10. [Maximum mark: 6]

A given polynomial function is defined as $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$. The roots of the polynomial equation $f(x) = 0$ are consecutive terms of a geometric sequence with a common ratio of $\frac{1}{2}$ and first term 2.

Given that $a_{n-1} = -63$ and $a_n = 16$ find

(a) the degree of the polynomial; [4]

(b) the value of a_0. [2]

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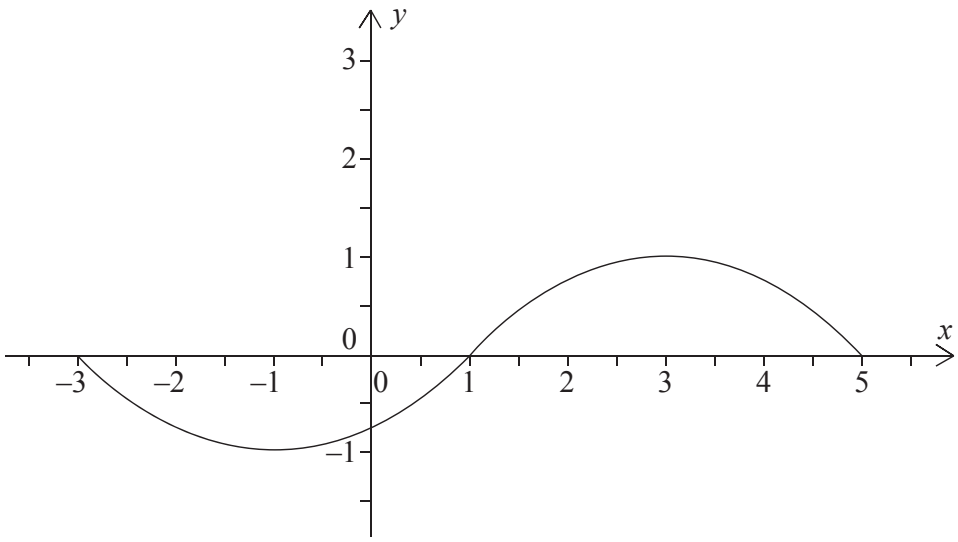
Do **not** write solutions on this page.

12. [Maximum mark: 21]

The following graph represents a function $y = f(x)$, where $-3 \le x \le 5$.
The function has a maximum at $(3, 1)$ and a minimum at $(-1, -1)$.



(a) The functions u and v are defined as $u(x) = x - 3$, $v(x) = 2x$ where $x \in \mathbb{R}$.

(i) State the range of the function $u \circ f$.

(ii) State the range of the function $u \circ v \circ f$.

(iii) Find the largest possible domain of the function $f \circ v \circ u$. [7]

(b) (i) Explain why f does not have an inverse.

(ii) The domain of f is restricted to define a function g so that it has an inverse g^{-1}. State the largest possible domain of g.

(iii) Sketch a graph of $y = g^{-1}(x)$, showing clearly the y-intercept and stating the coordinates of the endpoints. [6]

Consider the function defined by $h(x) = \dfrac{2x-5}{x+d}$, $x \neq -d$ and $d \in \mathbb{R}$.

(c) (i) Find an expression for the inverse function $h^{-1}(x)$.

(ii) Find the value of d such that h is a self-inverse function.

For this value of d, there is a function k such that $h \circ k(x) = \dfrac{2x}{x+1}$, $x \neq -1$.

(iii) Find $k(x)$. [8]