Trigonometric equations

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- values of trigonometric functions for standard angles ($0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$);

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- reduction formulae (eg. $\sin(\pi - x) = \sin x$ or $\sin(\frac{\pi}{2} - x) = \cos x$)

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- reduction formulae (eg. $\sin(\pi - x) = \sin x$ or $\sin(\frac{\pi}{2} - x) = \cos x$)
- trigonometric identities: Pythagorean identity, double angle identities, angle sum and difference identities, sum-to-product identities (the last one is not strictly speaking required by IB, but it will be required in my class as it often helps a lot).

Important note

This presentation is for your use only. Please do not share it publicly. In particular do not post it online anywhere.

Plan

We will cover the following topics:

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- basic trigonometric equations,

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- IB exam questions.

Basic trigonometric equations - example 1

We will start with the following equation:

$$\sin x = \frac{\sqrt{3}}{2}$$

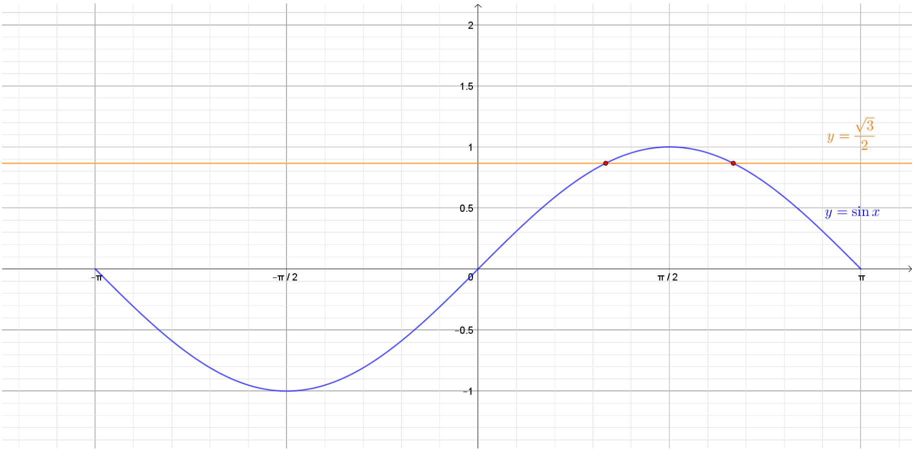
Basic trigonometric equations - example 1

We will start with the following equation:

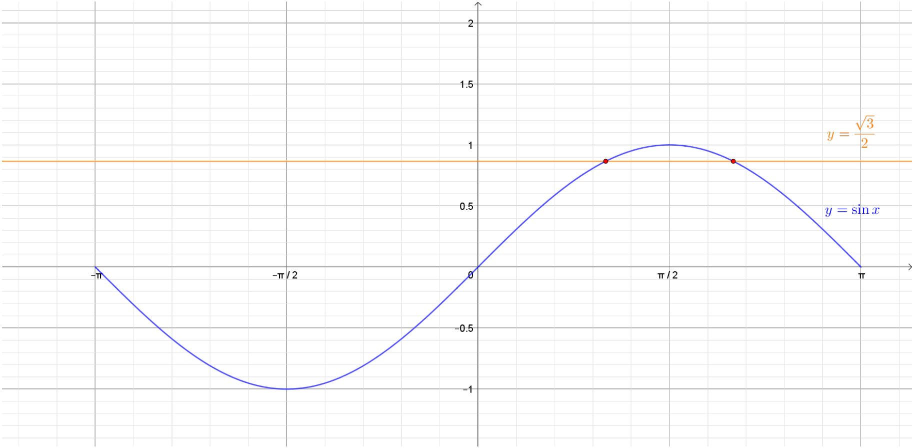
$$\sin x = \frac{\sqrt{3}}{2}$$

We want to draw one period of the sine function (eg. from $-\pi$ to π) and the line $y = \frac{\sqrt{3}}{2}$.

