Name:

1. (4 points) Given that $1 + 2i$ is a root of the polynomial
$$P(x) = 4x^4 - 24x^3 + 69x^2 - 114x + 85$$
find the other roots.

2. (6 points)

    (a) Show that:

$$\cos\left(\arcsin x + \arcsin\left(\frac{x}{2}\right)\right) = \frac{\sqrt{1-x^2}\sqrt{4-x^2}-x^2}{2}$$

    (b) **Hence** find the value of

$$\cos\left(\arcsin\left(\frac{3}{\sqrt{21}}\right) + \arcsin\left(\frac{3}{2\sqrt{21}}\right)\right)$$

    (c) **Hence** write down the value of

$$\arcsin\left(\frac{3}{\sqrt{21}}\right) + \arcsin\left(\frac{3}{2\sqrt{21}}\right)$$

3. (4 points) Solve:

$$3 + 3\cos x = 2\sin^2 x$$

for $0 \leqslant x \leqslant 4\pi$.

4. (6 points)

   (a) Show that $\dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x} = \dfrac{4}{\sin^2 2x}$.

   (b) Hence find the exact solutions to the equation

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{16}{3}$$

   for $-\pi < x < \pi$.