# Exponential equations

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We will learn how to solve basic exponential equations.

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We will deal with the equations of the form

$$a^{f(x)} = b^{g(x)}$$

where a, b > 0 and f, g are real-valued functions. In our example these will be polynomial functions and sometimes functions involving absolute value.

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• **step 1** Write both sides as a power of the same number.

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- step 2 Compare the exponents and solve.

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Example. Solve  $3^{2x-1} = 243$ 

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$$3^{2x-1} = 243$$
  
 $3^{2x-1} = 3^5$ 

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- step 2 Compare the exponents and solve.

Example. Solve  $3^{2x-1} = 243$ 

$$3^{2x-1} = 243$$
  
 $3^{2x-1} = 3^5$ 

Now we compare the exponents:

$$2x - 1 = 5$$
$$x = 3$$

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Solve

$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$

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$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$

We write both sides as powers of 2:

$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$
$$2^{-x-1} = 2^{2x+4}$$

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Solve

$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$

We write both sides as powers of 2:

$$\left(\frac{1}{2}\right)^{x+1} = 4^{x+2}$$
$$2^{-x-1} = 2^{2x+4}$$

Compare exponents:

$$-x - 1 = 2x + 4$$
$$x = -\frac{5}{3}$$

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Solve:

 $\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$ 

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Solve:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$

We write both sides as powers of 3:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$
$$3^{-2x+4} = 3^{\frac{x}{2}+3}$$

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Solve:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$

We write both sides as powers of 3:

$$\left(\frac{1}{9}\right)^{x-2} = (\sqrt{3})^{x+6}$$
$$3^{-2x+4} = 3^{\frac{x}{2}+3}$$

Compare exponents:

$$-2x + 4 = \frac{x}{2} + 3$$
$$x = \frac{2}{5}$$

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Solve

$$4\times 8^x = (2\sqrt{2})^{-x}$$

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Solution:

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Solve

$$4 \times 8^x = (2\sqrt{2})^{-x}$$

Solution:

$$4 \times 8^{x} = (2\sqrt{2})^{-x}$$
$$2^{2} \times 2^{3x} = 2^{-\frac{3}{2}x}$$
$$2^{3x+2} = 2^{-\frac{3}{2}x}$$
$$3x+2 = -\frac{3}{2}x$$
$$x = -\frac{4}{9}$$

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Solve

$$3 \times 81^{x-1} = (\sqrt[3]{3})^{-2x}$$

Solve

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Solution (try it on your own first):

Solve

$$3 \times 81^{x-1} = (\sqrt[3]{3})^{-2x}$$

Solution (try it on your own first):

$$3 \times 81^{x-1} = (\sqrt[3]{3})^{-2x}$$
$$3 \times 3^{4x-4} = 3^{-\frac{2x}{3}}$$
$$3^{4x-3} = 3^{-\frac{2x}{3}}$$
$$4x - 3 = -\frac{2x}{3}$$
$$x = \frac{9}{14}$$

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Solve:

$$4\times\left(\frac{1}{\sqrt{2}}\right)^x=\frac{1}{2}\times 16^{x-1}$$



Solve:

$$4\times\left(\frac{1}{\sqrt{2}}\right)^x=\frac{1}{2}\times 16^{x-1}$$

Solution:

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Solve:

$$4 \times \left(\frac{1}{\sqrt{2}}\right)^x = \frac{1}{2} \times 16^{x-1}$$

Solution:

$$4 \times \left(\frac{1}{\sqrt{2}}\right)^{x} = \frac{1}{2} \times 16^{x-1}$$
$$2^{2} \times 2^{-\frac{x}{2}} = 2^{-1} \times 2^{4x-4}$$
$$2^{2-\frac{x}{2}} = 2^{4x-5}$$
$$2 - \frac{x}{2} = 4x - 5$$
$$x = \frac{14}{9}$$

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Solve:

$$2^{|x+3|} = 1024$$

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Solve:

$$2^{|x+3|} = 1024$$

Solution:

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Solve:

$$2^{|x+3|} = 1024$$

Solution:

$$2^{|x+3|} = 1024$$
  

$$2^{|x+3|} = 2^{10}$$
  

$$|x+3| = 10$$
  

$$x+3 = -10 \quad \lor \quad x+3 = 10$$
  

$$x = -13 \quad \lor \quad x = 7$$

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Solve:

$$3^{|x-2|} = 9^x$$

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Solve:

$$3^{|x-2|} = 9^x$$

Solution:

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Solve:

$$3^{|x-2|} = 9^x$$

Solution:

$$3^{|x-2|} = 9^{x}$$
  
 $3^{|x-2|} = 3^{2x}$   
 $|x-2| = 2x$ 

Now we need to solve |x - 2| = 2x.

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Solve:

$$3^{|x-2|} = 9^x$$

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Solve:

$$3^{|x-2|} = 9^x$$

we need to solve |x - 2| = 2x.

Solve:

$$3^{|x-2|} = 9^x$$

we need to solve |x - 2| = 2x.

$$\begin{array}{c|c} x < 2 & x \ge 2 \\ -(x-2) = 2x & x-2 = 2x \\ x = \frac{2}{3} \\ \frac{2}{3} < 2 & -2 \ge 2 \end{array}$$

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Solve:

$$3^{|x-2|} = 9^x$$

we need to solve |x - 2| = 2x.

$$\begin{array}{c|c} x < 2 & x \ge 2 \\ -(x-2) = 2x & x-2 = 2x \\ x = \frac{2}{3} & x = -2 \\ \frac{2}{3} < 2 & -2 \ge 2 \end{array}$$

So the only solution is  $x = \frac{2}{3}$ .

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Solve:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

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Solve:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

Solution:

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Solve:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

Solution:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$
$$2^{x^2-1} = 2^{2x+2}$$
$$x^2 - 1 = 2x + 2$$
$$x^2 - 2x - 3 = 0$$
$$x - 3)(x + 1) = 0$$

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Solve:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$

Solution:

$$(\sqrt[3]{2})^{3x^2-3} = 4^{x+1}$$
$$2^{x^2-1} = 2^{2x+2}$$
$$x^2 - 1 = 2x + 2$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$

We get x = 3 or x = -1.

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In case of any question you can email me at T.J.Lechowski@gmail.com.

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