Name:

1. (4 points) Solve the inequality

$$|2x - 3| \leqslant |x - 1|$$

2. (4 points) Consider a function $f(x)$ with a domain D such that if $x \in D$, then $-x \in D$.

Let $g(x) = \dfrac{f(x) + f(-x)}{2}$

(a) Show that $g(x)$ is a an even function.

Let $h(x) = \dfrac{f(x) - f(-x)}{2}$

(b) Show that $h(x)$ is a an odd function.

Let $f(x) = \dfrac{1}{x+2}$ with the domain $D = \mathbb{R} - \{-2, 2\}$.

(c) Using parts (a) and (b) express $f(x)$ as a sum of an even and an odd function, simplify your answer.

3. (6 points) Consider a polynomial

$$P(x) = 4x^3 + 4x^2 - 3x - 3$$

(a) Show that -1 is a root of $P(x)$ and hence find all solutions to the equation $P(x) = 0$.

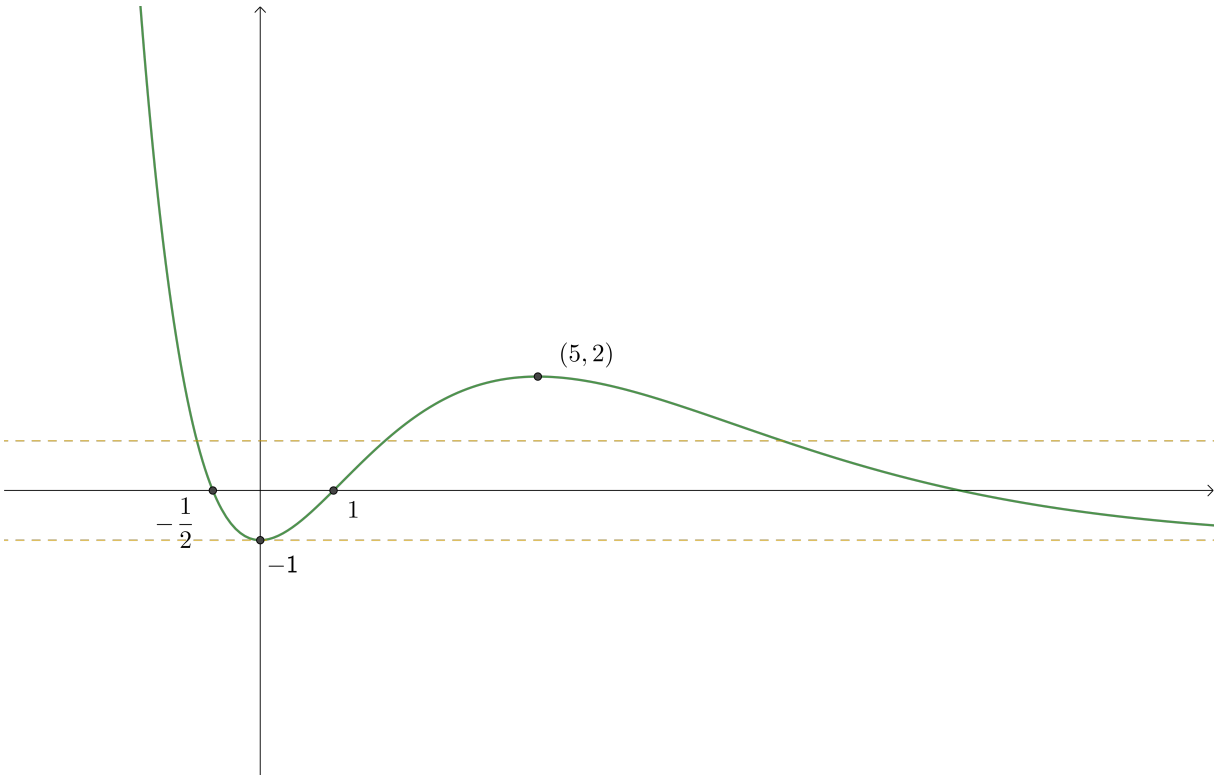
(b) Show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(c) Hence solve

$$4\sin^2\theta - 3 = \sin 3\theta$$

for $0 \leqslant \theta \leqslant 2\pi$.

