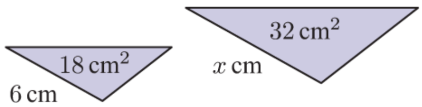
c



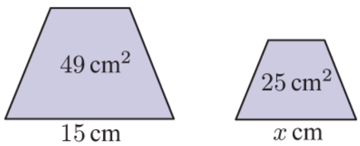
d



e

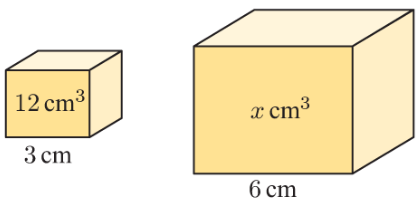


f

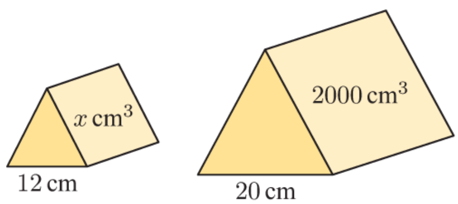


2 For each pair of similar figures, find x:

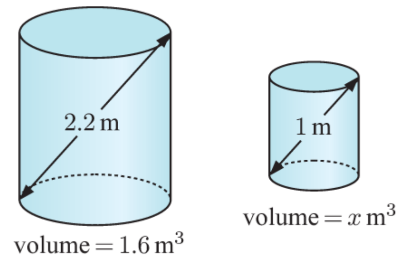
a



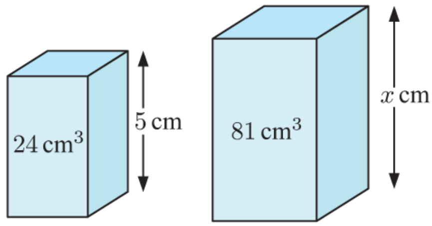
b



c



d



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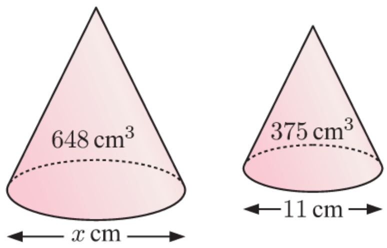

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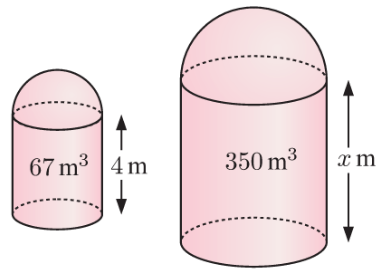

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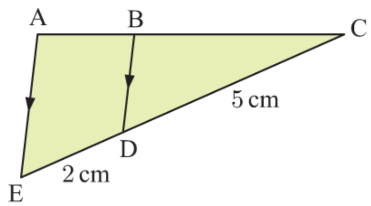
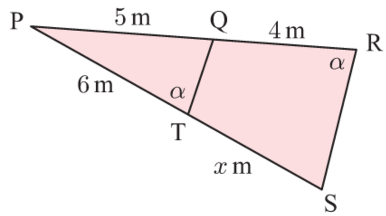


The area of $\triangle$BCD is 6.4 cm^2. Find the area of:

a $\triangle$ACE **b** quadrilateral ABDE.

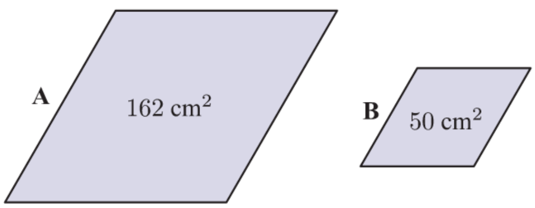
4



a Find the value of x.

b Quadrilateral QRST has area 22 m^2. Find the area of $\triangle$PQT.

5 Rhombuses **A** and **B** are similar.
Given that each side of **A** is 13.5 cm long, find the perimeter of **B**.



6 The *density* of an object is its ratio of mass to volume. Objects made of the same material have the same density, so their mass is in proportion to their volume.

a What will happen to the mass of a sphere if its radius is:

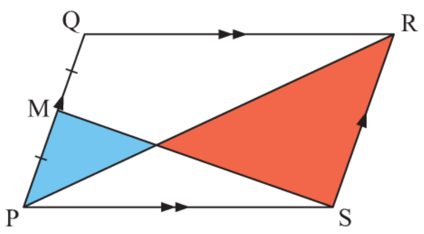
i doubled **ii** increased by 20%?

b What will happen to the mass of a cylinder if its radius and height are both:

i halved **ii** increased by 50%?

