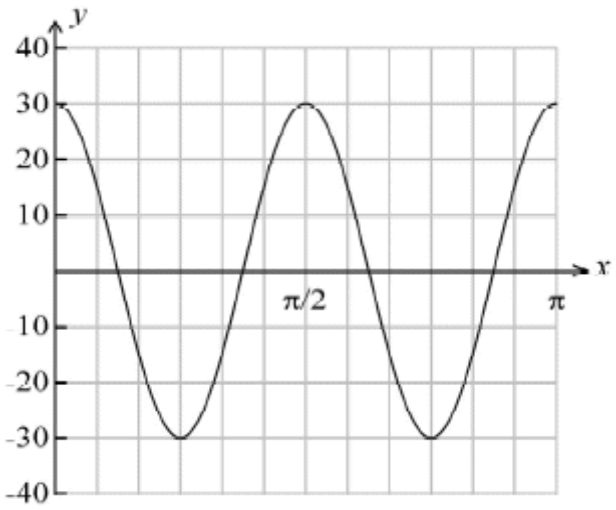
1. The graph of a function of the form $y = p \cos qx$ is given in the diagram below.



(a) Write down the value of p.

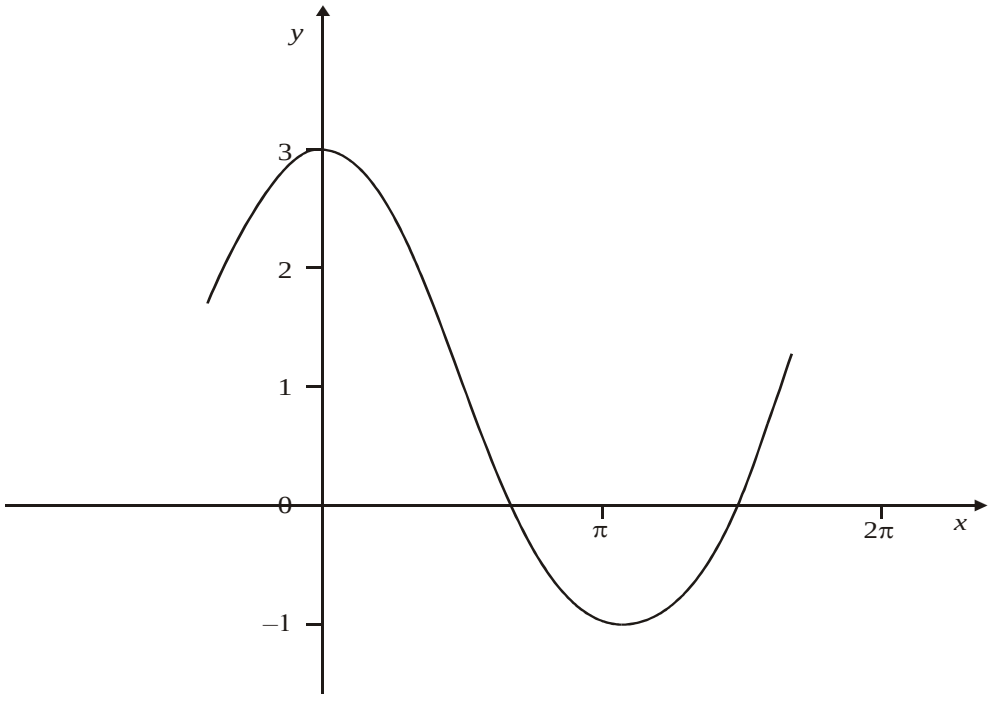
(2)

(b) Calculate the value of q.

(4)

(Total 6 marks)

2. Part of the graph of $y = p + q \cos x$ is shown below. The graph passes through the points $(0, 3)$ and $(\pi, -1)$.



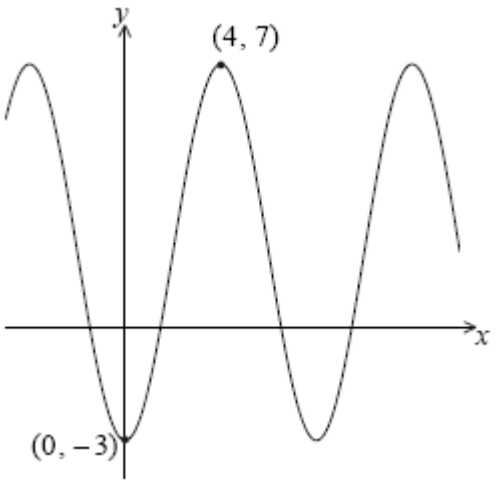
Find the value of

(a) p;

(b) q.

(Total 6 marks)

3. The graph of $y = p \cos qx + r$, for $-5 \leq x \leq 14$, is shown below.



There is a minimum point at (0, –3) and a maximum point at (4, 7).