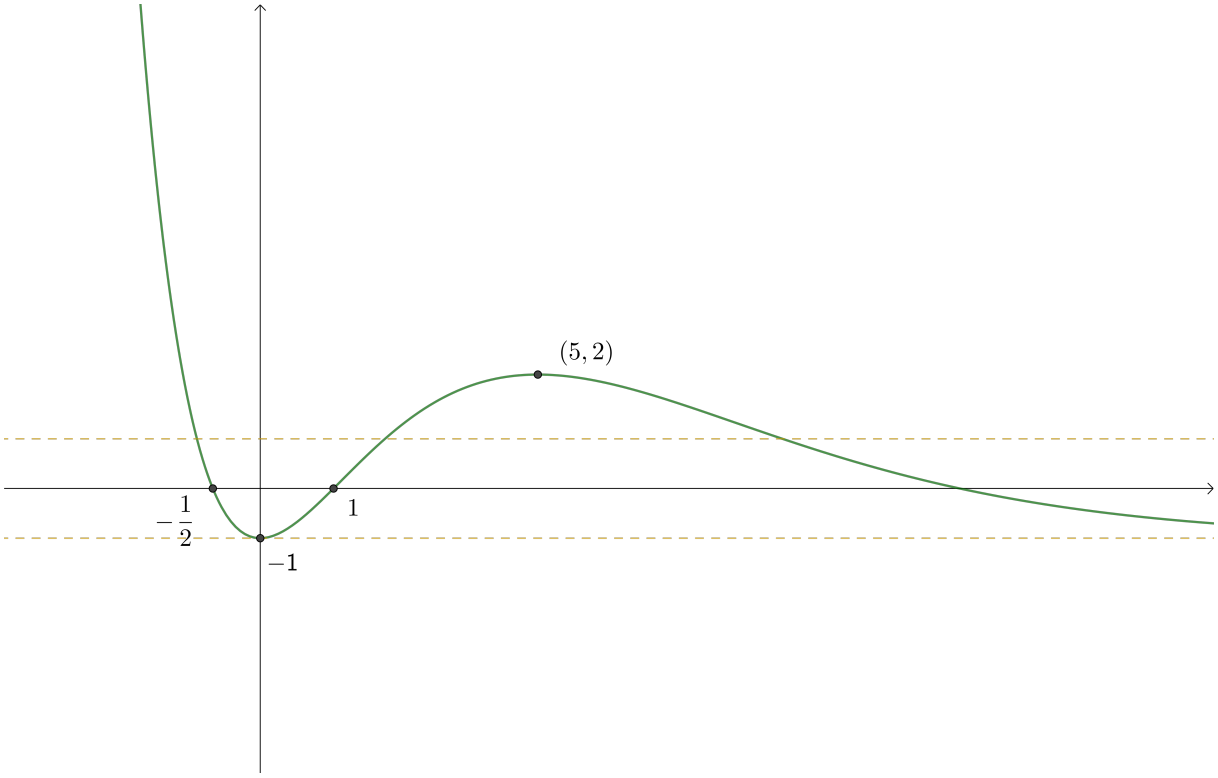
4. (6 points) Consider the following graph of $y = f(x)$

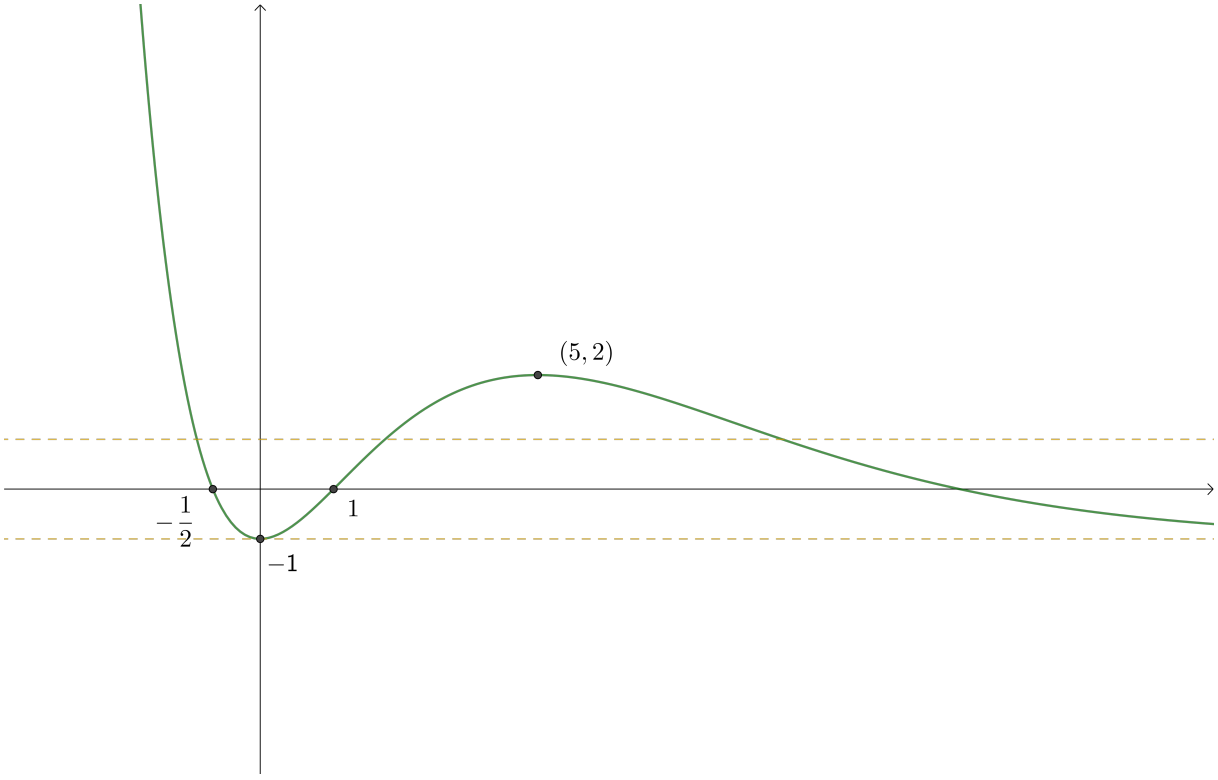


The dotted lines represent lines $y = 1$ and $y = -1$. The latter being the horizontal asymptote of the graph of $f(x)$. Use the diagrams on the next page to sketch the graphs of

(a) $g(x) = \dfrac{1}{f(\frac{1}{2}x)}$ (b) $h(x) = (f(|x|))^2$.

Clearly indicate axes intercepts, asymptotes and maxima and minima.


(5, 2)
$-\frac{1}{2}$
1
−1


(5, 2)
$-\frac{1}{2}$
1
−1