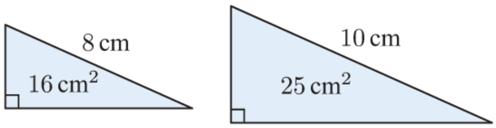
c Two similar cones made from the same material have surface areas 192 cm^2 and 75 cm^2. The volume of the larger cone is 200 cm^3. The mass of the smaller cone is 320 g.

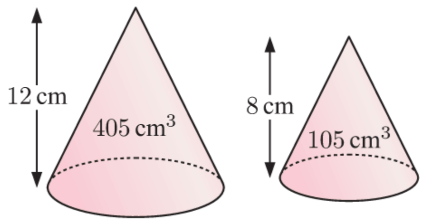
i Find the volume of the smaller cone. **ii** Find the mass of the larger cone.

7 

In parallelogram PQRS, M is the midpoint of [PQ]. Show that the red area is 4 times larger than the blue area.

8 Determine whether each of the pairs of figures below are similar.

a 

b 

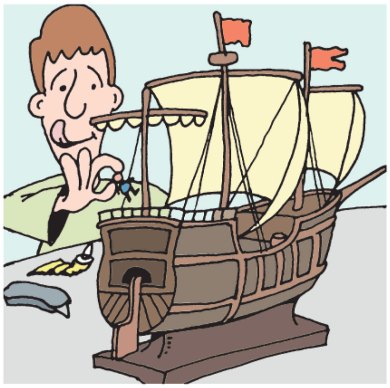
9 A scale model is made of a 300 year old sailing ship. The model is a 1 : 200 reduction of the original. Find:

a the height of the mast in the model if the original mast was 20 m high

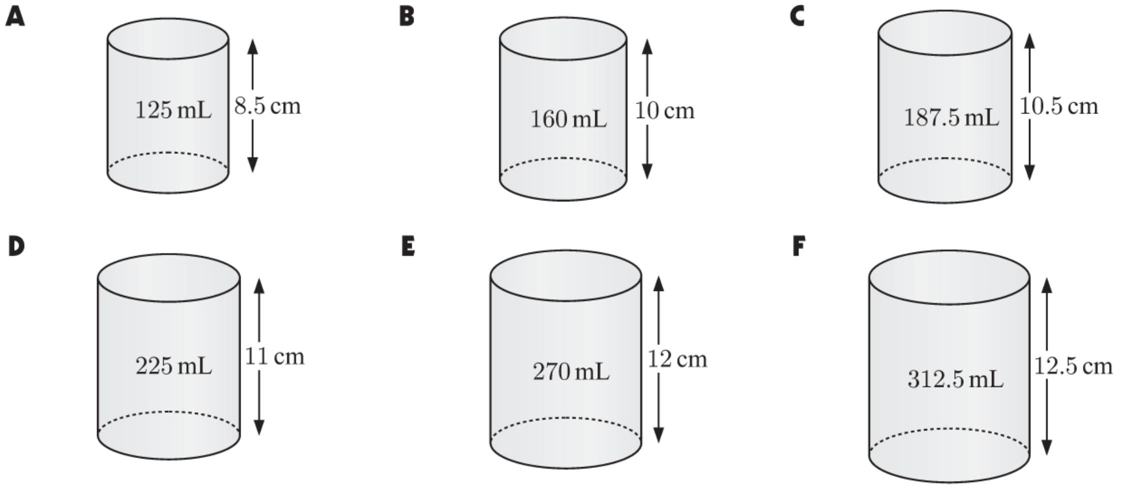
b the area of a sail in the model if the original sail was 120 m^2

c the height and radius of a keg in the model if the original was 1.2 m high and 0.9 m in diameter

d the capacity of the water tank in the model if the capacity of the original was 10 000 litres.



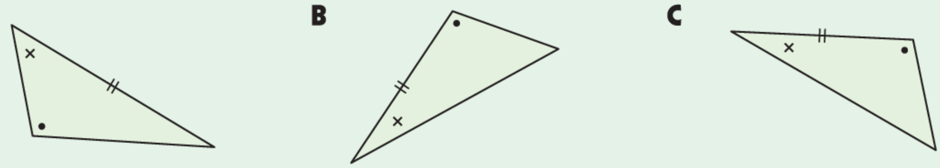
10 A glassware company manufactures cylindrical drinking glasses in six different sizes. Their heights and capacities are given below. Which two of the glasses are similar?



