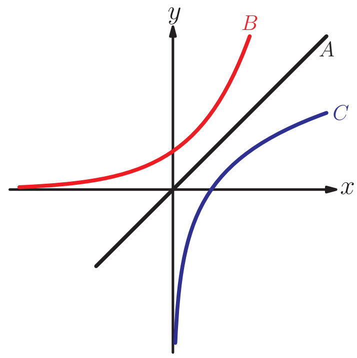
Mixed examination practice 5

Short questions

1. Find the inverses of the following functions:

(a) $f(x) = \log_3(x+3),\ x > 0$ (b) $g(x) = 3e^{x^3-1}$ *[5 marks]*

2. The diagram shows three graphs.



A is part of the graph of $y = x$.
B is part of the graph of $y = 2^x$.
C is the reflection of graph B in line A.
Write down:

(a) The equation of C in the form $y = f(x)$

(b) The coordinates of the point where C cuts the x-axis. *[5 marks]*

3. (a) Write down the equations of all asymptotes of the graph of $y = \frac{4x-3}{5-x}$.

(b) Find the inverse function of $f(x) = \frac{4x-3}{5-x}$. *[6 marks]*

4. The function f is given by $f(x) = x^2 - 6x + 10$, for $x \geq 3$.

(a) Write $f(x)$ in the form $(x-p)^2 + q$.

(b) Find the inverse function $f^{-1}(x)$.

(c) State the domain of $f^{-1}(x)$. *[6 marks]*

5. If $h(x) = x^2 - 6x + 2$:

(a) Write $h(x)$ in the form $(x-p)^2 + q$.

(b) Hence or otherwise find the range of $h(x)$.

(c) By using the largest possible domain of the form $x > k$ where, find the inverse function $h^{-1}(x)$. *[7 marks]*

6. The function $f(x)$ is defined by $f(x)=\dfrac{3-x}{x+1}, x\neq -1$.

(a) Find the range of f.

(b) Sketch the graph of $y=f(x)$.

(c) Find the inverse function of f in the form $f^{-1}(x)=\dfrac{ax+b}{cx+d}$. State its domain and range. *[11 marks]*

7. A function is defined by:

$$f(x)=\begin{cases}5-x, & x<0\\ pe^{-x}, & x\geq 0\end{cases}$$

(a) Given that $p=3$,

(i) Find the range of $f(x)$.

(ii) Find an expression for $f^{-1}(x)$ and state its domain.

(b) Find the value of p for which $f(x)$ is continuous. *[7 marks]*

8. The functions $f(x)$ and $g(x)$ are given by $f(x)=\sqrt{x-2}$ and $g(x)=x^2+x$. The function $f\circ g(x)$ is defined for $x\in\mathbb{R}$ except for the interval $]\,a,b[$.

(a) Calculate the value of a and of b.

(b) Find the range of $f\circ g$. *[7 marks]*

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Long questions

1. If $f(x)=x^2+1,\ x>3$ and $g(x)=5-x$:

(a) evaluate $f(3)$.

(b) Find and simplify an expression for $gf(x)$.

(c) State the geometric relationship between the graphs of $y=f(x)$ and $y=f^{-1}(x)$.

(d) (i) Find an expression for $f^{-1}(x)$.

(ii) Find the range of $f^{-1}(x)$.

(iii) Find the domain of $f^{-1}(x)$.

(e) Solve the equation $f(x)=g(3x)$. *[10 marks]*

2. If $f(x)=2x+1$ and $g(x)=\dfrac{x+3}{x-1},\ x\neq 1$

(a) find and simplify

(i) $f(7)$ (ii) the range of $f(x)$

(iii) $fg(x)$ (iv) $ff(x)$

(b) Explain why $gf(x)$ does not exist.

(c) (i) Find the form of $g^{-1}(x)$.

(ii) State the domain of $g^{-1}(x)$.

(iii) State the range of $g^{-1}(x)$. *[9 marks]*

3. The functions f and g are defined over the domain of all real numbers, $g(x) = e^x$.

(a) Write $f(x) = x^2 + 4x + 9 \ \ x \in \mathbb{R}$ in the form $f(x) = (x+p)^2 + q$.

