

1. (a) Let  $\log_c 3 = p$  and  $\log_c 5 = q$ . Find an expression in terms of  $p$  and  $q$  for
- (i)  $\log_c 15$ ;
  - (ii)  $\log_c 25$ .
- (b) Find the value of  $d$  if  $\log_d 6 = \frac{1}{2}$ .

**(Total 6 marks)**

2. Let  $\ln a = p$ ,  $\ln b = q$ . Write the following expressions in terms of  $p$  and  $q$ .

- (a)  $\ln a^3 b$
- (b)  $\ln \left( \frac{\sqrt{a}}{b} \right)$

**(Total 6 marks)**

3. Solve the equation  $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$ .

**(Total 4 marks)**

4. Solve the equation  $9^{x-1} = \left( \frac{1}{3} \right)^{2x}$ .

**(Total 4 marks)**

5. Given that  $p = \log_a 5$ ,  $q = \log_a 2$ , express the following in terms of  $p$  and/or  $q$ .

- (a)  $\log_a 10$
- (b)  $\log_a 8$
- (c)  $\log_a 2.5$

**(Total 6 marks)**

6. Let  $p = \log_{10} x$ ,  $q = \log_{10} y$  and  $r = \log_{10} z$ .

Write the expression  $\log_{10} \left( \frac{x}{y^2 \sqrt{z}} \right)$  in terms of  $p$ ,  $q$  and  $r$ .

(Total 6 marks)

7. (a) Find  $\log_2 32$ .

(1)

(b) Given that  $\log_2 \left( \frac{32^x}{8^y} \right)$  can be written as  $px + qy$ , find the value of  $p$  and of  $q$ .

(4)

(Total 5 marks)

8. Given that  $\log_5 x = y$ , express each of the following in terms of  $y$ .

(a)  $\log_5 x^2$

(b)  $\log_5 \left( \frac{1}{x} \right)$

(c)  $\log_{25} x$

(Total 6 marks)

9. Solve  $\log_2 x + \log_2(x - 2) = 3$ , for  $x > 2$ .

(Total 7 marks)

10. Find the **exact** solution of the equation  $9^{2x} = 27^{(1-x)}$ .

(Total 6 marks)

11. Solve the following equations.

(a)  $\ln(x + 2) = 3$ .

(b)  $10^{2x} = 500$ .

(Total 6 marks)

12. The functions  $f(x)$  and  $g(x)$  are defined by  $f(x) = e^x$  and  $g(x) = \ln(1 + 2x)$ .

(a) Write down  $f^{-1}(x)$ .

(b) (i) Find  $(f \circ g)(x)$ .

(ii) Find  $(f \circ g)^{-1}(x)$ .

(Total 6 marks)

13. Solve the following equations.

(a)  $\log_x 49 = 2$

(3)

(b)  $\log_2 8 = x$

(2)

(c)  $\log_{25} x = -\frac{1}{2}$

(3)

(d)  $\log_2 x + \log_2(x - 7) = 3$

(5)

(Total 13 marks)

14. Let  $f(x) = 3 \ln x$  and  $g(x) = \ln 5x^3$ .

(a) Express  $g(x)$  in the form  $f(x) + \ln a$ , where  $a \in \mathbb{Z}^+$ .

(4)

(b) The graph of  $g$  is a transformation of the graph of  $f$ . Give a full geometric description of this transformation.

(3)

(Total 7 marks)

15. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After  $n$  years the number of taxis,  $T$ , in the city is given by

$$T = 280 \times 1.12^n.$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After  $n$  years the number of people,  $P$ , in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90e^{-0.1n}}.$$

(i) Find the value of  $P$  at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of  $P$  be double its value at the end of 2000? Justify your answer.

(6)

(c) Let  $R$  be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if  $R < 70$ .

(i) Find the value of  $R$  at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

(5)

(Total 17 marks)