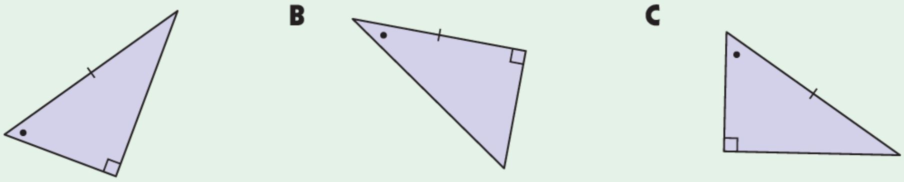
REVIEW SET 7A

1 In each set of three triangles, two are congruent. State which pair is congruent giving a reason for your answer. The triangles are not drawn to scale, but contain correct information.

a **A** **B** **C**



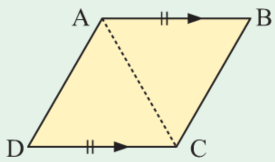
b **A** **B** **C**



2 Consider the quadrilateral ABCD.

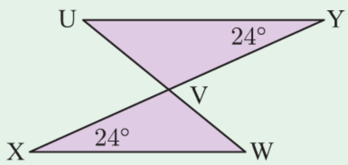
a Show that triangles ABC and CDA are congruent.

b Hence deduce that ABCD is a parallelogram.

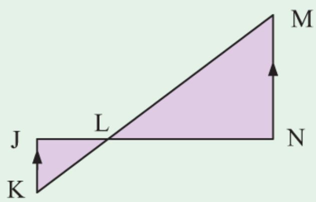


3 Show that the following figures possess similar triangles.

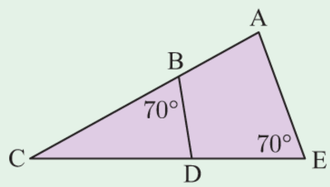
a



b

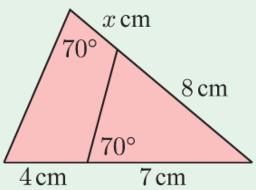


c

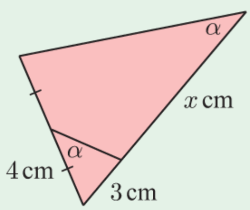


4 In each of the following figures, establish that a pair of triangles is similar, and hence find x:

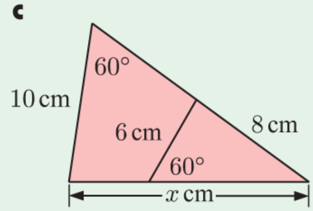
a



b



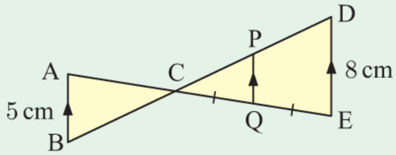
c



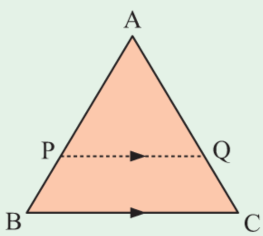
5 $\triangle$ABC has an area of 15 cm^2.

a Find the area of $\triangle$CDE.

b Find the area of PQED.

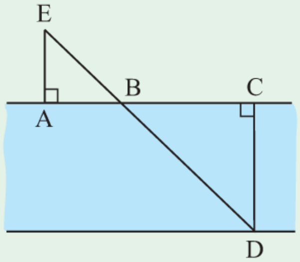


6

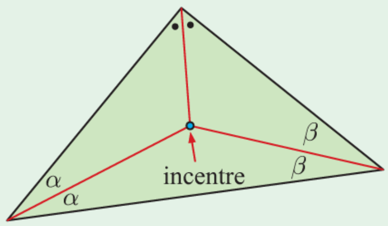


Triangle ABC is isosceles, with AB = AC.
[PQ] is parallel to [BC].
Show that CP = BQ.

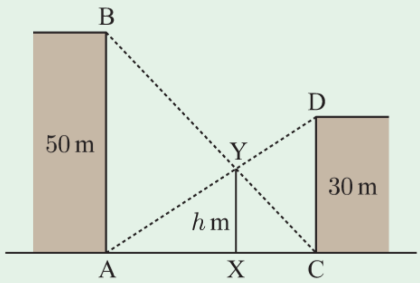
7 A, B, and C are pegs on the bank of a canal which has parallel straight sides. C and D are directly opposite each other. AB = 30 m and BC = 140 m.
When I walk from A directly away from the bank, I reach a point E, 25 m from A, such that E, B, and D line up. How wide is the canal?



