Homework

Example of an exam question:

Consider the function $f: x \to \sqrt{\frac{\pi}{4} - \arccos x}$.

(a) Find the largest possible domain of f.

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(b) Determine an expression for the inverse function, f^{-1}, and write down its domain.

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(Total 8 marks)

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which is of course equivalent to:

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You should say: "Hold on! What happened at (1)? Why is the inequality sign reversed?". Let's make a quick detour here and talk about inequalities on the next slides.

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The answer is: in general no. It is true only if we know that the function is increasing. For instance if $f(x) = 2x$ (an increasing function), then $x > 13 \implies 2x > 26$. And we use it all the time when solving equations. The functions $f(x) = \frac{x}{2}$ and $g(x) = x - 7$ are increasing, so when we solve an inequality dividing by 2 and subtracting 7 preserves the inequality.

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However for instance $h(x) = -3x$ is a decreasing function, so the larger the argument the smaller the value, which means that if $x > 13$, then $-3x < -39$.

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However for instance $h(x) = -3x$ is a decreasing function, so the larger the argument the smaller the value, which means that if $x > 13$, then $-3x < -39$. The inequality is reversed.

Some other functions are neither decreasing nor increasing. For instance $f(x) = x^2$, so if we have $x > -5$, then we don't know if $x^2 > (-5)^2$ or $x^2 < (-5)^2$, both are possible.

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However if we restrict the domain of $f(x) = x^2$ to only non-negative values, then this function is increasing, so if $x > 13$ (both are positive), then $x^2 > 169$.

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However if we restrict the domain of $f(x) = x^2$ to only non-negative values, then this function is increasing, so if $x > 13$ (both are positive), then $x^2 > 169$.

Similarly if we restrict the domain of $f(x) = x^2$ to only non-positive values, then this function is decreasing, so if $x < -7$ (both are negative), then $x^2 > 49$ (inequality reversed).

Going back to this point:

$$\arccos x \leq \frac{\pi}{4}$$

We apply cos() to both sides. Note however that we have restricted the domain of *cosine* to $[0, \pi]$ and in this domain it is a decreasing function, so the inequality sign should be reversed and we get:

$$x \geq \frac{\sqrt{2}}{2}$$

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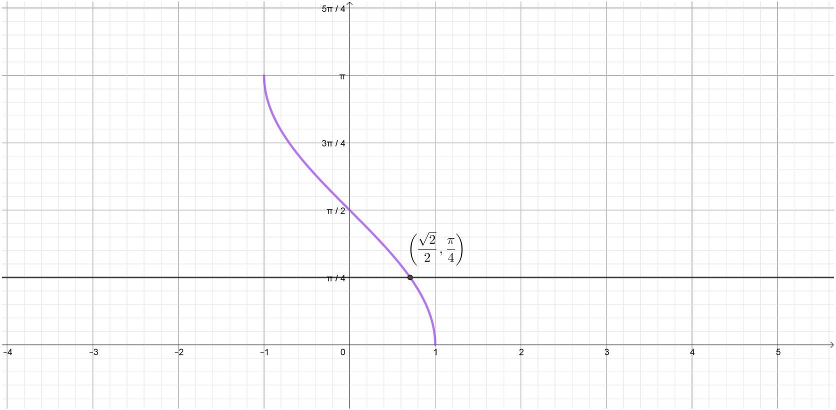
Finally our answer is that the domain is $\frac{\sqrt{2}}{2} \leq x \leq 1$.

The whole question looks much easier if we just draw the arccos x and the line $y = \frac{\pi}{4}$:



We want $\arccos x \leq \dfrac{\pi}{4}$, so we have $\dfrac{\sqrt{2}}{2} \leq x \leq 1$.

We move on to part (b). We start with: We have a function:

$$y = \sqrt{\frac{\pi}{4} - \arccos x}$$

squaring and rearranging gives:

$$\arccos x = \frac{\pi}{4} - y^2$$

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So we have:

$$f^{-1}(x) = \cos\left(\frac{\pi}{4} - x^2\right)$$

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This is fairly easy: the least $\arccos x$ can be is 0, the most it can be is $\frac{\pi}{4}$, so the range of our function is $0 \leq y \leq \sqrt{\frac{\pi}{4}}$.

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So the domain of

$$f^{-1}(x) = \cos\left(\frac{\pi}{4} - x^2\right)$$

is $0 \leq x \leq \sqrt{\frac{\pi}{4}}$.

In case of any questions you can email me at T.J.Lechowski@gmail.com.