Exercise 15D

1 Find the value of R and $\tan\alpha$ in each of the following identities.

a) $3\sin\theta + 4\cos\theta \equiv R\sin(\theta + \alpha)$
b) $5\sin\theta - 12\cos\theta \equiv R\sin(\theta - \alpha)$
c) $2\cos\theta + 5\sin\theta \equiv R\cos(\theta - \alpha)$
d) $2\cos\theta + 5\sin\theta \equiv R\sin(\theta + \alpha)$
e) $\cos\theta - \sin\theta \equiv R\cos(\theta + \alpha)$
f) $20\sin\theta - 15\cos\theta \equiv R\sin(\theta - \alpha)$
g) $\sqrt{3}\cos\theta + \sin\theta \equiv R\cos(\theta - \alpha)$
h) $2\cos\theta - 4\sin\theta \equiv R\cos(\theta + \alpha)$

2 Find the greatest and least values of each of the following expressions, and state, correct to one decimal place, the smallest non negative value of θ for which each occurs.

a) $12\sin\theta + 5\cos\theta$
b) $2\cos\theta + \sin\theta$
c) $7 + 3\sin\theta - 4\cos\theta$
d) $10 - 2\sin\theta + \cos\theta$
e) $\dfrac{1}{2 + \sin\theta + \cos\theta}$
f) $\dfrac{1}{7 - 2\cos\theta + \sqrt{5}\sin\theta}$
g) $\dfrac{3}{5\cos\theta - 12\sin\theta + 16}$
h) $\dfrac{2}{7 - 4\sqrt{3}\cos\theta + \sin\theta}$

3 Solve each of the following equations for $0° \leqslant \theta \leqslant 360°$, giving your answers correct to one decimal place.

a) $\sin\theta + \sqrt{3}\cos\theta = 1$
b) $4\sin\theta - 3\cos\theta = 2$
c) $\sin\theta + \cos\theta = \dfrac{1}{\sqrt{2}}$
d) $5\sin\theta + 12\cos\theta = 7$
e) $7\sin\theta - 4\cos\theta = 3$
f) $\cos\theta - 3\sin\theta = 2$
g) $5\cos\theta + 2\sin\theta = 4$
h) $9\cos 2\theta - 4\sin 2\theta = 6$
i) $\dfrac{\sqrt{5}}{2}\sec\theta - \tan\theta = 2$
j) $\cot\theta - \sqrt{13}\operatorname{cosec}\theta = 5$
k) $24 = 10\operatorname{cosec}\theta - 7\cot\theta$
l) $\sqrt{2}\tan 2\theta - \sqrt{3}\sec 2\theta = \sqrt{2}$

*4 Show that $1 - \sqrt{2} \leqslant 2\cos^2\theta + \sin 2\theta \leqslant 1 + \sqrt{2}$ for all values of θ.

*5 a) Given that $6\cos^2\theta - 8\sin\theta\cos\theta \equiv A + R\cos(2\theta + \alpha)$, find the values of the constants A, R and α.
b) Hence solve the equation $6\cos^2\theta - 8\sin\theta\cos\theta = 5$, for $0° \leqslant \theta \leqslant 360°$.
c) Deduce the solutions to the equation $6\sin^2\phi + 8\cos\phi\sin\phi = 5$, for $-90° \leqslant \phi \leqslant 270°$.

Exercise 15D

1 a) 5, $\frac{4}{3}$ b) 13, $\frac{12}{5}$ c) $\sqrt{29}$, $\frac{5}{2}$ d) $\sqrt{29}$, $\frac{2}{5}$ e) $\sqrt{2}$, 1 f) 25, $\frac{3}{4}$ g) 2, $\frac{1}{\sqrt{3}}$ h) $2\sqrt{5}$, 2 2 a) 13, 67.4°; −13, 247.4°

2 b) $\sqrt{5}$, 26.6°, $-\sqrt{5}$, 206.6° c) 12, 143.1°; 2, 323.1° d) $10 + \sqrt{5}$, 296.6°; $10 - \sqrt{5}$, 116.6°

2 e) $\dfrac{1}{2-\sqrt{2}}\left(=\dfrac{2+\sqrt{2}}{2}\right)$, 225°; $\dfrac{1}{2+\sqrt{2}}\left(=\dfrac{2-\sqrt{2}}{2}\right)$, 45° f) $\frac{1}{4}$, 311.8°; $\frac{1}{10}$, 131.8° g) 1, 112.6°; $\frac{3}{29}$, 292.6° h) $\pm\infty$, near 351.8°

3 a) 90°, 330° b) 60.4°, 193.3° c) 105°, 345° d) 80.0°, 325.2° e) 51.6°, 187.9° f) 237.7°, 339.2° g) 63.8°, 339.8°

3 h) 14.2°, 141.8°, 194.2°, 321.8° i) 86.6°, 326.6° j) 236.3°, 326.3° k) 7.3°, 140.2° l) 52.5°, 82.5°, 232.5°, 262.5°

5 a) $A = 3$, $R = 5$, $\alpha = \tan^{-1}\frac{4}{3}$ b) 6.6°, 120.2°, 186.6°, 300.2° c) −83.4°, 30.2°, 96.6°, 210.2°