

LEVEL 1

Unit Penalty applies in parts (a) and (d) and Financial Penalty applies in parts (b) and (e).

(a) $BD^2 = 190^2 + 120^2 - 2(190)(120)\cos 75^\circ$ (M1)(A1)

Note: Award (M1) for substituted cosine formula, (A1) for correct substitution.

$= 197 \text{ m}$ (A1)(G2)

Note: If radians are used award a maximum of (M1)(A1)(A0).

(b) $\text{cost} = 196.717 \dots \times 17$ (M1)

$= 3344 \text{ USD}$ (A1)(ft)(G2)

Note: Accept 3349 from 197.

(c) $\frac{\sin(\text{ABD})}{70} = \frac{\sin(115^\circ)}{196.7}$ (M1)(A1)

Note: Award (M1) for substituted sine formula, (A1) for correct substitution.

$= 18.81^\circ \dots$ (A1)(ft)

$= 18.8^\circ$ (AG)

Notes: Both the unrounded and rounded answers must be seen for the final (A1) to be awarded. Follow through from their (a). If 197 is used the unrounded answer is $18.78^\circ \dots$

(d) $\text{angle BDA} = 46.2^\circ$ (A1)

$\text{Area} = \frac{70 \times (196.717 \dots) \times \sin(46.2^\circ)}{2}$ (M1)(A1)

Note: Award (M1) for substituted area formula, (A1) for correct substitution.

$\text{Area of ABD} = 4970 \text{ m}^2$ (A1)(ft)(G2)

Notes: If 197 used answer is 4980.

Notes: Follow through from (a) only. Award (G2) if there is no working shown and 46.2° not seen. If 46.2° seen without subsequent working, award (A1)(G2).

(e) $4969.38 \dots \times 120$ (M1)

$= 596\,327 \text{ USD}$ (A1)(ft)(G2)

Notes: Follow through from their (d).

(f) $300\,000 \left(1 + \frac{r}{100}\right)^{15} = 600\,000$ or equivalent (A1)(M1)(A1)

Notes: Award (A1) for 600 000 seen or implied by alternative formula, (M1) for substituted CI formula, (A1) for correct substitutions.

$r = 4.73$ (A1)(ft)(G3)

Notes: Award G3 for 4.73 with no working. Award G2 for 4.7 with no working.

2)

Unit penalty applies in parts (b) (c) and (d).

(a) Angle ABC = 50° (A1)

(b) $\frac{AC}{\sin 50^\circ} = \frac{25}{\sin 55^\circ}$ (M1)(A1)(ft)

Notes: Award (M1) for substitution into the correct formula, (A1)(ft) for correct substitution. Follow through from their angle ABC.

AC = 23.4 m (A1)(ft)(G2)

(c) Area of Δ ABC = $\frac{1}{2} \times 23.379... \times 25 \times \sin 75^\circ$ (M1)(A1)(ft)

Notes: Award (M1) for substitution into the correct formula, (A1)(ft) for correct substitution. Follow through from their AC.

OR

Area of triangle ABC = $\frac{29.479... \times 19.151...}{2}$ (A1)(ft)(M1)

Note: (A1)(ft) for correct values of AB (29.479...) and CN (19.151...). Follow through from their (a) and/or (b). Award (M1) for substitution of their values of AB and CN into the correct formula.

Area of Δ ABC = 282 m² (A1)(ft)(G2)

Note: Accept 283 m² if 23.4 is used.

(d) $NM = \frac{25 \times \sin 50^\circ}{2}$ (M1)(M1)

Note: Award (M1) for $25 \times \sin 50^\circ$ or equivalent for the length of CN, (M1) for dividing their CN by 2.

NM = 9.58 m (A1)(ft)(G2)

Note: Follow through from their angle ABC.

Notes: Premature rounding of CN leads to the answers 9.60 or 9.6.

Award at most (M1)(M1)(A0) if working seen. Do not penalize with (AP).

CN may be found in (c).

Note: The working for this part of the question may be in part (b).

- (e) Angle $NCB = 40^\circ$ seen (A1)(ft)

Note: Follow through from their (a).