Basic trigonometric equations - example 1

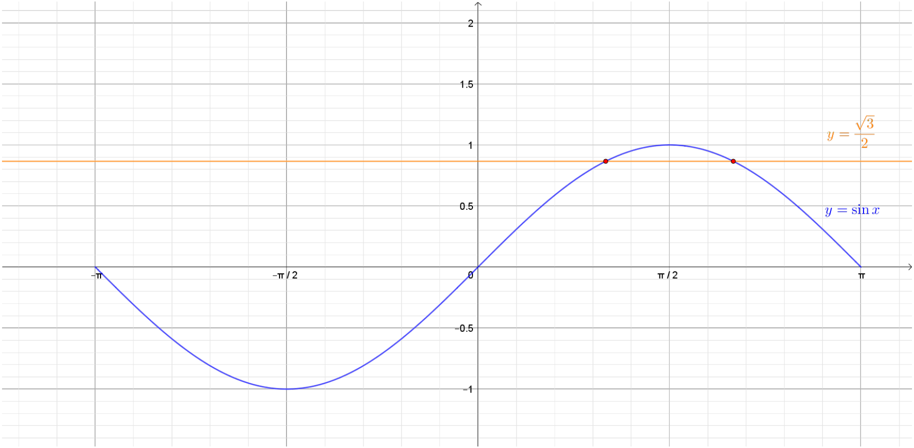


Basic trigonometric equations - example 1



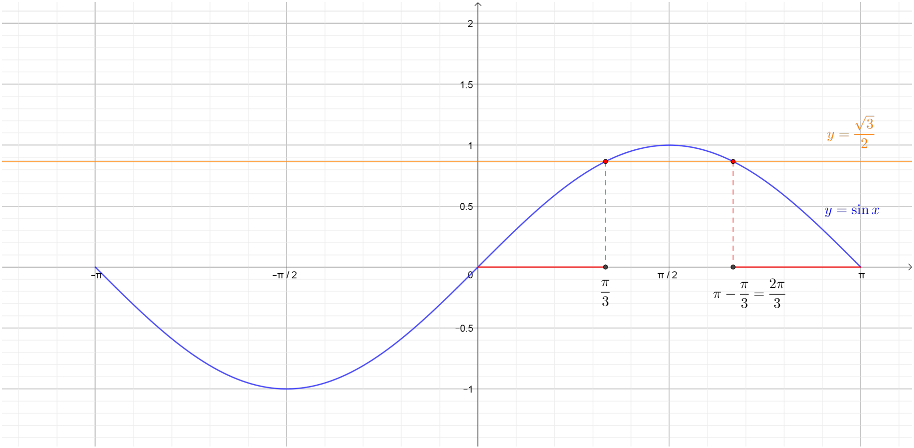
We can see two solutions (red points).

Basic trigonometric equations - example 1



We can see two solutions (red points). We should know one of those (from tables of values of standard angles), we can find the other one using symmetries of the graph.

Basic trigonometric equations - example 1



Our solutions are $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

$$x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$, so k is an integer.

Basic trigonometric equations - example 1

So the solutions to

$$\sin x = \frac{\sqrt{3}}{2}$$

are:

$$x = \frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$, so k is an integer.

Where does the $2k\pi$ come from?

So the solutions to

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are:

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where $k \in \mathbb{Z}$, so k is an integer.

Where does the $2k\pi$ come from? We only drew one period of *sine*, the values repeat themselves every 2π, so adding or subtracting any multiple of 2π to x will not change the value of the function.

Basic trigonometric equations - example 2

Now we want to solve:

$$\cos x = \frac{\sqrt{2}}{2}$$

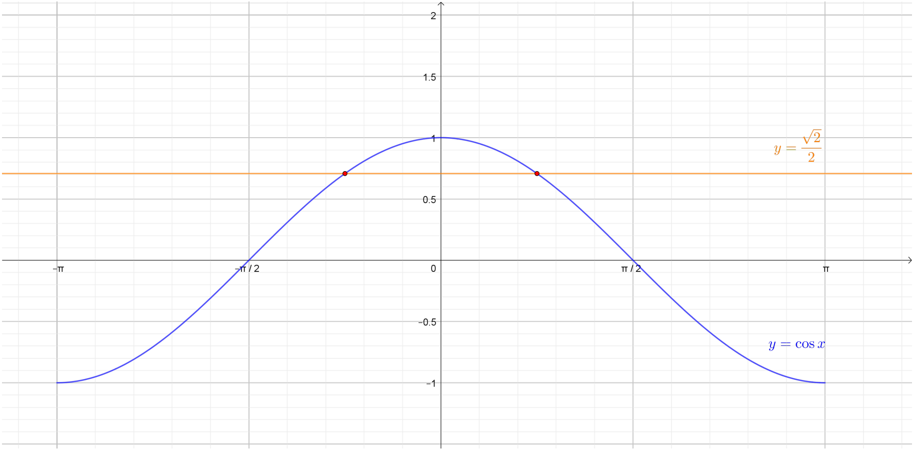
Basic trigonometric equations - example 2

Now we want to solve:

$$\cos x = \frac{\sqrt{2}}{2}$$

We draw one period of the cosine function (again it can be from $-\pi$ to π) and the line $y = \frac{\sqrt{2}}{2}$.

