Complex numbers – quick review

1. Given that $\frac{z}{z+2} = 2 - \text{i}$, $z \in \mathbb{C}$, find z in the form $a + \text{i}b$.

(Total 4 marks)

2. Consider the complex numbers $z = 1 + 2\text{i}$ and $w = 2 + a\text{i}$, where $a \in \mathbb{R}$.

Find a when

(a) $|w| = 2|z|$;

(3)

(b) $\text{Re}(zw) = 2\,\text{Im}(zw)$.

(3)
(Total 6 marks)

3. Solve the simultaneous equations

$$\text{i}z_1 + 2z_2 = 3$$
$$z_1 + (1 - \text{i})z_2 = 4$$

giving z_1 and z_2 in the form $x + \text{i}y$, where x and y are real.

(Total 9 marks)

Conjugate roots

4. Consider the polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$.

Given that $1 + \mathrm{i}$ and $1 - 2\mathrm{i}$ are zeros of $p(x)$, find the values of a, b, c and d.

(Total 7 marks)

5. (a) Show that the complex number i is a root of the equation

$$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0.$$

(2)

(b) Find the other roots of this equation.

(4)

(Total 6 marks)

6. Given that $2 + \mathrm{i}$ is a root of the equation $x^3 - 6x^2 + 13x - 10 = 0$ find the other two roots.

(Total 5 marks)

7. Consider the equation $z^3 + az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$. The points in the Argand diagram representing the three roots of the equation form the vertices of a triangle whose area is 9. Given that one root is $-1 + 3\mathrm{i}$, find

(a) the other two roots;

(4)

(b) a, b and c.

(3)

(Total 7 marks)