From triangle MCP:

$$MP^2 = (9.5756\dots)^2 + 12.5^2 - 2 \times 9.5756\dots \times 12.5 \times \cos(40^\circ) \quad (M1)(A1)(ft)$$

$$MP = 8.034\dots \text{ m} \quad (A1)(ft)(G3)$$

Notes: Award (M1) for substitution into the correct formula, (A1)(ft) for their correct substitution. Follow through from their (d).

Award (G3) for correct value of MP seen without working.

OR

From right triangle MCP

$$CP = 12.5 \text{ m seen} \quad (A1)$$

$$MP^2 = (12.5)^2 - (9.575\dots)^2 \quad (M1)(A1)(ft)$$

$$MP = 8.034\dots \text{ m} \quad (A1)(G3)(ft)$$

Notes: Award (M1) for substitution into the correct formula, (A1)(ft) for their correct substitution. Follow through from their (d).

Award (G3) for correct value of MP seen without working.

OR

From right triangle MCP

$$\text{Angle MCP} = 40^\circ \text{ seen} \quad (A1)(ft)$$

$$\frac{MP}{12.5} = \sin(40^\circ) \text{ or equivalent} \quad (M1)(A1)(ft)$$

$$MP = 8.034\dots \text{ m} \quad (A1)(G3)(ft)$$

Notes: Award (M1) for substitution into the correct formula, (A1)(ft) for their correct substitution. Follow through from their (a).

Award (G3) for correct value of MP seen without working.

The goat cannot reach point P as $MP > 7 \text{ m}$. (A1)(ft)

Note: Award (A1)(ft) only if their value of MP is compared to 7 m, and conclusion is stated.

3)

(a) (i) $A(0, 4)$ *Accept* $x = 0, y = 4$ (A1)

(ii) $B(8, 0)$ *Accept* $x = 8, y = 0$ (A1)(ft)

Note: Award (A0) if coordinates are reversed in (i) and (A1)(ft) in (ii).

(b) $AB = \sqrt{8^2 + 4^2} = \sqrt{80}$ (M1)

$AB = 8.944$ (A1)

$= 8.94$ (AG)

(c) (i) $y = -0.5x + 4$ (M1)

Gradient $AB = -0.5$ (A1)

Note: Award (A2) if -0.5 seen.

OR

Gradient $AB = \frac{(0-4)}{(8-0)}$ (M1)

$= -\frac{1}{2}$ (A1)

Note: Award (M1) for correct substitution in the gradient formula. Follow through from their answers to part (a).

Gradient $CN = 2$ (A1)(ft)(G2)

Note: Special case: Follow through for gradient CN from their gradient AB .

(ii) $CN: y = 2x + c$
 $7 = 2(4) + c$ (M1)

Note: Award (M1) for correct substitution in equation of a line.

$y = 2x - 1$ (A1)(ft)(G2)

Note: Accept alternative forms for the equation of a line including $y - 7 = 2(x - 4)$. Follow through from their gradient in (i).

Note: If $c = -1$ seen but final answer is not given, award (A1)(d).

(d) $x + 2(2x - 1) = 8$ or equivalent (M1)

$N(2, 3)$ ($x = 2, y = 3$) (A1)(A1)(ft)(G3)

Note: Award (M1) for attempt to solve simultaneous equations or a sketch of the two lines with an indication of the point of intersection.

(e) Cosine rule: $\cos(\hat{A}CB) = \frac{5^2 + 8.06^2 - 8.944^2}{2 \times 5 \times 8.06}$ (M1)(A1)

Note: Award (M1) for use of cosine rule with numbers from the problem substituted, (A1) for correct substitution.

$\hat{A}CB = 82.9^\circ$ (A1)(G2)

Note: If alternative right-angled trigonometry method used award (M1) for use of trig ratio in both triangles, (A1) for correct substitution of their values in each ratio, (A1) for answer.

Note: Accept 82.8° with use of 8.94.

(f) Area ACB = $\frac{5 \times 8.06 \sin(82.9)}{2}$ (M1)(A1)(ft)

Note: Award (M1) for substituted area formula, (A1) for correct substitution. Follow through from their angle in part (e).

