Level 1

1.

(a) $0.8 = 0.5 + 0.6 - \mathrm{P}(A \cap B)$ (M1)

$\mathrm{P}(A \cap B) = 0.3$ (A1) (C2)

Note: *Award (M1) for correct substitution, (A1) for correct answer.*

(b) $\mathrm{P}(A \mid B) = \dfrac{0.3}{0.6}$ (M1)

$= 0.5$ (A1)(ft) (C2)

Note: *Award (M1) for correct substitution in conditional probability formula. Follow through from their answer to part (a), provided probability is not greater than one.*

(c) $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ or $0.3 = 0.5 \times 0.6$ (R1)

OR

$\mathrm{P}(A \mid B) = \mathrm{P}(A)$ (R1)

they are independent. (Yes) (A1)(ft) (C2)

Note: *Follow through from their answers to parts (a) or (b). Do not award (R0)(A1).*

[6]

2.

(a) $\frac{90}{200}$ (0.45, 45 %) (A1)(A1) (C2)

Note: *Award (A1) for numerator, (A1) for denominator.*

(b) $\frac{60}{90}$ ($0.\bar{6}$, 0.667, $66.\bar{6}$%, 66.6...%, 66.7 %) (A1)(A1)(ft) (C2)

Notes: *Award (A1) for numerator, (A1)(ft) for denominator, follow through from their numerator in part (a). Last mark is lost if answer is not a probability.*

(c) $\frac{90}{200}+\frac{100}{200}-\frac{60}{200}$ (M1)

Note: *Award (M1) for correct substitution in the combined events formula. Follow through from their answer to part (a).*

$= \frac{130}{200}$ (0.65, 65 %) (A1)(ft)

OR

$\frac{60}{200}+\frac{40}{200}+\frac{30}{200}$ (M1)

Note: *Award (M1) for adding the correct fractions.*

$= \frac{130}{200}$ (0.65, 65 %) (A1)

OR

$= 1-\frac{70}{200}$ (M1)

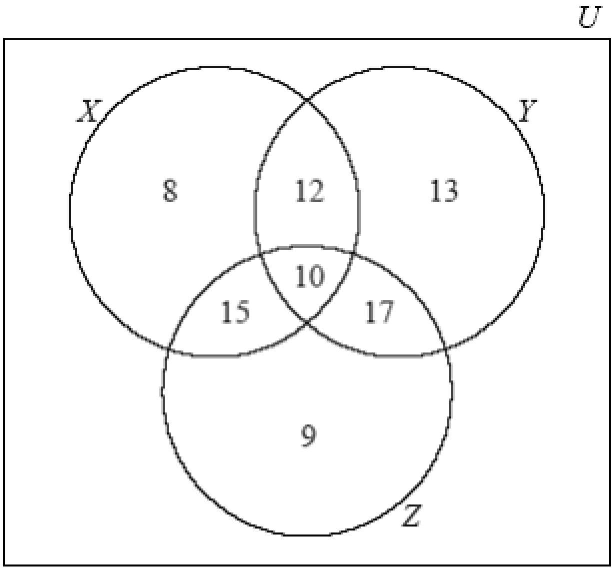
Note: *Award (M1) for subtraction of correct fraction from 1.*

$= \frac{130}{200}$ (0.65, 65%) (A1) (C2)

[6]

3.

(a)



(A1) for rectangle and three intersecting circles
(A1) for 10, (A1) for 8, 13 and 9, (A1) for 12, 15 and 17 (A4)

(b) $100 - (9 + 12 + 13 + 15 + 10 + 17 + 8) = 16$ (M1)(A1)(ft)(G2)

***Note:** Follow through from their diagram.*

(c) $\frac{51}{100}$ (0.51) (A1)(ft)

$= 51\%$ (A1)(ft)(G2)

***Note:** Follow through from their diagram.*

(d) **Note:** The following statements are correct. Please note that the connectives are important. It is not the same (had cereal) <u>and</u> (not bread) and (had cereal) <u>or</u> (not bread). The parentheses are not needed but are there to facilitate the understanding of the propositi ons.

(had cereal) and (did not have bread)
(had cereal only) or (had cereal and fruit only)
(had either cereal or (fruit and cereal)) and (did not have bread) (A1)(A1)

***Notes:** If the statements are correct but the connectives are wrong then award at most (A1)(A0).*
For the statement (had only cereal) and (cereal and fruit) award (A1)(A0).
For the statement had cereal and fruit award (A0)(A0).