(b) Hence sketch the graph of $y = x^2 + 4x + 9$, labelling carefully all axes intercepts and the coordinates of the turning point.

(c) State the range of $f(x)$ and $g(x)$.

(d) Hence or otherwise find the range of $h(x) = e^{2x} + 4e^x + 9$. *[10 marks]*

4. Given that $(2x+3)(4-y) = 12$ for $x, y \in \mathbb{R}$:

(a) Write y in terms of x, giving your answer in the form $y = \dfrac{ax+b}{cx+d}$.

(b) Sketch the graph of y against x.

(c) Let $g(x) = 2x + k$ and $h(x) = \dfrac{8x}{2x+3}$.

(i) Find $h(g(x))$.

(ii) Write down the equations of the asymptotes of the graph of $y = h(g(x))$.

(iii) Show that when $k = -\dfrac{19}{2}$, $h(g(x))$ is a self-inverse function. *[17 marks]*

5. (a) Show that if $g(x) = \dfrac{1}{x}$ then $gg(x) = x$.

(b) A function satisfies the identity $f(x) + 2f\left(\dfrac{1}{x}\right) = 2x + 1$.

By replacing all instances of x with $\dfrac{1}{x}$, find another identity that $f(x)$ satisfies.

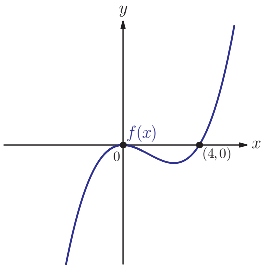
(c) By solving these two identities simultaneously, express $f(x)$ in terms of x.

[10 marks]

Mixed examination practice 6

Short questions

1. The graph of $y = f(x)$ is shown.



Sketch on separate diagrams the graphs of

(a) $y = 3f(x-2)$

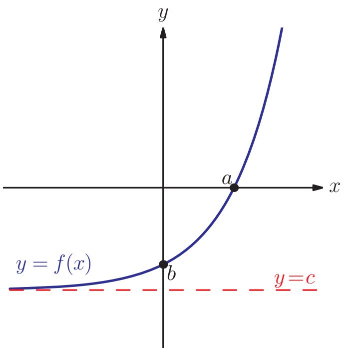
(b) $\dfrac{1}{f(x)}$

Indicate clearly the positions of any x-intercepts and asymptotes. *[6 marks]*

2. The graph of $y = x^3 - 1$ is transformed by applying a translation with vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ followed by a vertical stretch with scale factor 2. Find the equation of the resulting graph in the form $y = ax^3 + bx^2 + cx + d$. *[4 marks]*

3. Solve the inequality $|2x-1| < x$. *[6 marks]*

4. The diagram shows the graph of $y = f(x)$.



On separate diagrams sketch the following graphs, labelling appropriately.

(a) $y = |f(x)|$

(b) $y = f(|x|) - 1$ *[5 marks]*

5. (a) Sketch the graph of $y = -\dfrac{3}{x}$.

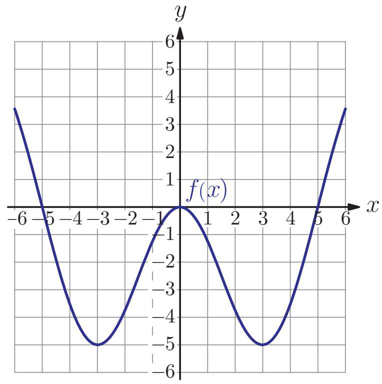
(b) Describe two transformations which transform the graph of $y = \dfrac{1}{x}$ to the graph of $y = -\dfrac{3}{x}$.

(c) Let $f(x) = -\dfrac{3}{x}, x \neq 0$. Write down an equation for $f^{-1}(x)$. *[4 marks]*

6. The graph of $y = f(x)$ is shown.

(a) On the same diagram sketch the graph of $y = \dfrac{1}{f(x)}$.

(b) State the coordinates of the maximum points. *[5 marks]*



7. Find two transformations whose composition transforms the graph of $y = (x-1)^2$ to the graph of $y = 3(x+2)^2$. *[4 marks]*

8. (a) Describe two transformations whose composition transforms the graph of $y = f(x)$ to the graph of $y = 3f\left(\dfrac{x}{2}\right)$.