OR

Area ACB = $\frac{AB \times CN}{2} = \frac{8.94 \times \sqrt{(4-2)^2 + (7-3)^2}}{2}$ (M1)(M1)(ft)

Note: Award (M1) substituted area formula with their values, (M1) for substituted distance formula. Follow through from coordinates of N.

Area ACB = 20.0 (A1)(ft)(G2)

Note: Accept 20

4)

Unit penalty (UP) applies in parts (a), (c) and (e).

(a) $AB^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 150^\circ$ (M1)(A1)

$AB = 17.4 \text{ km}$ (A1)(G2)

Note: Award (M1) for substitution into correct formula, (A1) for correct substitution, (A1) for correct answer.

(b) $\frac{8}{\sin \hat{CAB}} = \frac{17.4}{\sin 150^\circ}$ (M1)(A1)

$\hat{CAB} = 13.3^\circ$ (A1)(ft)(G2)

Notes: Award (M1) for substitution into correct formula, (A1) for correct substitution, (A1) for correct answer. Follow through from their answer to part (a).

(c) $AD = 8.70 \text{ km}$ (8.7 km) (A1)(ft)

Note: Follow through from their answer to part (a).

(d) $DT = \tan(13.29\dots^\circ) \times 8.697\dots = 2.0550\dots$ (M1)(A1)
 $= 2.06$ (AG)

Notes: Award (M1) for correct substitution in the correct formula, award (A1) for the unrounded answer seen. If 2.06 not seen award at most (M1)(A0).

(e) $\sqrt{8.70^2 + 2.06^2} + 8.70 + 2.06$ (A1)(M1)

$= 19.7 \text{ km}$ (A1)(ft)(G2)

Note: Award (A1) for AT, (M1) for adding the three sides of the triangle ADT, (A1)(ft) for answer. Follow through from their answer to part (c).

(f) $\frac{19.7}{70} \times 60 + 10$ (M1)(M1)

$= 26.9$ (A1)(ft)

Note: Award (M1) for time on road in minutes, (M1) for adding 10, (A1)(ft) for unrounded answer. Follow through from their answer to (e).

$= 27$ (nearest minute) (A1)(ft)(G3)

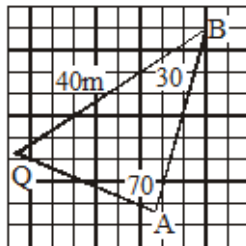
Note: Award (A1)(ft) for their unrounded answer given to the nearest minute.

LEVEL 2

1)

(a) $\frac{PQ}{40} = \tan 36^\circ$
 $\Rightarrow PQ \approx 29.1 \text{ m (3 sf)}$ (A1) (C1)

(b)



$\hat{AQB} = 80^\circ$ (A1)

$\frac{AB}{\sin 80^\circ} = \frac{40}{\sin 70^\circ}$ (M1)

Note: Award (M1) for correctly substituting.

$\Rightarrow AB = 41.9 \text{ m (3 sf)}$ (A1) (C3)

[4]

2)

(a) finding $\hat{ABC} = 110^\circ (= 1.92 \text{ radians})$ (A1)

evidence of choosing cosine rule (M1)

e.g. $AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos \hat{ABC}$

correct substitution

A1

e.g. $AC^2 = 25^2 + 40^2 - 2(25)(40) \cos 110^\circ$

$AC = 53.9 \text{ (km)}$

A1 N3

(b) **METHOD 1**

correct substitution into the sine rule

A1

e.g. $\frac{\sin \hat{BAC}}{40} = \frac{\sin 110^\circ}{53.9}$

$\hat{BAC} = 44.2^\circ$

A1

bearing = 074°

A1 N1

METHOD 2

correct substitution into the cosine rule

A1

e.g. $\cos \hat{BAC} = \frac{40^2 - 25^2 - 53.9^2}{-2(25)(53.9)}$

$\hat{BAC} = 44.3^\circ$

A1

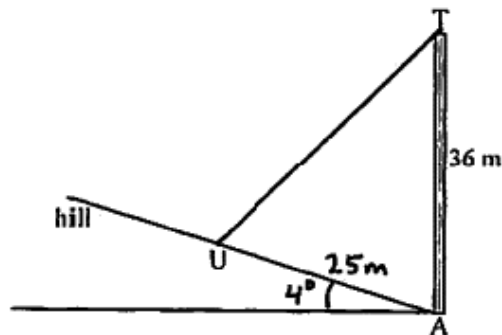
bearing = 074°

A1 N1

[7]