(e) $\frac{54}{100}$ (0.54, 54 %) (A1)(ft)(A1)(ft)(G2)

Note: Award (A1)(ft) for numerator, follow through from their diagram, (A1)(ft) for denominator. Follow through from total or denominator used in part (c).

(f) $\frac{10}{100} \times \frac{9}{99} = \frac{1}{110}$ (0.00909, 0.909%) (A1)(ft)(M1)(A1)(ft)(G2)

***Notes:** Award (A1)(ft) for their correct fractions, (M1) for multiplying two fractions, (A1)(ft) for their correct answer. Answer 0.009 with no working receives no marks. Follow through from denominator in parts (c) and (e) and from their diagram.*

[15]

4.

(a) $P(A \cap B) = 0$ (A1) (C1)

(b) $P(A \cap B) = P(A) \times P(B)$

$= \frac{4}{13} \times \frac{5}{13}$ (M1)

***Note:** Award (M1) for product of two fractions, decimals or percentages.*

$P(A \cap B) = \frac{20}{169}$ (= 0.118) (A1) (C2)

(c) $\frac{7}{13} = \frac{4}{13} + \frac{5}{13} - P(A \cap B)$ (M1)(M1)

***Notes:** Award (M1) for $\frac{4}{13} + \frac{5}{13}$ seen, (M1) for subtraction of $\frac{7}{13}$ shown.*

OR

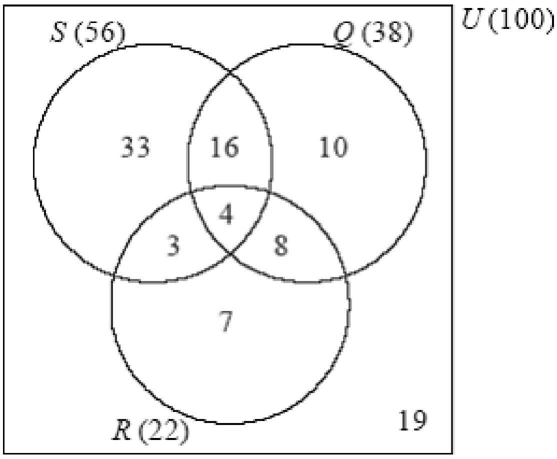
Award (M1) for Venn diagram with 2 intersecting circles, (A1) for correct probabilities in diagram.

$P(A \cap B) = \frac{2}{13}$ (= 0.154) (A1) (C3)

[6]

5.

(a)



(A1)(A1)(A1)(A1)(A1)

Note: *Award (A1) for rectangle (U not required), (A1) for 3 intersecting circles, (A1) for 4 in central intersection, (A1) for 16, 3, 8 and (A1) for 33, 10, 7 (ft) if subtraction is carried out, or for S(56), Q(38) and R(22) seen by the circles.*

(b) $100 - 81$ (M1)

19 (A1)(ft)(G2)

Note: *Award (M1) for subtracting their total from 100.*

(c) $33 + 10 + 7$ (M1)

Note: *Award (M1) for adding their values from (a).*

$\left(\frac{50}{100}\right) \times 100\%$ (A1)(ft)

50% (50) (A1)(ft)(G3)

(d) P (own a cat given they own a bird) = $\frac{12}{22}\left(0.545, \frac{6}{11}\right)$ (A1)(ft)(A1)(ft)

Note: *Award (A1)(ft) for the numerator, (A1)(ft) for the denominator.*

[12]

LEVEL 2

1.

(a)	(i)	$s = 1$	A1	N1	
	(ii)	evidence of appropriate approach	(M1)		
		e.g. 21–16, $12 + 8 - q = 15$			
		$q = 5$	A1	N2	
	(iii)	$p = 7, r = 3$	A1A1	N2	5
(b)	(i)	$\text{P(art}\vert\text{music)} = \frac{5}{8}$	A2	N2	
	(ii)	**METHOD 1**			
		$P(\text{art}) = \frac{12}{16}\left(= \frac{3}{4}\right)$	A1		
		evidence of correct reasoning	R1		
		e.g. $\frac{3}{4} \neq \frac{5}{8}$			
		the events are not independent	AG	N0	
		METHOD 2			
		$\text{P(art)} \times \text{P(music)} = \frac{96}{256}\left(= \frac{3}{8}\right)$	A1		
		evidence of correct reasoning	R1		
		e.g. $\frac{12}{16} \times \frac{8}{16} \neq \frac{5}{16}$			
		the events are not independent	AG	N0	4
(c)	P(first takes only music) $= \frac{3}{16} =$ (seen anywhere)		A1		
	P(second takes only art)$= \frac{7}{15}$ (seen anywhere)		A1		
	evidence of valid approach		(M1)		
	e.g. $\frac{3}{16} \times \frac{7}{15}$				
	P(music and art)$= \frac{21}{240}\left(= \frac{7}{80}\right)$		A1	N2	4

[13]

2.

