Name:

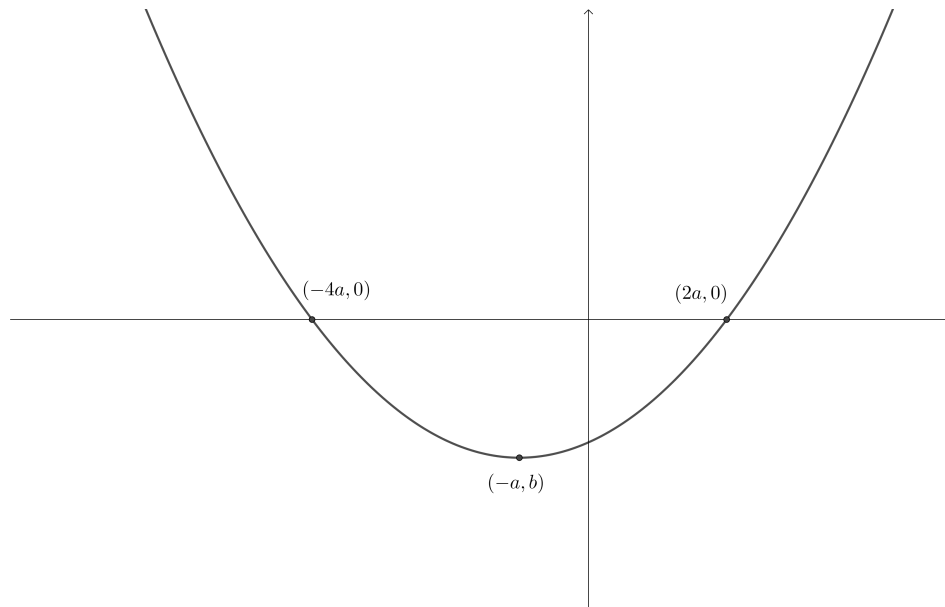1. (5 points) Consider the polynomial $P(x) = x^3 - 2x^2 - x + 2$.

   (a) Show that $x = 1$ is a root of $P(x)$.

   (b) Hence, or otherwise, factorize $P(x)$ into product of linear factors.

   Consider another polynomial $Q(x)$. The remainders when $Q(x)$ is divided by $(x - 1)$, $(x + 1)$ and $(x - 2)$ are 2, -8 and 10 respectively.

   (c) Find the remainder when $Q(x)$ is divided by $P(x)$.

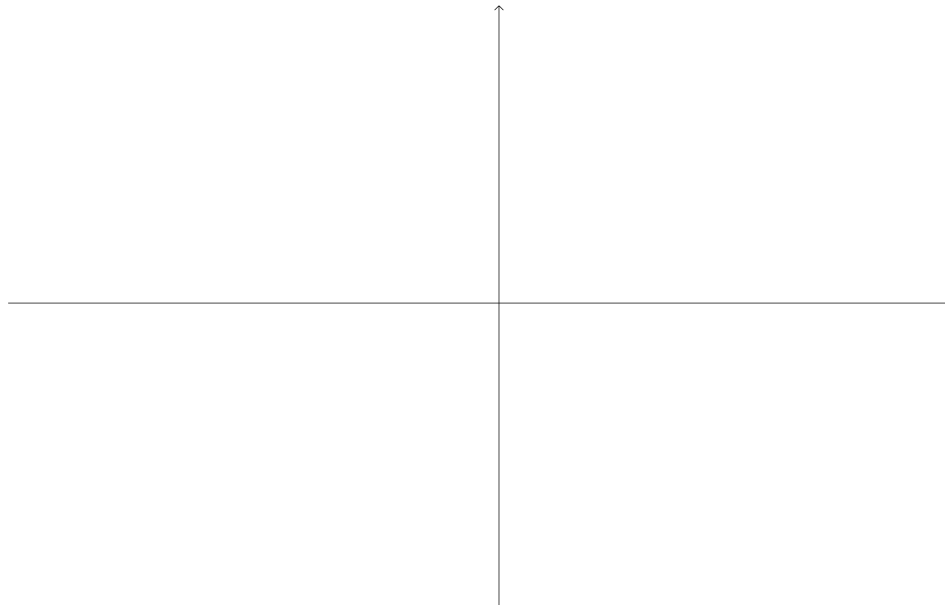2. (5 points) The diagram below shows the graph of a function $f(x)$.

with $a > 0$ and $b < -1$.

Use the diagram below to sketch the graphs of

(i) $g(x) = |f(-2x)|$

(ii) $h(x) = \dfrac{1}{f(x - 2a)}$

Clearly indicate all the x-axis intercepts, maxima and minima and asymptotes.
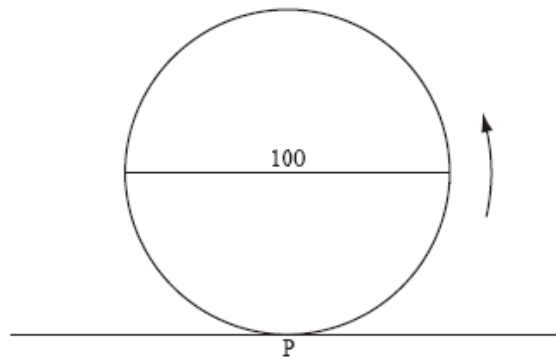
3. (4 points) Consider the function

$$f(x) = \sqrt{\arcsin x + \frac{\pi}{6}}$$

(a) Find the domain and range of $f(x)$.

(b) Find the $f^{-1}(x)$, the inverse of $f(x)$.

(c) Write down the domain and range of $f^{-1}(x)$.

4. (7 points) The following diagram represents a large Ferris wheel, with a diameter of 100 metres.

Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an counter-clockwise direction. One revolution takes 20 minutes.

(a) Write down the height of P above ground level after

      (i) 10 minutes;

      (ii) 15 minutes;

Let $h(t)$ metres be the height of P above ground level after t minutes.

(b) Given that h can be expressed in the form $h(t) = a \cos bt + c$, find a, b and c.

(c) Sketch the graph of $h(t)$ for $0 \leqslant t \leqslant 40$.