3)

(a)



A1A1A1 N3 3

Note: Award A1 for labelling 4° with horizontal, A1 for labelling [AU] 25 metres, A1 for drawing [TU].

(b) $\angle TAU = 86^\circ$

(A1)

evidence of choosing cosine rule

(M1)

correct substitution

A1

e.g. $x^2 = 25^2 + 36^2 - 2(25)(36) \cos 86^\circ$

$x = 42.4$

A1 N3 4

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4)

(a) Sine rule $\frac{PR}{\sin 35} = \frac{9}{\sin 120}$

(M1)(A1)

$$PR = \frac{9 \sin 35}{\sin 120}$$

$$= 5.96 \text{ km}$$

(A1) 3

(b) **EITHER**

Sine rule to find PQ

$$PQ = \frac{9 \sin 25}{\sin 120}$$

$$= 4.39 \text{ km}$$

(M1)(A1)

(A1)

OR

Cosine rule: $PQ^2 = 5.96^2 + 9^2 - (2)(5.96)(9) \cos 25$

$$= 19.29$$

(M1)(A1)

$PQ = 4.39 \text{ km}$

(A1)

Time for Tom = $\frac{4.39}{8}$

(A1)

Time for Alan = $\frac{5.96}{a}$

(A1)

Then $\frac{4.39}{8} = \frac{5.96}{a}$

(M1)

$a = 10.9$

(A1) 7

(c) $RS^2 = 4QS^2$ (A1)

$4QS^2 = QS^2 + 81 - 18 \times QS \times \cos 35$ (M1)(A1)

$3QS^2 + 14.74QS - 81 = 0$ (or $3x^2 + 14.74x - 81 = 0$) (A1)

$\Rightarrow QS = -8.20$ or $QS = 3.29$ (G1)

therefore $QS = 3.29$ (A1)

OR

$\frac{QS}{\sin \hat{S}RQ} = \frac{2QS}{\sin 35}$ (M1)

$\Rightarrow \sin \hat{S}RQ = \frac{1}{2} \sin 35$ (A1)

$\hat{S}RQ = 16.7^\circ$ (A1)

Therefore, $\hat{Q}SR = 180 - (35 + 16.7)$
 $= 128.3^\circ$ (A1)

$\frac{9}{\sin 128.3} = \frac{QS}{\sin 16.7} \left(= \frac{SR}{\sin 35} \right)$ (M1)

$QS = \frac{9 \sin 16.7}{\sin 128.3} \left(= \frac{9 \sin 35}{2 \sin 128.3} \right)$
 $= 3.29$ (A1)

6

[16]

5)

Note: Do not penalize missing units in this question.

$$\begin{aligned}
 \text{(a) } AB^2 &= 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 75^\circ && \text{(A1)} \\
 &= 12^2(2 - 2 \cos 75^\circ) && \text{(A1)} \\
 &= 12^2 \times 2(1 - \cos 75^\circ) \\
 AB &= 12\sqrt{2(1 - \cos 75^\circ)} && \text{(AG) } 2
 \end{aligned}$$

Note: The second (A1) is for transforming the initial expression to any simplified expression from which the given result can be clearly seen.

$$\begin{aligned}
 \text{(b) } \hat{POB} &= 37.5^\circ && \text{(A1)} \\
 BP &= 12 \tan 37.5^\circ && \text{(M1)} \\
 &= 9.21 \text{ cm} && \text{(A1)}
 \end{aligned}$$