(a) Find the value of

(i) p;

(ii) q;

(iii) r.

(6)

(b) The equation $y = k$ has exactly **two** solutions. Write down the value of k.

(1)

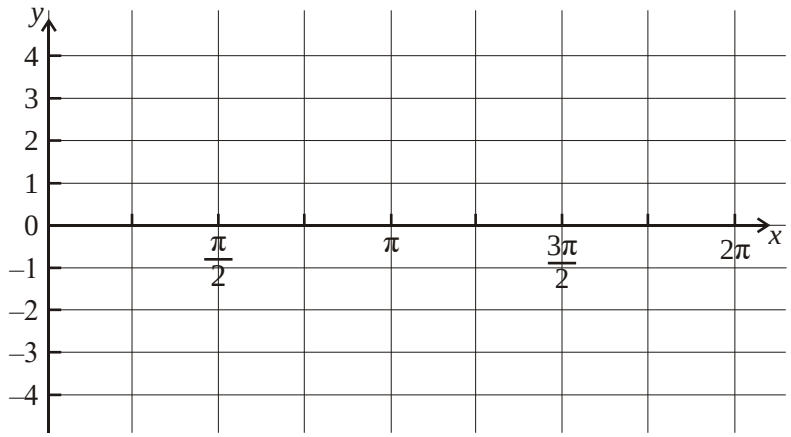
(Total 7 marks)

4. Consider $g(x) = 3 \sin 2x$.

(a) Write down the period of g.

(1)

(b) On the diagram below, sketch the curve of g, for $0 \leq x \leq 2\pi$.



(3)

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \leq x \leq 2\pi$.

(2)

(Total 6 marks)

5. The depth, y metres, of sea water in a bay t hours after midnight may be represented by the function

$$y = a + b\cos\left(\frac{2\pi}{k}t\right), \text{ where } a, b \text{ and } k \text{ are constants.}$$

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of

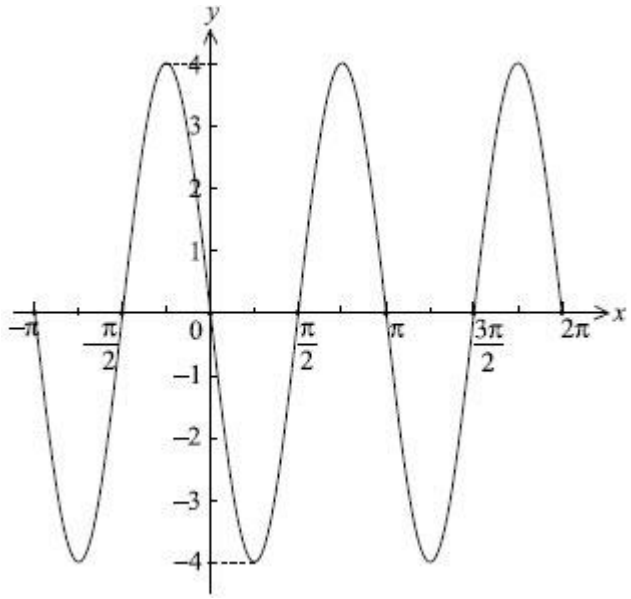
(a) a;

(b) b;

(c) k.

(Total 4 marks)

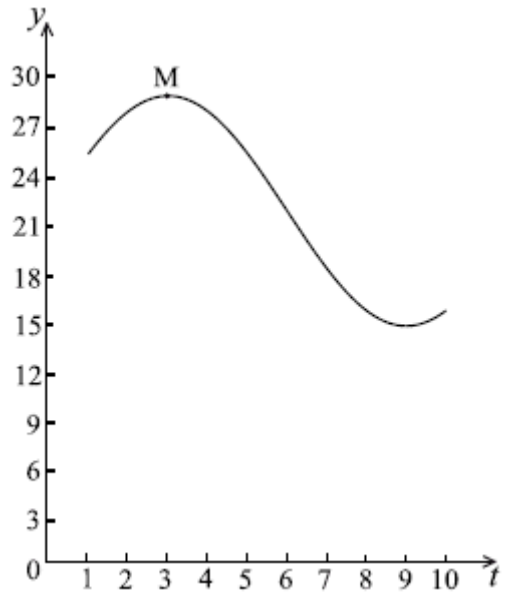
6. Let $f(x) = a \sin b(x - c)$. Part of the graph of f is given below.



Given that a, b and c are positive, find the value of a, of b and of c.

(Total 6 marks)

7. Let $f(t) = a \cos b\,(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

(7)

The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′.

(2)

The graph of g is the image of the graph of f under P.

(c) Find $g(t)$ in the form $g(t) = 7 \cos B(t - C) + D$.