(a)	evidence of valid approach involving A and B	(M1)		
	e.g. $P(A \cap \text{pass}) + P(B \cap \text{pass})$, tree diagram			
	correct expression	(A1)		
	e.g. $P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9$			
	$P(\text{pass}) = 0.84$	A1	N2	3
(b)	evidence of recognizing complement (seen anywhere)	(M1)		
	e.g. $P(B) = x$, $P(A) = 1 - x$, $1 - P(B)$, $100 - x$, $x + y = 1$			
	evidence of valid approach	(M1)		
	e.g. $0.8(1 - x) + 0.9x$, $0.8x + 0.9y$			
	correct expression	A1		
	e.g. $0.87 = 0.8(1 - x) + 0.9x$, $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$, $0.8x + 0.9y = 0.87$			
	70 % from B	A1	N2	4

[7]

3.

(a)	(i)	$p = 0.2$	A1	N1
	(ii)	$q = 0.4$	A1	N1
	(iii)	$r = 0.1$	A1	N1
(b)		$P(A \mid B') = \frac{2}{3}$	A2	N2

Note: *Award A1 for an unfinished answer such as* $\frac{0.2}{0.3}$.

(c)	valid reason	R1	
	e.g. $\frac{2}{3} \neq 0.5$, $0.35 \neq 0.3$		
	thus, A and B are not independent	AG	N0

[6]

4.

(a)	appropriate approach *e.g.* tree diagram or a table	(M1)	
	$\mathrm{P(win)} = \mathrm{P}(H \cap W) + \mathrm{P}(A \cap W)$	(M1)	
	$= (0.65)(0.83) + (0.35)(0.26)$	A1	
	$= 0.6305$ (or 0.631)	A1	N2
(b)	evidence of using complement *e.g.* $1 - p$, 0.3695	(M1)	
	choosing a formula for conditional probability *e.g.* $\mathrm{P}(H \mid W') = \dfrac{\mathrm{P}(W' \cap H)}{\mathrm{P}(W')}$	(M1)	
	correct substitution *e.g.* $\dfrac{(0.65)(0.17)}{0.3695} \left(= \dfrac{0.1105}{0.3695}\right)$	A1	
	$\mathrm{P(home)} = 0.299$	A1	N3

[8]

5.

(a)	(i)	evidence of substituting into $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ *e.g.* $75 + 55 - 100$, Venn diagram	(M1)	
		30	A1	N2
	(ii)	45	A1	N1
(b)	(i)	**METHOD 1**		
		evidence of using complement, Venn diagram *e.g.* $1 - p$, $100 - 30$	(M1)	
		$\frac{70}{100}\left(=\frac{7}{10}\right)$	A1	N2
		METHOD 2		
		attempt to find P(only one sport), Venn diagram *e.g.* $\frac{25}{100}+\frac{45}{100}$	(M1)	
		$\frac{70}{100}\left(=\frac{7}{10}\right)$	A1	N2
	(ii)	$\frac{45}{70}\left(=\frac{9}{14}\right)$	A2	N2
(c)		valid reason in words or symbols *e. g.* $P(A \cap B) = 0$ if mutually exclusive, $P(A \cap B)$ if not mutually exclusive	(R1)	
		correct statement in words or symbols *e.g.* $P(A \cap B) = 0.3$, $P(A \cup B) \neq P(A) + P(B)$, $P(A) + P(B) > 1$, some students play both sports, sets intersect	A1	N2
(d)		valid reason for independence *e.g.* $P(A \cap B) = P(A) \times P(B)$, $P(B \mid A) = P(B)$	(R1)	
		correct substitution *e.g.* $\frac{30}{100} \neq \frac{75}{100} \times \frac{55}{100}$, $\frac{30}{55} \neq \frac{75}{100}$	A1A1	N3

[12]

6.

