# Intro to logarithms

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So the expressions like $\log_1 3$, $\log_{-2} 5$ or $\log_4(-1)$ are not defined in real numbers (similarly to expressions like $\sqrt{-6}$).

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Secondly $\log_a b = c$ means $a$ raised to the power of $c$ is equal to $b$. So if we want to calculate $\log_a b$, we need to find a number to which we need to raise $a$ to to get $b$.

We will practice the above definition in this presentation.

# Example 1

Calculate $\log_{\frac{1}{3}} 81$.

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So $\log_{\frac{1}{3}} 81 = -4$.

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We can change all terms into powers of 4 (or 2)

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So $\log_{\frac{1}{4}} 16 = -2$.

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$$2^{\frac{3}{2}x} = 2^4$$

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so $x = \frac{8}{3}$.

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So we have $\log_{2\sqrt{2}} 16 = \frac{8}{3}$.

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Which gives:

$$5^x = 5^{3.5}$$

so $x = 3.5$.

So we have $\log_5 125\sqrt{5} = 3.5$.

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We need to find the power to which to raise $3\sqrt{3}$ to get $81\sqrt[3]{3}$. We need to solve the following exponential equation:

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Change into powers of 3:

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this gives $x = \frac{26}{9}$.

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So we have $\log_{3\sqrt{3}} 81\sqrt[3]{3} = \frac{26}{9}$.

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We need to find the power to which to raise $\frac{1}{4}$ to get $\frac{2\sqrt[5]{64}}{\sqrt{8}}$. We write the corresponding exponential equation:

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so:

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which gives $x = -\frac{7}{20}$.

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so:

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which gives $x = -\frac{7}{20}$.

So in the end $\log_{\frac{1}{4}} \frac{2\sqrt[5]{64}}{\sqrt{8}} = -\frac{7}{20}$.

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$$\log_2 x = -3$$

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So $x = \frac{1}{8}$.

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So $x = \frac{1}{9}$.

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So $x = 3$ ($x \neq -3$, as it has to be positive).

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So $x = 4$.

In case of any questions you can email me at t.j.lechowski@gmail.com.