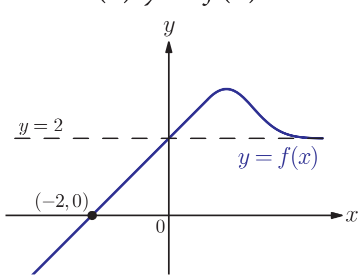
(b) Sketch the graph of $y = 3\ln\left(\dfrac{x}{2}\right)$.

(c) Sketch the graph of $y = 3\ln\left(\dfrac{x}{2}+1\right)$ marking clearly the positions of any asymptotes and x-intercepts. *[7 marks]*

9. The diagram shows a part of the graph of $y = f(x)$

On separate diagrams sketch the graphs of

(a) $y = \dfrac{1}{f(x)}$ (b) $y = xf(x)$ *[6 marks]*



10. For which values of the real number x is $|x+k| = |x| + k$, where k is a positive real number? *[4 marks]*

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Long questions

1 (a) Describe two transformations which transform the graph of $y = x^2$ to the graph of $y = 3x^2 - 12x + 12$.

(b) Describe two transformations which transform the graph of $y = x^2 + 6x - 1$ to the graph of $y = x^2$.

(c) Hence describe a sequence of transformations which transform the graph of $y = x^2 + 6x - 1$ to the graph of $y = 3x^2 - 12x + 12$.

(d) Sketch the graph of $y = \dfrac{1}{3x^2 - 12x + 12}$. *[12 marks]*

2. Given that $f(x) = \dfrac{3x - 5}{x - 2}$

(a) Write down the equation of the horizontal asymptote of the graph of $y = f(x)$.

(b) Find the value of constants p and q such that $f(x) = p + \dfrac{q}{x - 2}$.

(c) Hence describe a single transformation which transforms the graph of $y = \dfrac{1}{x}$ to the graph of $y = f(x)$.

(d) Find an expression for $f^{-1}(x)$ and state its domain.

(e) Describe the transformation which transforms the graph of $y = f(x)$ to the graph of $y = f^{-1}(x)$. *[11 marks]*

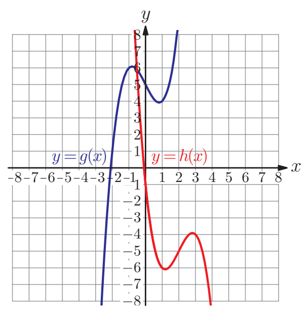
3. (a) Describe a transformation which transforms the graph of $y = f(x)$ to the graph of $y = f(x + 2)$.

(b) Sketch on the same diagram the graphs of

(i) $y = \ln(x + 2)$ (ii) $y = \dfrac{1}{\ln(x + 2)}$.

Mark clearly any asymptotes and x-intercepts on your sketches.

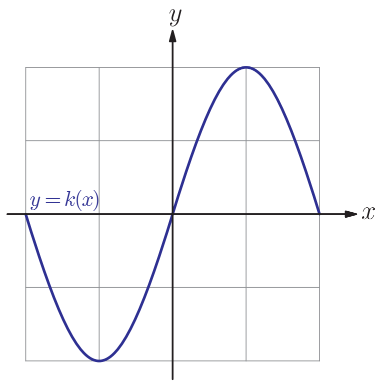
(c) The graph of the function $y = g(x)$ has been translated and then reflected in the x-axis to produce the graph of $y = h(x)$.



(i) State the translation vector.

(ii) If $g(x) = x^3 - 2x + 5$, find constants a, b, c and d such that $h(x) = ax^3 + bx^2 + cx + d$.

(d) The diagram below shows the graph of $y = k(x)$.



On the same diagram, sketch the graph of $y = (k(x))^2$. *[14 marks]*

4. $f(x) = x^2 - 7x + 10$ $\qquad g(x) = x^2 - 7|x| + 10$

(a) Sketch the graph of $y = f(x)$.

(b) Show that $g(x) = f(|x|)$.

(c) Sketch the graph of $y = g(x)$.