8 The three angle bisectors of a triangle meet at a point called the **incentre** of the triangle.
Show that the incentre is equidistant from each edge of the triangle.
Hint: Draw a perpendicular line from each edge to the incentre.



9



The vertical walls of two buildings are 50 m and 30 m tall. A vertical flagpole [XY] stands between the buildings such that B, Y, and C are collinear, and A, Y, and D are collinear.

a Show that $\frac{h}{50} + \frac{h}{30} = 1$.

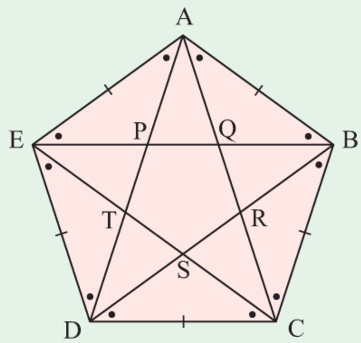
b Hence, find the height of the flagpole.

10 When the diagonals of the regular pentagon ABCDE are drawn, a smaller pentagon PQRST is formed.

a Explain why all of the angles marked • are of equal size.

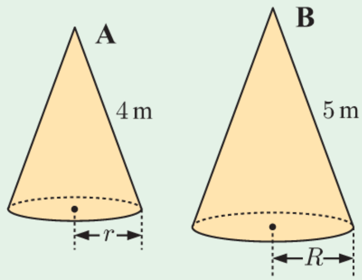
b Hence show that the interior angles of PQRST are of equal size.

c Hence show that PQRST is a regular pentagon.



11 A sphere of lead with radius 10 cm is melted into 125 identical smaller spheres. Find the radius of each new sphere.

12



The slant heights of two similar cones are 4 m and 5 m respectively.

a Find the ratio $R : r$.

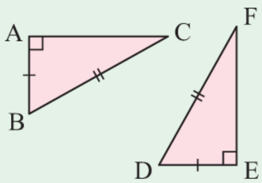
b Find the ratio of the surface areas for the curved part of each figure.

c Find the ratio of the volumes of the cones.

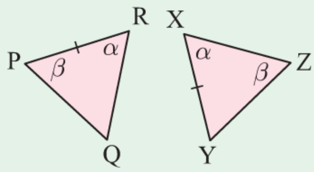
REVIEW SET 7B

1 Are these triangles congruent? If so, state the congruence relationship and give a brief reason.

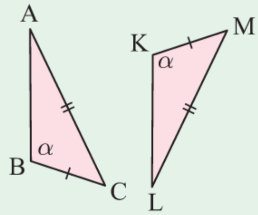
a



b



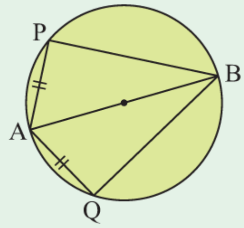
c



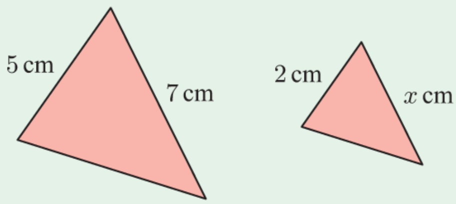
2 [AB] is a diameter of the circle.

  a Show that the figure contains congruent triangles.

  b What other facts can then be deduced about the figure?