OR

$$\hat{BPA} = 105^\circ \quad \hat{BAP} = 37.5^\circ \quad \text{(A1)}$$

$$\frac{AB}{\sin 105^\circ} = \frac{BP}{\sin 37.5^\circ} \quad \text{(M1)}$$

$$BP = \frac{AB \sin 37.5^\circ}{\sin 105^\circ} = 9.21(\text{cm}) \quad \text{(A1) } 3$$

$$\begin{aligned}
 \text{(c) (i) Area } \triangle OBP &= \frac{1}{2} \times 12 \times 9.21 \left(\text{or } \frac{1}{2} \times 12 \times 12 \tan 37.5^\circ \right) && \text{(M1)} \\
 &= 55.3 \text{ (cm}^2\text{)} \text{ (accept } 55.2 \text{ cm}^2\text{)} && \text{(A1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) Area } \triangle ABP &= \frac{1}{2} (9.21)^2 \sin 105^\circ && \text{(M1)} \\
 &= 41.0 \text{ (cm}^2\text{)} \text{ (accept } 40.9 \text{ cm}^2\text{)} && \text{(A1) } 4
 \end{aligned}$$

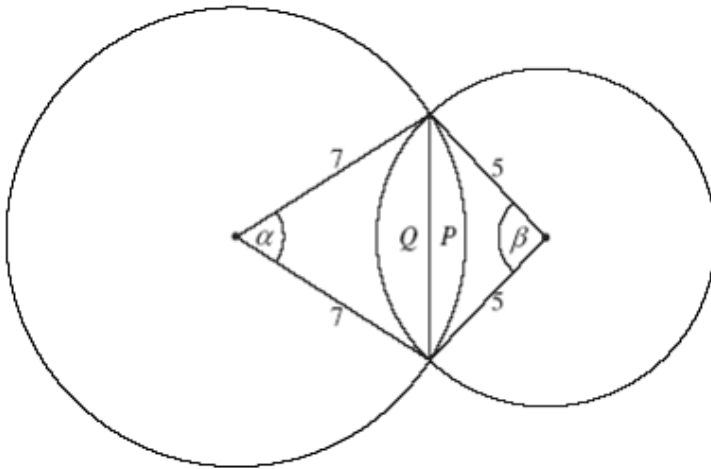
$$\begin{aligned}
 \text{(d) Area of sector} &= \frac{1}{2} \times 12^2 \times 75 \times \frac{\pi}{180} \left(\text{or } \frac{75}{360} \times \pi \times 12^2 \right) && \text{(M1)} \\
 &= 94.2 \text{ (cm}^2\text{)} \text{ (accept } 30\pi \text{ or } 94.3 \text{ (cm}^2\text{))} && \text{(A1) } 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(e) Shaded area} &= 2 \times \text{area } \triangle OPB - \text{area sector} && \text{(M1)} \\
 &= 16.4 \text{ (cm}^2\text{)} \text{ (accept } 16.2 \text{ cm}^2, 16.3 \text{ cm}^2\text{)} && \text{(A1) } 2
 \end{aligned}$$

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LEVEL 3

1)



$$\alpha = 2 \arcsin \left(\frac{4.5}{7} \right) (\Rightarrow \alpha = 1.396... = 80.010^\circ \dots)$$

M1(A1)

$$\beta = 2 \arcsin \left(\frac{4.5}{5} \right) (\Rightarrow \beta = 2.239... = 128.31^\circ \dots)$$

(A1)

Note: Allow use of cosine rule.

$$\text{area } P = \frac{1}{2} \times 7^2 \times (\alpha - \sin \alpha) = 10.08\dots$$

M1(A1)

$$\text{area } Q = \frac{1}{2} \times 5^2 \times (\beta - \sin \beta) = 18.18\dots$$

(A1)

Note: The M1 is for an attempt at area of sector minus area of triangle.