(4)

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f.

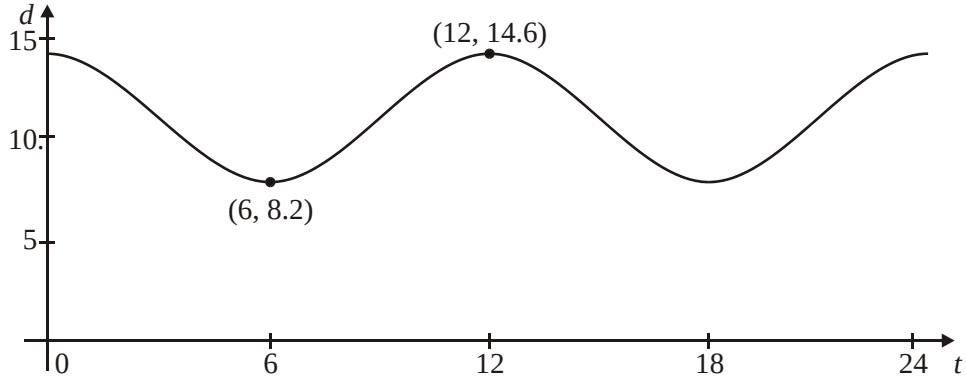
(3)

(Total 16 marks)

8. A formula for the depth d metres of water in a harbour at a time t hours after midnight is

$$d = P + Q\cos\left(\frac{\pi}{6}t\right), \quad 0 \le t \le 24,$$

where P and Q are positive constants. In the following graph the point (6, 8.2) is a minimum point and the point (12, 14.6) is a maximum point.



(a) Find the value of

(i) Q;

(ii) P.

(3)

(b) Find the **first** time in the 24-hour period when the depth of the water is 10 metres.

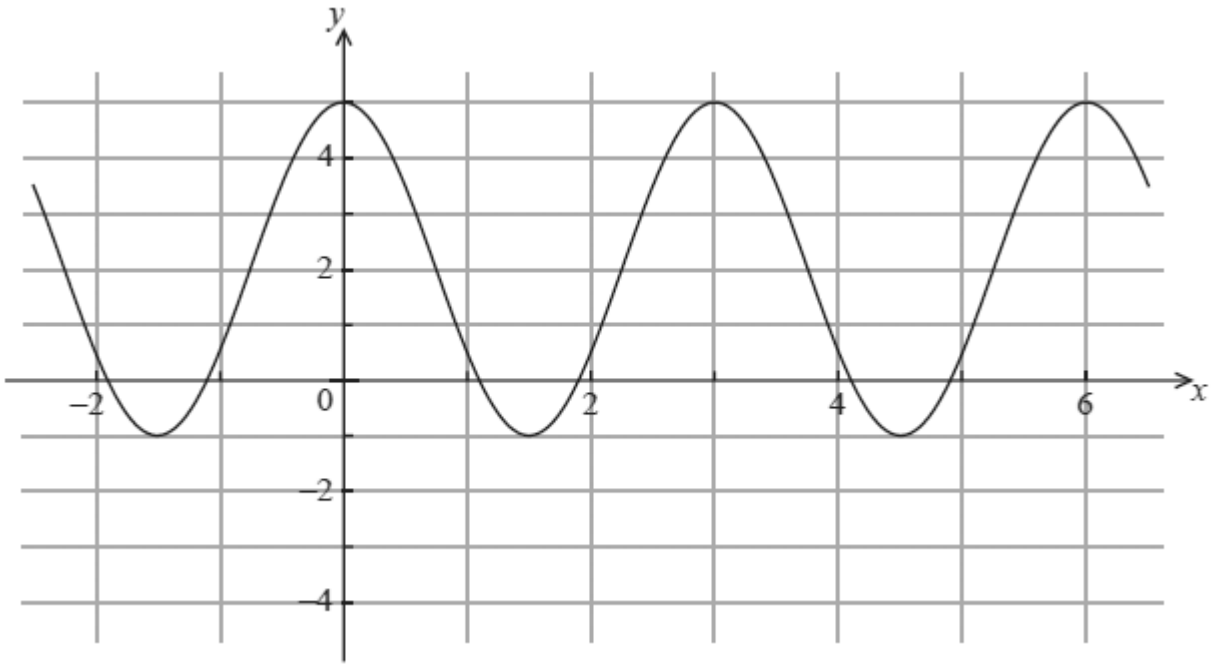
(3)

(c) (i) Use the symmetry of the graph to find the **next** time when the depth of the water is 10 metres.

(ii) Hence find the time intervals in the 24-hour period during which the water is less than 10 metres deep.

(4)

9. The graph below shows $y = a \cos (bx) + c$.



Find the value of a, the value of b and the value of c.

(Total 4 marks)