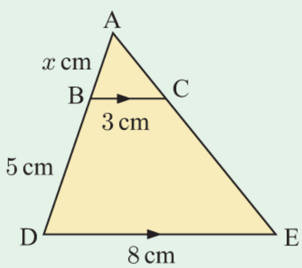


3 The figures alongside are similar. Find x.

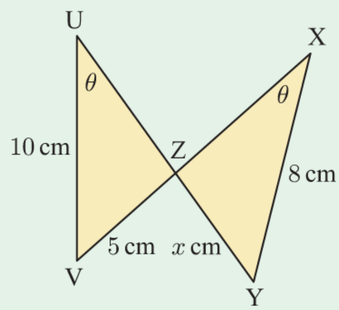


4 In the following figures, establish that a pair of triangles is similar, then find x:

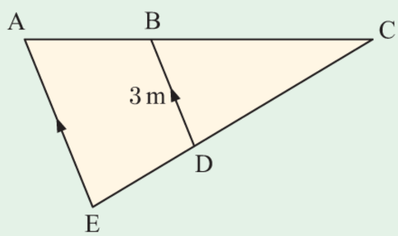
a



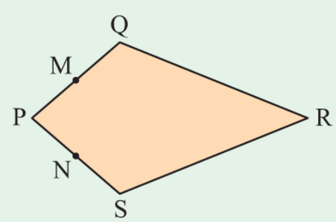
b



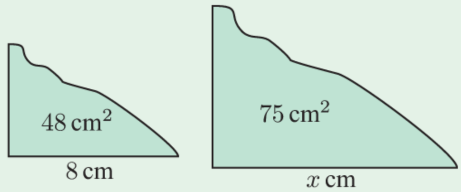
5 Triangle BCD has area 8 m^2, and quadrilateral ABDE has area 12 m^2.
Find the length of [AE].



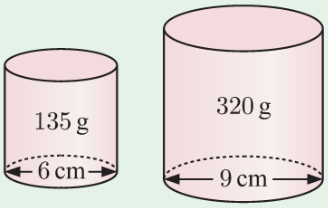
6 PQRS is a kite, with PQ = PS and QR = SR. M and N are the midpoints of [PQ] and [PS] respectively.
Prove that triangle MNR is isosceles.



7 For the following similar figures, find x.



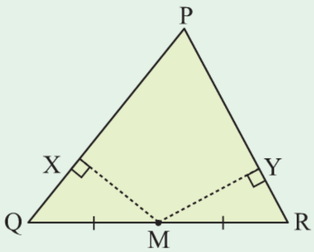
8 The cylinders below are made from the same material, so their densities are the same. Determine whether the cylinders are similar.



9 In $\triangle$PQR, M is the midpoint of [QR]. [MX] is drawn perpendicular to [PQ], and [MY] is drawn perpendicular to [PR].
Suppose these perpendiculars are equal in length.

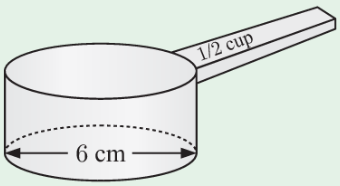
a Prove that $\triangle$MQX is congruent to $\triangle$MRY.

b Hence, prove that $\triangle$PQR is isosceles.

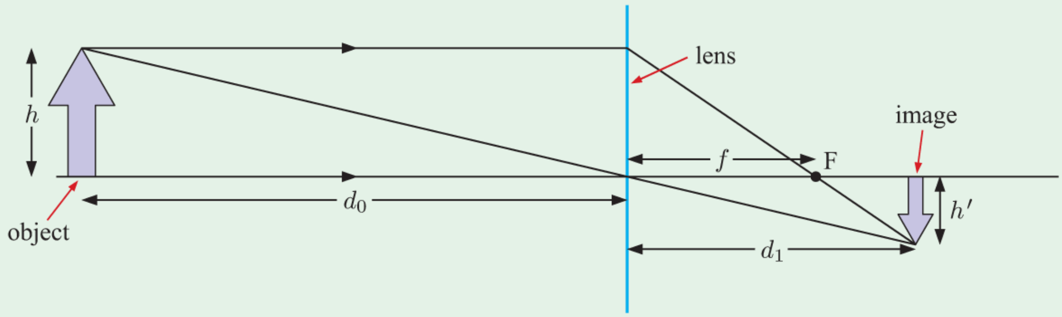


10 In a measuring cup set, the $\frac{1}{2}$ cup measure is 6 cm wide.
The set also contains a 1 cup measure and a $\frac{1}{3}$ cup measure, both of which are similar in shape to the $\frac{1}{2}$ cup measure.
Find the width of:

a the 1 cup measure **b** the $\frac{1}{3}$ cup measure.



11



In optics, the **thin lens equation** states that $\dfrac{1}{f} = \dfrac{1}{d_0} + \dfrac{1}{d_1}$, where:

f is the distance between the lens and the focal point F
d_0 is the distance between the object and the lens
d_1 is the distance between the image and the lens.

a Use similar triangles to show that: **i** $\dfrac{h'}{h} = \dfrac{d_1}{d_0}$ **ii** $\dfrac{h'}{h} = \dfrac{d_1 - f}{f}$

b Hence, show that $\dfrac{1}{f} = \dfrac{1}{d_0} + \dfrac{1}{d_1}$.