(d) Solve the equation $g(x) = x^2$.

(e) Solve the equation $g(x) = -2$. *[12 marks]*

5. If $f(x) = 3x^2 + bx + 10$ and the graph $y = f(x)$ has a line of symmetry when $x = 3$

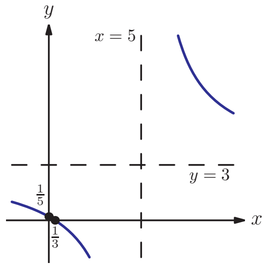
(a) find b.

(b) If $f(x) = f(d - x)$ for all x, find the value of d.

(c) $g(x) = f(x + p) + q$ and $g(x)$ is an even function which passes through the origin. Find p and q. *[14 marks]*

(d) Find the set values which satisfy $g(x) = g(|x|)$.

7.



8. (a) $y \neq \frac{a}{2}$

(b) $f^{-1}(x) = \frac{8x+3}{2x-a}, x \neq \frac{a}{2}$

(c) 8

Mixed examination practice 5

Short questions

1. (a) $3^x - 3$

(b) $\sqrt[3]{\ln\left(\frac{x}{3}\right)+1}$

2. (a) $y = \log_2 x$ (b) (1,0)

3. (a) $x = 5, y = -4$

(b) $f^{-1}(x) = \frac{5x+3}{x+4}$

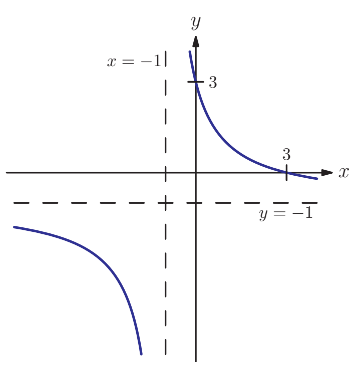
4. (a) $(x-3)^2 + 1$ (b) $\sqrt{x-1} + 3$

(c) $x \geq 1$

5. (a) $(x-3)^2 - 7$ (b) $y \geq -7$ (c) $\sqrt{x+7} + 3$

6. (a) $y \in \mathbb{R}, y \neq -1$

(b)



(c) $f^{-1}(x) = \frac{3-x}{x+1}, x \neq -1, y \neq -1$

7. (a) (i) $]0, 3] \cup]5, \infty[$

(ii) $f^{-1}(x) = \begin{cases} \ln\left(\frac{3}{x}\right), & 0 < x \leq 3 \\ 5 - x, & x > 5 \end{cases}$

Domain: $]0, 3] \cup]5, \infty[$

(b) $p = 5$

8. (a) $a = -2, b = 1$ (b) $y \geq 0$

Long questions

1. (a) 10

(b) $4 - x^2$

(c) Reflection in the line $y = x$

(d) (i) $\sqrt{x-1}$ (ii) $y > 3$

(iii) $x > 10$

(e) $x = -4, 1$

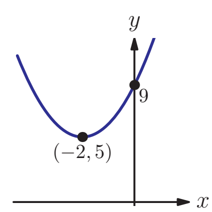
2. (a) (i) 15 (ii) $y \in \mathbb{R}$

(iii) $\frac{3x+5}{x-1}$ (iv) $4x + 3$

(b) $f(x)$ can be 1, which is not in the domain of g.

(c) (i) $\frac{x+3}{x-1}$ (ii) $x \neq 1$ (ii) $y \neq 1$

3. (a) $(x+2)^2 + 5$ (b)

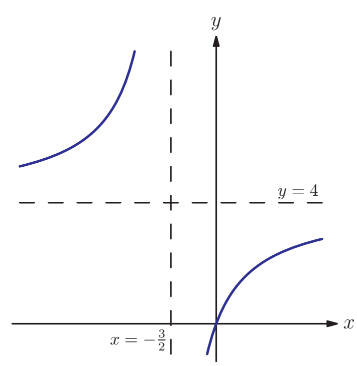


(c) Range of $f(x)$ is $y \geq 5$, Range of $g(x)$ is $y > 0$

(d) $y > 9$

4. (a) $y = \frac{8x}{2x+3}$

(b)



