1.

(a) (i) $\log_c 15 = \log_c 3 + \log_c 5$ (A1)

$= p + q$ A1 N2

(ii) $\log_c 25 = 2\log_c 5$ (A1)

$= 2q$ A1 N2

(b) **METHOD 1**

$d^{\frac{1}{2}} = 6$ M1

$d = 36$ A1 N1

METHOD 2

For changing base M1

eg $\frac{\log_{10} 6}{\log_{10} d} = \frac{1}{2}$, $2\log_{10} 6 = \log_{10} d$

$d = 36$ A1 N1

[6]

2

(a) $\ln a^3 b = 3\ln a + \ln b$ (A1)(A1)

$\ln a^3 b = 3p + q$ A1 N3

(b) $\ln \frac{\sqrt{a}}{b} = \frac{1}{2}\ln a - \ln b$ (A1)(A1)

$\ln \frac{\sqrt{a}}{b} = \frac{1}{2}p - q$ A1 N3

[6]

3.

METHOD 1

$\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = 2 - 1 + \frac{1}{2}$ (M1)

$\Rightarrow \frac{3}{2} = \log_9 x$ (A1)

$\Rightarrow x = 9^{\frac{3}{2}}$ (M1)

$\Rightarrow x = 27$ (A1) (C4)

METHOD 2

$\log 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9\left[81\left(\frac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

4.

$9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

$3^{2x-2} = 3^{-2x}$ (M1) (A1)

$2x - 2 = -2x$ (A1)

$x = \frac{1}{2}$ (A1) (C4)

[4]

5.

(a) $\log_a 10 = \log_a (5 \times 2)$ (M1)

$= \log_a 5 + \log_a 2$

$= p + q$ A1 N2

(b) $\log_a 8 = \log_a 2^3$ (M1)

$= 3 \log_a 2$

$= 3q$ A1 N2

(c) $\log_a 2.5 = \log_a \frac{5}{2}$ (M1)

$= \log_a 5 - \log_a 2$

$= p - q$ A1 N2

[6]

6.

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - \log_{10} y^2 - \log_{10}\sqrt{z}$ (A1)(A1)(A1)

$\log_{10} y^2 = 2 \log_{10} y$ (A1)

$\log_{10}\sqrt{z} = \frac{1}{2}\log z$ (A1)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - 2\log y - \frac{1}{2}\log z$

$= p - 2q - \frac{1}{2}r$ (A1) (C2)(C2)(C2)

7.

(a) 5 A1 N1

$\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$ (A1)

$= x\log_2 32 - y\log_2 8$ (A1)

$\log_2 8 = 3$ (A1)

$p = 5$, $q = -3$ (accept $5x - 3y$) A1 N3

8.

(a) $\log_5 x^2 = 2\log_5 x$ (M1)

$= 2y$ (A1) (C2)

(b) $\log_5 \frac{1}{x} = -\log_5 x$ (M1)

$= -y$ (A1) (C2)

(c) $\log_{25} x = \frac{\log_5 x}{\log_5 25}$ (M1)

$= \frac{1}{2}y$ (A1) (C2)

[6]

9.

recognizing $\log a + \log b = \log ab$ (seen anywhere) (A1)

e.g. $\log_2(x(x-2))$, $x^2 - 2x$

recognizing $\log_a b = x \Leftrightarrow a^x = b$ (seen anywhere) (A1)

e.g. $2^3 = 8$

correct simplification A1

e.g. $x(x-2) = 2^3$, $x^2 - 2x - 8$

evidence of correct approach to solve (M1)

e.g. factorizing, quadratic formula

correct working A1

e.g. $(x-4)(x+2)$, $\frac{2 \pm \sqrt{36}}{2}$

$x = 4$ A2 N3

[7]

10.

METHOD 1

$9 = 3^2,\ 27 = 3^3$ (A1)(A1)

expressing as a power of 3, $(3^2)^{2x} = (3^3)^{1-x}$ (M1)

$3^{4x} = 3^{3-3x}$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

METHOD 2

$2x \log 9 = (1 - x)\log 27$ (M1)(A1)(A1)

$\frac{2x}{1-x} = \frac{\log 27}{\log 9}\left(= \frac{3}{2}\right)$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

Notes: *Candidates may use a graphical method.*
Award (M1)(A1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 0.4285...., and (A1) for $\frac{3}{7}$.