METHOD 1

for independence $P(A \cap B) = P(A) \times P(B)$ (R1)
expression for $P(A \cap B)$, indicating $P(B) = 2P(A)$ (A1)
e.g. $P(A) \times 2P(A)$, $x \times 2x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)
correct substitution A1
e.g. $0.52 = x + 2x - 2x^2$, $0.52 = P(A) + 2P(A) - 2P(A)P(A)$
correct solutions to the equation (A2)
e.g. 0.2, 1.3 (accept the single answer 0.2)
$P(B) = 0.4$ A1 N6

METHOD 2

for independence $P(A \cap B) = P(A) \times P(B)$ (R1)

expression for $P(A \cap B)$, indicating $P(A) = \frac{1}{2}P(B)$ (A1)

e.g. $P(B) \times \frac{1}{2}P(B)$, $x \times \frac{1}{2}x$

substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)
correct substitution A1
e.g. $0.52 = 0.5x + x - 0.5x^2$, $0.52 = 0.5P(B) + P(B) - 0.5P(B)P(B)$

correct solutions to the equation (A2)
e.g. 0.4, 2.6 (accept the single answer 0.4)
$P(B) = 0.4$ (accept $x = 0.4$ if x set up as $P(B)$) A1 N6

[7]

7.

(a) (i) correct calculation (A1)

e.g. $\frac{9}{20}+\frac{5}{20}-\frac{2}{20}$, $\frac{4+2+3+3}{20}$

P(male or tennis) = $\frac{12}{20}\left(=\frac{3}{5}\right)$ A1 N2

(ii) correct calculation (A1)

e.g. $\frac{6}{20}\div\frac{11}{20}$, $\frac{3+3}{11}$

P(not football | female) = $\frac{6}{11}$ A1 N2

(b) P(first not football) = $\frac{11}{20}$, P(second not football) = $\frac{10}{19}$ A1

P(neither football) = $\frac{11}{20}\times\frac{10}{19}$ A1

P(neither football) = $\frac{110}{380}\left(=\frac{11}{38}\right)$ A1 N1

[7]

8.

(a)	$P(A \cap B) = P(A) \times P(B)$ $(= 0.6x)$		A1	N1
(b)	(i)	evidence of using $P(A \cup B) = P(A) + P(B) - P(A)P(B)$	(M1)	
		correct substitution	A1	
		e.g. $0.80 = 0.6 + x - 0.6x$, $0.2 = 0.4x$		
		$x = 0.5$	A1	N2
	(ii)	$P(A \cap B) = 0.3$	A1	N1
(c)	valid reason, with reference to $P(A \cap B)$ *e.g.* $P(A \cap B) \neq 0$		R1	N1

[6]

9.

(a) $\frac{3}{4}$ A1 N1

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{5} + \frac{3}{4} - \frac{7}{8}$ A1

$= \frac{11}{40}$ (0.275) A1 N2

(c) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} \left(= \frac{\frac{11}{40}}{\frac{3}{4}} \right)$ A1

$= \frac{11}{30}$ (0.367) A1 N1

[6]

10.

(a) $\frac{46}{97}$ $(= 0.474)$ A1A1 N2

(b) $\frac{13}{51}$ $(= 0.255)$ A1A1 N2

(c) $\frac{59}{97}$ $(= 0.608)$ A2 N2

[6]

LEVEL 3

1.

(a) (i) $P(A \cup B) = P(A) + P(B) = 0.7$ A1

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$= P(A) + P(B) - P(A)P(B)$ (M1)

$= 0.3 + 0.4 - 0.12 = 0.58$ A1

(b) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.3 + 0.4 - 0.6 = 0.1$ A1

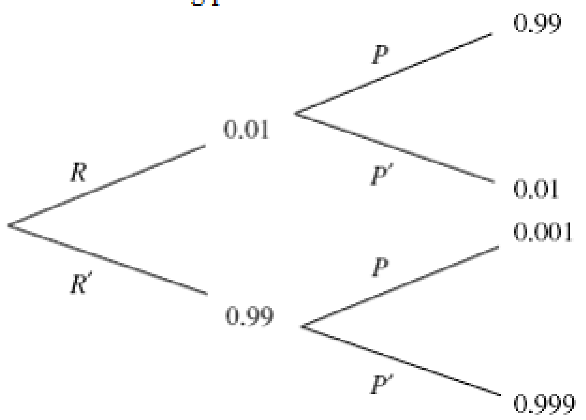
$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ (M1)

$= \dfrac{0.1}{0.4} = 0.25$ A1

[7]

2.