(c) (i) $\frac{16x+8k}{4x+2k+3}$ (ii) $x = -\frac{2k+3}{4}, y = 4$

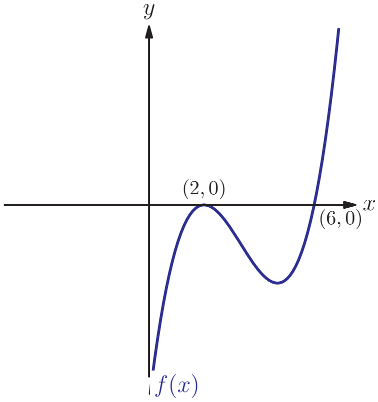
(iii) $f(x) = f^{-1}(x) = \frac{16x-76}{4x-16}$

5. (b) $f\left(\frac{1}{x}\right) + 2f(x) = \frac{2}{x} + 1$ (c) $\frac{1}{3}\left(\frac{4}{x} - 2x + 1\right)$

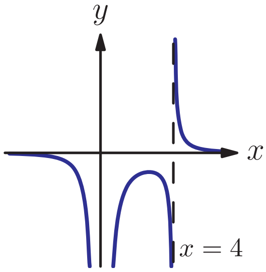
Mixed examination practice 6

Short questions

1. (a)



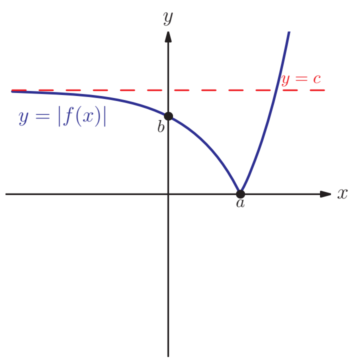
(b)



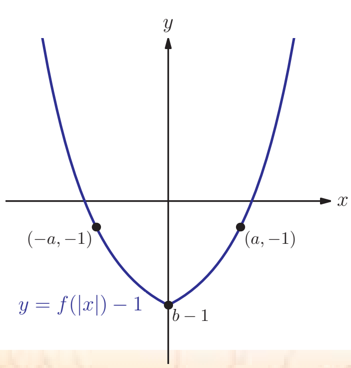
2. $y=2x^2-12x^2+24x-18$

3. $\frac{1}{3}<x<1$

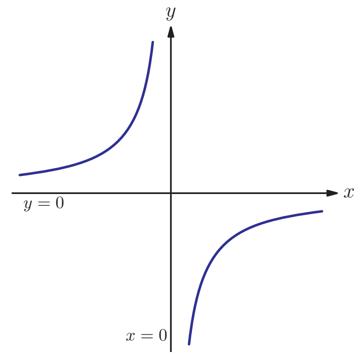
4. (a)



(b)



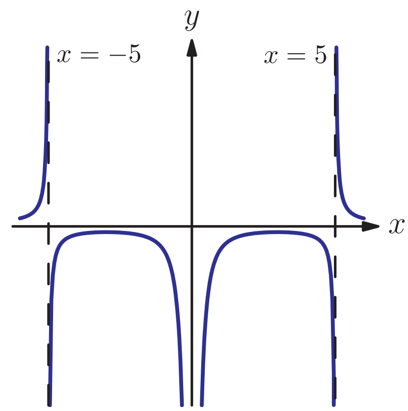
5. (a)



(b) Vertical stretch with scale factor 3 and reflection in the x-axis (or y-axis)

(c) $f^{-1}(x)=-\frac{3}{x}$

6. (a)