Note: The use of degrees correctly converted is acceptable.

$$\text{area} = 28.3(\text{cm}^2)$$

A1

[7]

2)

$$\begin{aligned} \text{area of triangle POQ} &= \frac{1}{2} 8^2 \sin 59^\circ \\ &= 27.43 \end{aligned}$$

M1

(A1)

$$\begin{aligned} \text{area of sector} &= \pi 8^2 \frac{59}{360} \\ &= 32.95 \end{aligned}$$

M1

(A1)

$$\begin{aligned} \text{area between arc and chord} &= 32.95 - 27.43 \\ &= 5.52 (\text{cm}^2) \end{aligned}$$

A1

[5]

3)

$$A = \frac{\theta}{2}(R^2 - r^2) \quad \text{A1}$$

$$B = \frac{\theta}{2}r^2 \quad \text{A1}$$

from $A : B = 2:1$, we have $R^2 - r^2 = 2r^2$ M1

$$R = \sqrt{3}r \quad \text{(A1)}$$

hence exact value of the ratio $R : r$ is $\sqrt{3} : 1$ A1 N0

[5]

4)

$$AC = AB = 10 \text{ (cm)} \quad \text{A1}$$

triangle OBC is equilateral (M1)

$$BC = 6 \text{ (cm)} \quad \text{A1}$$

EITHER

$$\hat{BAC} = 2 \arcsin \frac{3}{10} \quad \text{M1A1}$$

$$\hat{BAC} = 34.9^\circ \text{ (accept 0.609 radians)} \quad \text{A1}$$

OR

$$\cos \hat{BAC} = \frac{10^2 + 10^2 - 6^2}{2 \times 10 \times 10} = \frac{164}{200} \quad \text{M1A1}$$

$$\hat{BAC} = 34.9^\circ \text{ (accept 0.609 radians)} \quad \text{A1}$$

Note: Other valid methods may be seen.

[6]

5)

$$PR = h \tan 55^\circ, QR = h \tan 50^\circ \text{ where } RS = h \quad \text{M1A1A1}$$

Use the cosine rule in triangle PQR. (M1)

$$20^2 = h^2 \tan^2 55^\circ + h^2 \tan^2 50^\circ - 2h \tan 55^\circ h \tan 50^\circ \cos 45^\circ \quad \text{A1}$$

$$h^2 = \frac{400}{\tan^2 55^\circ + \tan^2 50^\circ - 2 \tan 55^\circ \tan 50^\circ \cos 45^\circ} \quad \text{(A1)}$$

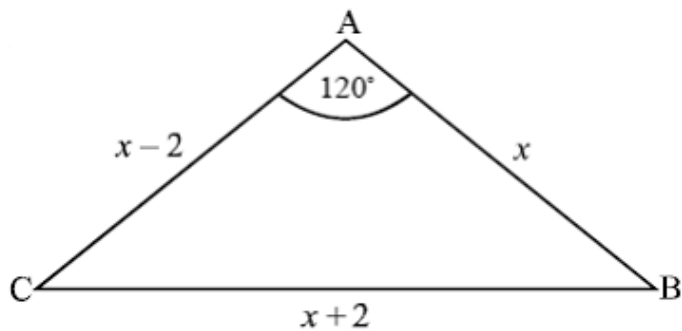
$$= 379.9... \quad \text{(A1)}$$

$$h = 19.5 \text{ (m)} \quad \text{A1}$$

[8]

6)

(a)



(M1)

$$(x + 2)^2 = (x - 2)^2 + x^2 - 2(x - 2)x \cos 120^\circ$$

M1A1

$$x^2 + 4x + 4 = x^2 - 4x + 4 + x^2 + x^2 - 2x$$

(M1)

$$0 = 2x^2 - 10x$$

A1

$$0 = x(x - 5)$$

A1

$$x = 5$$

(b) $\text{Area} = \frac{1}{2} \times 5 \times 3 \times \sin 120^\circ$

M1A1

$$= \frac{1}{2} \times 15 \times \frac{\sqrt{3}}{2}$$

A1

$$= \frac{15\sqrt{3}}{4}$$

AG

(c) $\sin A = \frac{\sqrt{3}}{2}$

$$\frac{15\sqrt{3}}{4} = \frac{1}{2} \times 5 \times 7 \times \sin B \Rightarrow \sin B = \frac{3\sqrt{3}}{14}$$

M1A1

Similarly $\sin C = \frac{5\sqrt{3}}{14}$

A1

$$\sin A + \sin B + \sin C = \frac{15\sqrt{3}}{14}$$

A1

[13]