Homework

Suppose we have a triangle ABC with the lengths of AB and BC and the size of the angle BAC given (with $\angle BAC$ acute).

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How many such triangles are possible and what does this depend on?

The answer to this problem is discussed on the next slides.

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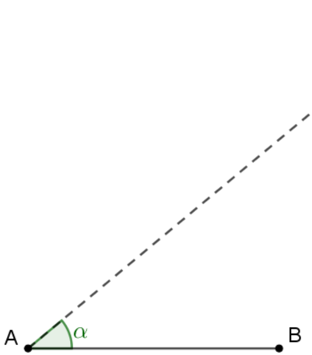

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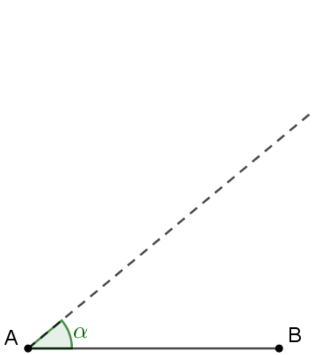

Now we will consider several cases.

Case 1

If the length of BC is *too small* then we won't be able to complete the construction:

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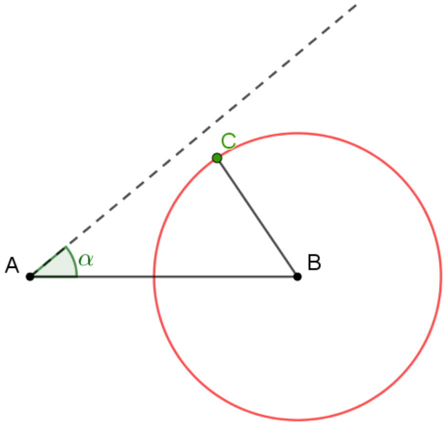

Case 2

If the length of BC is *exactly right* then we will have one triangle:

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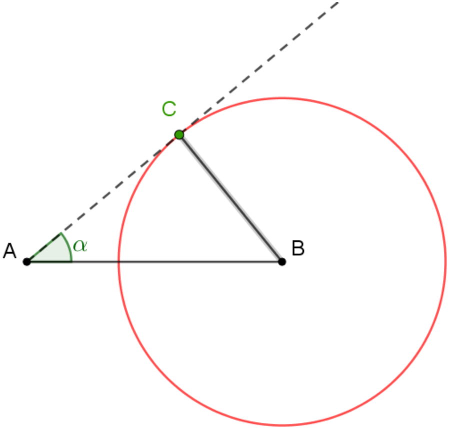

Case 3

If the length of BC is *big enough* then we will have two triangles:

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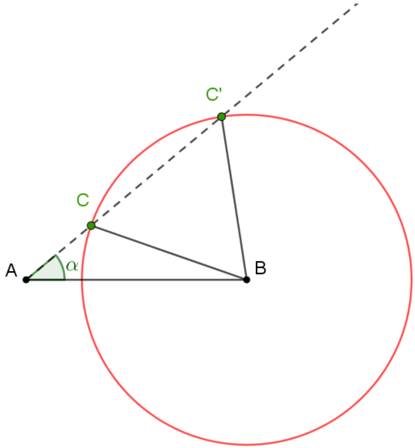

Case 4

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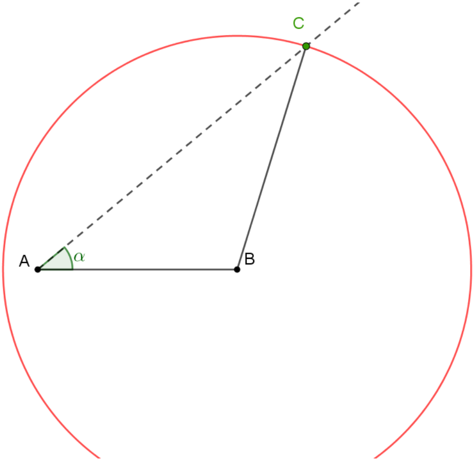

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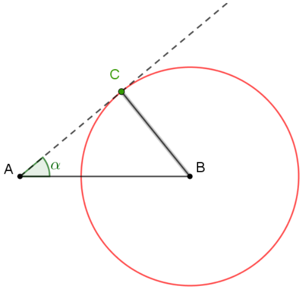
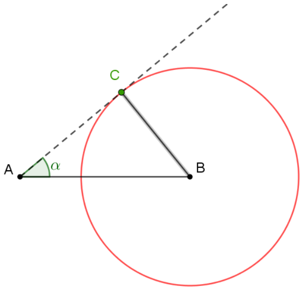
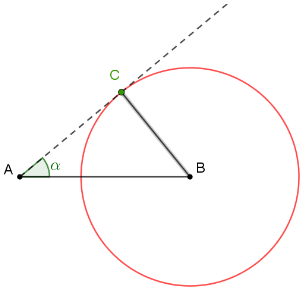




In this case the half-line AC is tangent to the circle of radius $|BC|$ centred at B. This means that the angle ACB is a right angle.



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$$|BC| = |AB| \sin \alpha$$

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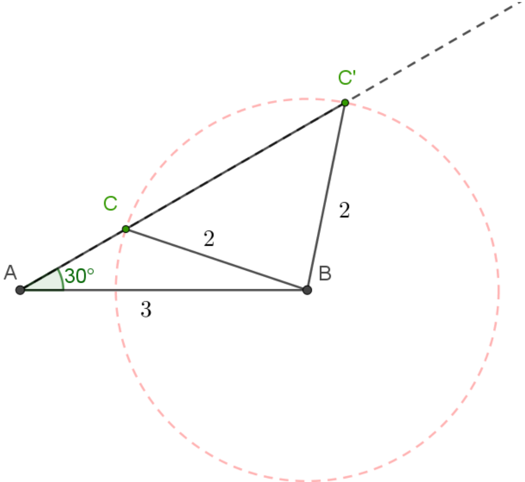
So for example if we're given $|AB| = 3, |BC| = 2$ and $\angle BAC = 30^\circ$, then because $3 \sin 30^\circ < 2 < 3$, we know that there will be two triangle satisfying these conditions.

Two triangles satisfying $|AB| = 3, |BC| = 2$ and $\angle BAC = 30°$:



Note that throughout this discussion we've assumed that $\angle BAC$ is acute. If it's not, then the whole thing is much simpler, but I'll leave it to you to think about it.

In case of any questions you can email me at T.J.Lechowski@gmail.com.