

1. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$.

(a) (i) Find $f'(x)$.

(ii) Find the gradient of the curve $f(x)$ when $x = 3$.

(4)

(b) Find the x -coordinates of the points on the curve where the gradient is equal to -12 .

(3)

(c) (i) Calculate the x -coordinates of the local maximum and minimum points.

(ii) Hence find the coordinates of the local minimum.

(6)

(d) For what values of x is the value of $f(x)$ increasing?

(2)

(Total 15 marks)

2. The cost of producing a mathematics textbook is \$15 (US dollars) and it is then sold for \$ x .

(a) Find an expression for the profit made on each book sold.

(1)

A total of $(100\,000 - 4000x)$ books is sold.

(b) Show that the profit made on all the books sold is

$$P = 160\,000x - 4000x^2 - 1500\,000.$$

(3)

(c) (i) Find $\frac{dP}{dx}$.

(2)

(ii) Hence calculate the value of x to make a maximum profit

(2)

(d) Calculate the number of books sold to make this maximum profit.

(2)

(Total 10 marks)

3. The height (cm) of a daffodil above the ground is given by the function $h(w) = 24w - 2.4w^2$, where w is the time in weeks after the plant has broken through the surface ($w \geq 0$).
- (a) Calculate the height of the daffodil after two weeks. (2)
- (b) (i) Find the rate of growth, $\frac{dh}{dw}$. (2)
- (ii) The rate of growth when $w = k$ is 7.2 cm per week. Find k . (3)
- (iii) When will the daffodil reach its maximum height? What height will it reach? (4)
- (c) Once the daffodil has reached its maximum height, it begins to fall back towards the ground. Show that it will touch the ground after 70 days. (3)
- (Total 14 marks)**

4. A closed box has a square base of side x and height h .

(a) Write down an expression for the volume, V , of the box. (1)

(b) Write down an expression for the total surface area, A , of the box. (1)

The volume of the box is 1000 cm^3

(c) Express h in terms of x . (2)

(d) Hence show that $A = 4000x^{-1} + 2x^2$. (2)

(e) Find $\frac{dA}{dx}$. (2)

(f) Calculate the value of x that gives a minimum surface area. (4)

(g) Find the surface area for this value of x . (3)

(Total 15 marks)