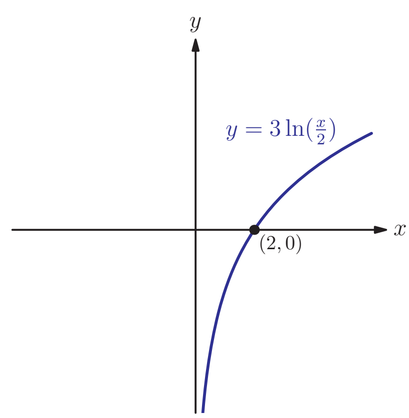


(b) $\left(-3,-\frac{1}{5}\right),\left(3,-\frac{1}{5}\right)$

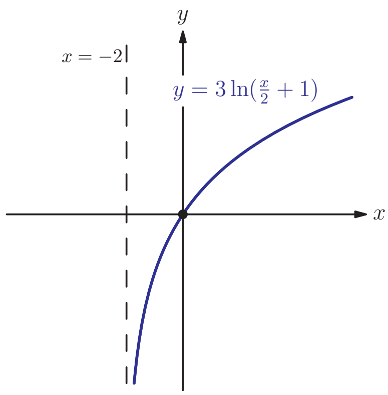
7. Translation by $\begin{pmatrix}-3\\0\end{pmatrix}$ and vertical stretch with scale factor (sf)3.

8. (a) Horizontal stretch with sf 2; vertical stretch with sf 3

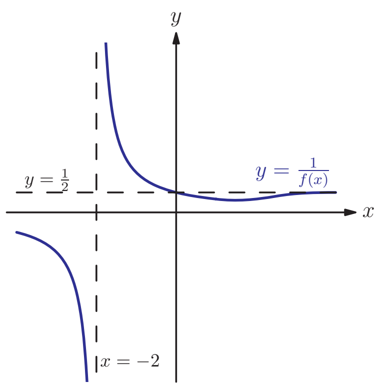
(b)



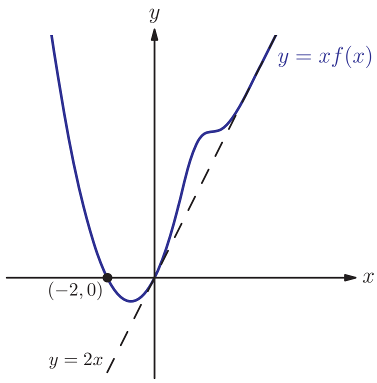
(c)



9. (a)



(b)



10. $x \geq 0$

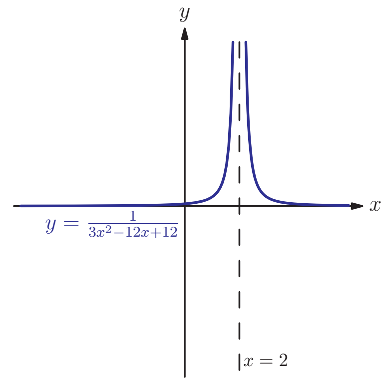
Long questions

1. (a) Translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and vertical stretch with sf 3.

(b) Translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and translation by $\begin{pmatrix} 0 \\ 10 \end{pmatrix}$

(c) Translation by $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ and vertical stretch with scale factor 3.

(d)



2. (a) $y = 3$

(b) $p = 3, q = 1$

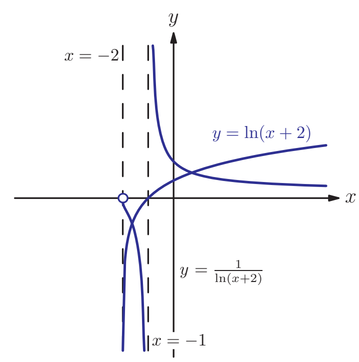
(c) Translation with vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(d) $f^{-1}(x) = \dfrac{2x-5}{x-3}, x \neq 3$

(e) Reflection in the line $y = x$

3. (a) Translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

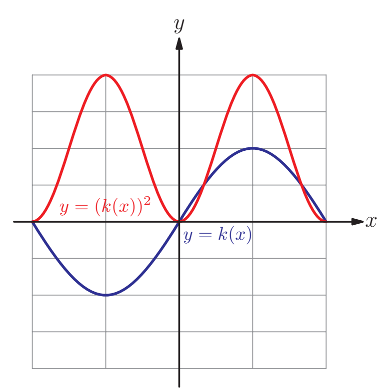
(b)



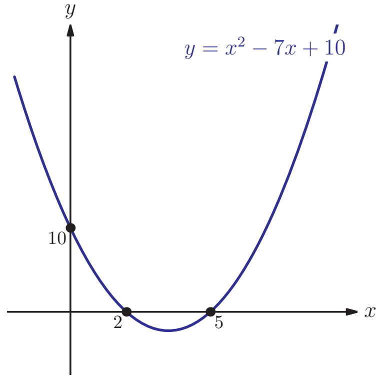
(c) (i) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(ii) $a = -1, b = 6, c = -10,$
$d = -1$