[6]

11.

(a) $e^{\ln(x+2)} = e^3$ (M1)

$x + 2 = e^3$ (A1)

$x = e^3 - 2\ (= 18.1)$ A1 N3

(b) $\log_{10}(10^{2x}) = \log_{10} 500$ (accept lg and log for $\log_{10}$) (M1)

$2x = \log_{10} 500$ (A1)

$x = \frac{1}{2}\log_{10} 500 \quad \left(= \frac{\log 500}{\log 100} = 1.35\right)$ A1 N3

Note: *In both parts (a) and (b), if candidates use a graphical approach, award **M1** for a sketch, A1 for indicating appropriate points of intersection, and A1 for the answer.*

[6]

12.

(a) $f^{-1}(x) = \ln x$ A1 N1

(b) (i) Attempt to form composite $(f \circ g)(x) = f(\ln(1 + 2x))$ (M1)

$(f \circ g)(x) = e^{\ln(1+2x)} = (= 1 + 2x)$ A1 N2

(ii) Simplifying $y = e^{\ln(1+2x)}$ to $y = 1 + 2x$ (may be seen in part (i) or later) (A1)

Interchanging x and y (may happen any time) M1

eg $x = 1 + 2y$ $x - 1 = 2y$

$(f \circ g)^{-1}(x) = \frac{x-1}{2}$ A1 N2

[6]

13.

(a) $x^2 = 49$ (M1)
$x = \pm 7$ (A1)
$x = 7$ A1 N3

(b) $2^x = 8$ (M1)
$x = 3$ A1 N2

(c) $x = 25^{-\frac{1}{2}}$ (M1)

$x = \frac{1}{\sqrt{25}}$ (A1)

$x = \frac{1}{5}$ A1 N3

(d) $\log_2(x(x-7)) = 3$ (M1)
$\log_2(x^2 - 7x) = 3$
$2^3 = 8$ $(8 = x^2 - 7x)$ (A1)
$x^2 - 7x - 8 = 0$ A1
$(x - 8)(x + 1) = 0$ $(x = 8, x = -1)$ (A1)
$x = 8$ A1 N3

[13]

14.

(a)	attempt to apply rules of logarithms	(M1)		
	e.g. $\ln a^b = b\ln a$, $\ln ab = \ln a + \ln b$			
	correct application of $\ln a^b = b\ln a$ (seen anywhere)	A1		
	e.g. $3\ln x = \ln x^3$			
	correct application of $\ln ab = \ln a + \ln b$ (seen anywhere)	A1		
	e.g. $\ln 5x^3 = \ln 5 + \ln x^3$			
	so $\ln 5x^3 = \ln 5 + 3\ln x$			
	$g(x) = f(x) + \ln 5$ (accept $g(x) = 3\ln x + \ln 5$)	A1	N1	4
(b)	transformation with correct name, direction, and value	A3		
	e.g. translation by $\begin{pmatrix} 0 \\ \ln 5 \end{pmatrix}$, shift up by $\ln 5$, vertical translation of $\ln 5$			3

[7]

15.

(a)	(i)	$n = 5$	(A1)	
		$T = 280 \times 1.12^5$		
		$T = 493$	A1	N2
	(ii)	evidence of doubling *e.g.* 560	(A1)	
		setting up equation *e.g.* $280 \times 1.12^n = 560$, $1.12^n = 2$	A1	
		$n = 6.116...$	(A1)	
		in the year 2007	A1	N3
(b)	(i)	$P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(5)}}$	(A1)	
		$P = 39\,635.993...$	(A1)	
		$P = 39\,636$	A1	N3
	(ii)	$P = \dfrac{2\,560\,000}{10 + 90\,\mathrm{e}^{-0.1(7)}}$		
		$P = 46\,806.997...$	A1	
		not doubled	A1	N0
		valid reason for **their** answer *e.g.* $P < 51200$	R1	
(c)	(i)	correct value *e.g.* $\dfrac{25600}{280}, 91.4, 640:7$	A2	N2
	(ii)	setting up an inequality (accept an equation, or reversed inequality) *e.g.* $\dfrac{P}{T} < 70$, $\dfrac{2\,560\,000}{(10 + 90\mathrm{e}^{-0.1n})280 \times 1.12^n} < 70$	M1	
		finding the value 9.31....	(A1)	
		after 10 years	A1	N2

[17]