R is 'rabbit with the disease'
P is 'rabbit testing positive for the disease'



(a) $P(P) = P(R \cap P) + P(R' \cap P)$

$= 0.01 \times 0.99 + 0.99 \times 0.001$ M1

$= 0.01089$ $(= 0.0109)$ A1

Note: Award M1 for a correct tree diagram with correct probability values shown.

(b) $P(R' \mid P) = \dfrac{0.001 \times 0.99}{0.001 \times 0.99 + 0.01 \times 0.99} \left(= \dfrac{0.00099}{0.01089}\right)$ M1A1

$\dfrac{0.00099}{0.01089} < \dfrac{0.001}{0.01} = 10\ \%$ (or other valid argument) R1

[5]

3.

P (six in first throw) = $\frac{1}{6}$ (A1)

P (six in third throw) = $\frac{25}{36}\times\frac{1}{6}$ (M1)(A1)

P (six in fifth throw) = $\left(\frac{25}{36}\right)^2\times\frac{1}{6}$

P(A obtains first six) = $\frac{1}{6}+\frac{25}{36}\times\frac{1}{6}+\left(\frac{25}{36}\right)^2\times\frac{1}{6}+\ldots$ (M1)

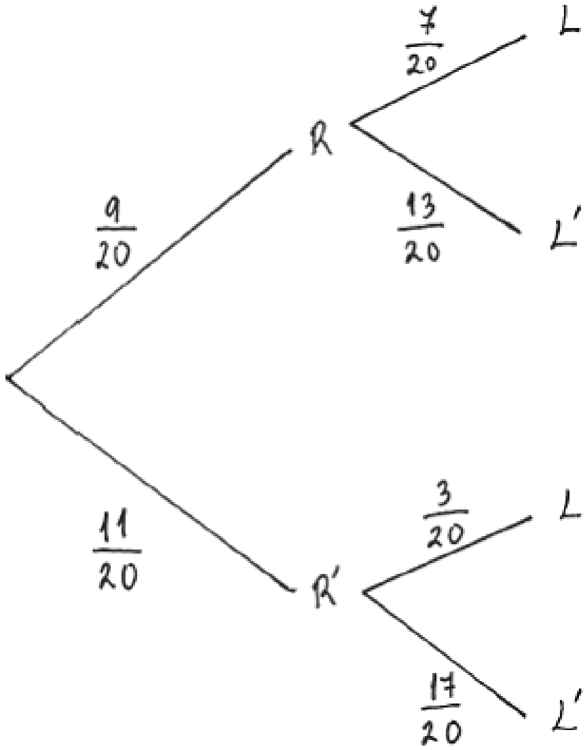
recognizing that the common ratio is $\frac{25}{36}$ (A1)

P(A obtains first six) = $\frac{\frac{1}{6}}{1-\frac{25}{36}}$ (by summing the infinite GP) M1

$=\frac{6}{11}$ A1

[7]

4.



(A1)

$\mathrm{P}(R' \cap L) = \frac{11}{20} \times \frac{3}{20}$ A1

$\mathrm{P}(L) = \frac{9}{20} \times \frac{7}{20} + \frac{11}{20} \times \frac{3}{20}$ A1

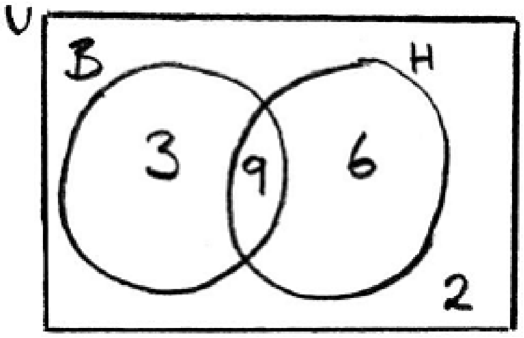
$P(R'|L) = \frac{\mathrm{P}(R' \cap L)}{\mathrm{P}(L)}$ (M1)

$= \frac{33}{96} \left(= \frac{11}{32} \right)$ A1

[5]

5.

(a)



A1A1

Note: Award A1 for a diagram with two intersecting regions and at least the value of the intersection.