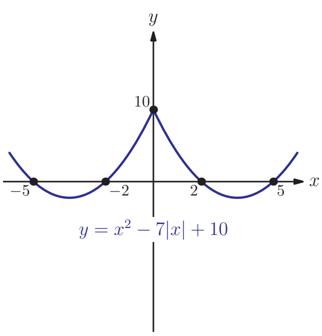
(d)



4. (a)



(c)

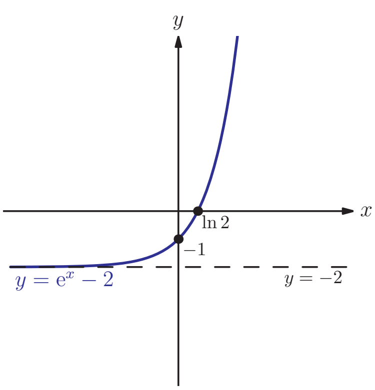


(d) $x = \pm\frac{10}{7}$ (e) $x = \pm 3, \pm 4$

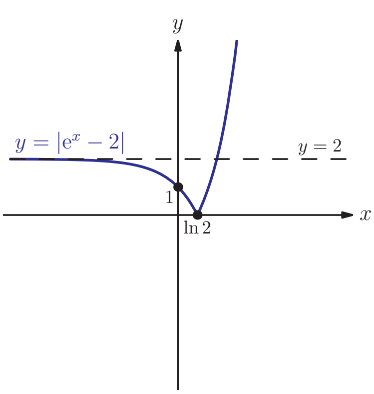
5. (a) -18 (b) 6

(c) $p = 3, q = 17$ (d) $x \in \mathbb{R}$

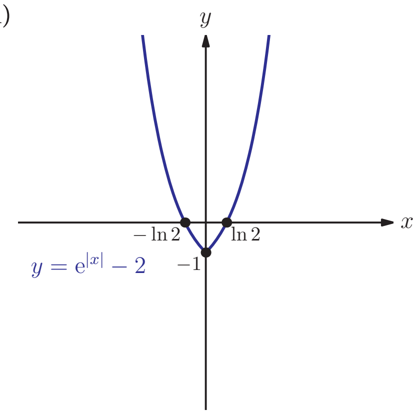
6. (a)



(b) (i)



(ii)



(c) $x = \ln\left(2 - \sqrt{3}\right), x \geq \ln 2$

Chapter 7

Exercise 7A

1. (a) (i) 3.1, 8.1, 13.1, 18.1, 23.1
   (ii) 10, 6.2, 2.4, −1.4, −5.2
   (b) (i) 0, 1, 4, 13, 40
   (ii) 1, −1, −19, −181, −1639
   (c) (i) 2, 3, 6, 18, 108
   (ii) $2, 1, \frac{1}{2}, \frac{1}{2}, 1$
   (d) (i) 3, 4, 8, 9, 13
   (ii) −3, 3, −5, 7, −9
   (e) (i) 0, 4, 8, 12, 16
   (ii) 13, 11, 9, 7, 5

2. (a) (i) 5, 8, 11, 14, 17
   (ii) −4.5, −3, −1.5, 0, 1.5
   (b) (i) 0, 7, 26, 63, 124
   (ii) 5, 20, 45, 80, 125
   (c) (i) 3, 9, 27, 81, 243
   (ii) $4, 2, 1, \frac{1}{2}, \frac{1}{4}$
   (d) (i) 1, 4, 27, 256, 3125
   (ii) 1, 0, −1, 0, 1

3. (a) (i) $u_{n+1} = u_n + 3,\ u_1 = 7$
   (ii) $u_{n+1} = u_n - 0.8,\ u_1 = 1$
   (b) (i) $u_{n+1} = 2u_n,\ u_1 = 3$
   (ii) $u_{n+1} = 1.5u_n,\ u_1 = 12$