Mixed examination practice 2

Short questions

1. Solve $\log_5\left(\sqrt{x^2+49}\right)=2$. *[4 marks]*

2. If $a=\log x$, $b=\log y$ and $c=\log z$ (all logs base 10) find in terms of a, b, c and integers:

(a) $\log\dfrac{x^2\sqrt{y}}{z}$ (b) $\log\sqrt{0.1x}$ (c) $\log_{100}\left(\dfrac{y}{z}\right)$ *[6 marks]*

3. Solve the simultaneous equations:

$$\ln x+\ln y^2=8$$

$$\ln x^2+\ln y=6$$ *[6 marks]*

4. If $y=\ln x-\ln(x+2)+\ln(4-x^2)$, express x in terms of y. *[6 marks]*

5. Find the exact value of x satisfying the equation

$$2^{3x-2}\times 3^{2x-3}=36^{x-1}$$

giving your answer in simplified form $\dfrac{\ln p}{\ln q}$, where $p,q\in\mathbb{Z}$. *[5 marks]*

6. Given $\log_a b^2=c$ and $\log_b a=c-1$ for some value c, where $0<a<b$, express a in terms of b. *[6 marks]*

7. Solve the equation $9\log_5 x=25\log_x 5$, expressing your answers in the form $5^{\frac{p}{q}}$, where $p,q\in\mathbb{Z}$. *[6 marks]*

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8. Find the exact solution to the equation $\ln x=4\log_x \mathrm{e}$. *[5 marks]*

Long questions

1. The speed of a parachutist (V) in metres per second, t seconds after jumping is modelled by the expression:

$$V=42(1-\mathrm{e}^{-0.2t})$$

(a) Sketch a graph of V against t.

(b) What is the initial speed?

(c) What is the maximum speed that the parachutist could reach?

When the parachutist reaches 22 ms^{-1} he opens the parachute.

(d) How long is he falling before he opens his parachute? *[9 marks]*

2. Scientists think that the global population of tigers is falling exponentially. Estimates suggest that in 1970 there were 37 000 tigers but by 1980 the number had dropped to 22 000.

(a) Form a model of the form $T = ka^n$ connecting the number of tigers (T) with the number of years after 1970 (n).

(b) What does the model predict that the population will be in 2020?

(c) When the population reaches 1000 the tiger population will be described as 'near extinction'. In which year will this happen?

In the year 2000 a worldwide ban on the sale of tiger products was implemented, and it is believed that by 2010 the population of tigers had recovered to 10 000.

(d) If the recovery has been exponential find a model of the form $T = ka^m$ connecting the number of tigers (T) with the number of years after 2000 (m).

(e) If in each year since 2000 the rate of growth has been the same, find the percentage increase each year. *[12 marks]*

3. (a) If $\ln y = 2\ln x + \ln 3$ find y in terms of x.

(b) If the graph of $\ln y$ against $\ln x$ is a straight line with gradient 4 and y-intercept 6, find the relationship between x and y.

(c) If the graph of $\ln y$ against x is a straight line with gradient 3 and it passes through the point (1, 2), express y in terms of x.

(d) If the graph of e^y against x^2 is a straight line through the origin with gradient 4, find the gradient of the graph of y against $\ln x$. *[10 marks]*

Mixed examination practice 2

Short questions

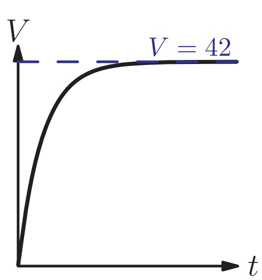
1. $x = \pm 24$
2. (a) $2a + \frac{b}{2} - c$

   (b) $\frac{a-1}{2}$

   (c) $\frac{b-c}{2}$
3. $x = e^{\frac{4}{3}} = 3.79, y = e^{\frac{10}{3}} = 28.0$
4. $x = 1 \pm \sqrt{1 - e^y}$
5. $x = \frac{\ln 3}{\ln 2}$
6. $a = b^{-2}$
7. $x = 5^{\frac{5}{3}}$ or $5^{\frac{-5}{3}}$
8. $x = e^2$ or e^{-2}

Long questions

1. (a)



   (b) 0 ms^{-1}
   (c) 42 ms^{-1}
   (d) 3.71s

2. (a) $k = 37000, a = \left(\frac{22}{37}\right)^{0.1} = 0.949$

   (b) 2750
   (c) 2039
   (d) $k = 7778, a = \left(\frac{10000}{7778}\right)^{0.1} = 1.025$

   (e) 2.5%

3. (a) $y = 3x^2$
   (b) $y = e^6 x^4$
   (c) $y = 2e^{3x-3}$
   (d) 2

Chapter 3

Exercise 3A

1. (a) Order 3, lead coefficient 3
   (b) Order 5, lead coefficient −1
   (c) No
   (d) No
   (e) No
   (f) No
   (g) Order 7, lead coefficient 2
   (h) Order 0, lead coefficient 1
2. (a) (i) $6x^3 + 8x^2 - 29x + 14$
   (ii) $3x^3 + 16x^2 + 23x + 6$
   (b) (i) $2x^4 - 15x^3 + 4x^2 + 4x - 1$
   (ii) $2x^4 - 7x^3 - 30x^2 + 6x + 15$
   (c) (i) $b^4 + b^3 - 3b^2 + 14b - 4$
   (ii) $r^4 - 11r^3 + 33r^2 - 62r + 14$
   (d) (i) $-x^6 + 2x^5 + 5x^4 - 10x^3 - x^2 + 5$
   (ii) $-x^6 + 2x^4 + x^3 - x^2 - x$
3. (a) (i) $x^2 + 5x - 1$
   (ii) $x^2 + x - 6$
   (b) (i) $3x^2 + 2x - 2$
   (ii) $5x^2 - 2$
   (c) (i) $x^3 - 2x^2 + 3x + 7$
   (ii) $x^3 - x^2 + x + 7$
   (d) (i) $x^2 + 5$
   (ii) $x - 2$
4. (a) (i) $x^3 + x^2 + 3$
   (ii) $x^3 + x^2 + 2$
   (b) (i) $2x^2 + 3$
   (ii) $x - 3$
5. (a) (i) $a = 4, b = -6$
   (ii) $a = 3, b = 1$
   (b) (i) $a = b = 2$
   (ii) $a = 0, b = -3$
   (c) (i) $a = 2, b = -2$
   (ii) $a = 2, b = 5$
   (d) (i) $a = -4, b = -6$
   (ii) $a = 10, b = 3$
   (e) (i) $a = \pm 2, b = 2$
   (ii) $a = \pm 2, b = \mp 5$
7. (a) Yes (b) No