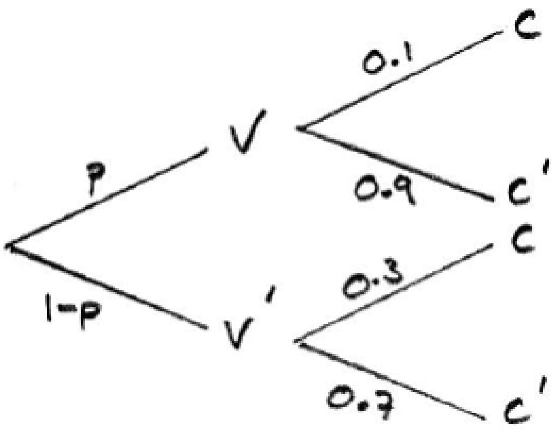
(b) $\frac{9}{20}$ A1

(c) $\frac{9}{12}\left(=\frac{3}{4}\right)$ A1

[4]

6.

(a)



using the law of total probabilities: (M1)

$0.1p + 0.3(1 - p) = 0.22$ A1

$0.1p + 0.3 - 0.3p = 0.22$

$0.2p = 0.08$

$p = \frac{0.08}{0.2} = 0.4$

$p = 40\%$ (accept 0.4) A1

(b) required probability $= \frac{0.4 \times 0.1}{0.22}$ M1

$= \frac{2}{11}$ (0.182) A1

[5]

7.

EITHER

Using $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ (M1)

$0.6P(B) = P(A \cap B)$ A1

Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to obtain
$0.8 = 0.6 + P(B) - P(A \cap B)$ A1

Substituting $0.6P(B) = P(A \cap B)$ into above equation M1

OR

As $P(A \mid B) = P(A)$ then A and B are independent events M1R1

Using $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$ A1

to obtain $0.8 = 0.6 + P(B) - 0.6 \times P(B)$ A1

THEN

$0.8 = 0.6 + 0.4P(B)$ A1

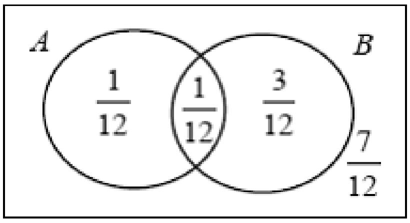
$P(B) = 0.5$ A1 N1

[6]

8.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ M1

$= \frac{2}{12} + \frac{4}{12} - \frac{5}{12} = \frac{1}{12}$ A1



M1A1

$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{\frac{7}{12}}{\frac{8}{12}} = \frac{7}{8}$ M1A1

[6]

9.

(a) $P(RR) = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right)$ (M1)

$= \frac{1}{10}$ A1 N2

(b) $P(RR) = \frac{4}{4+n} \times \frac{3}{3+n} = \frac{2}{15}$ A1

Forming equation $12 \times 15 = 2(4 + n)(3 + n)$ (M1)

$12 + 7n + n^2 = 90$ A1

$\Rightarrow n^2 + 7n - 78 = 0$ A1

$n = 6$ AG N0

(c) **EITHER**

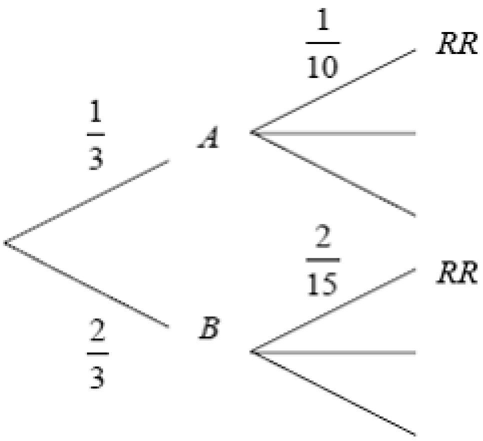
$P(A) = \frac{1}{3}$ $P(B)\frac{2}{3}$ A1

$P(RR) = P(A \cap RR) + P(B \cap RR)$ (M1)

$= \left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{15}\right)$

$= \frac{11}{90}$ A1 N2

OR



A1

$P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{15}$ M1

$= \frac{11}{90}$ A1 N2

(d)	$P(1 \text{ or } 6) = P(A)$	M1	
	$P(A \mid RR) = \dfrac{P(A \cap RR}{P(RR)}$	(M1)	
	$= \dfrac{\left[\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)\right]}{\frac{11}{90}}$	M1	
	$= \dfrac{3}{11}$	A1	N2

[13]