Basic trigonometric equations - example 2



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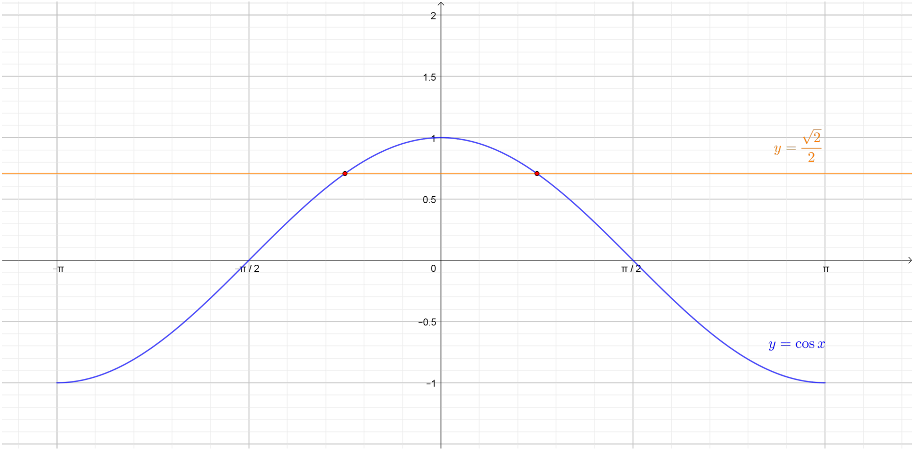

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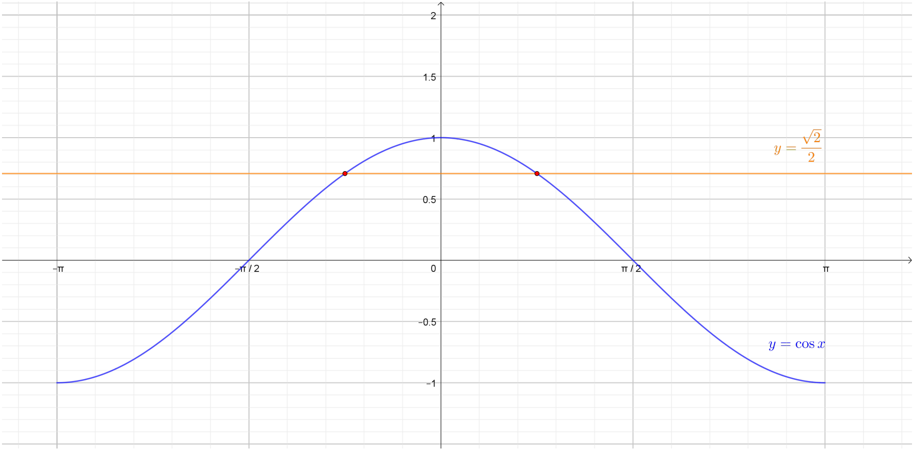

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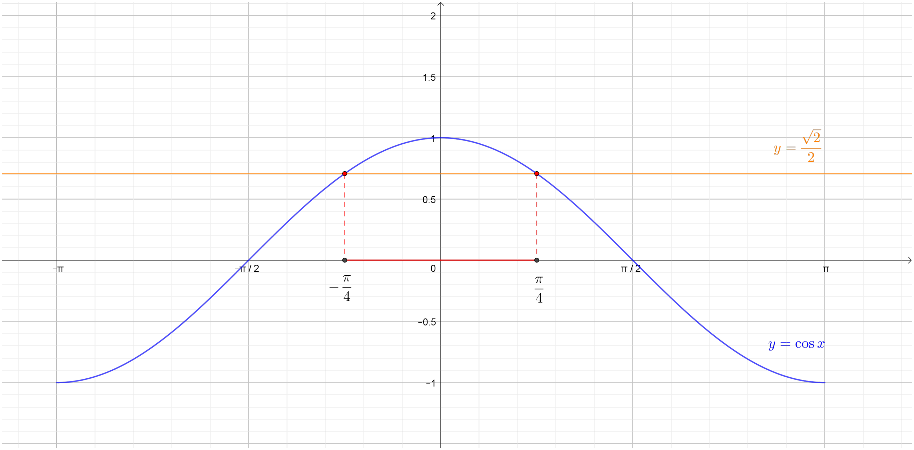
Basic trigonometric equations - example 2



One solution is $x = \frac{\pi}{4}$, the other is of course $x = -\frac{\pi}{4}$.

Basic trigonometric equations - example 2

Finally we get that the solutions to the equation

$$\cos x = \frac{\sqrt{2}}{2}$$

are:

Basic trigonometric equations - example 2

Finally we get that the solutions to the equation

$$\cos x = \frac{\sqrt{2}}{2}$$

are:

$$x = \frac{\pi}{4} + 2k\pi \qquad \text{or} \qquad x = -\frac{\pi}{4} + 2k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 3

Solve:

$$\tan x = \frac{\sqrt{3}}{3}$$

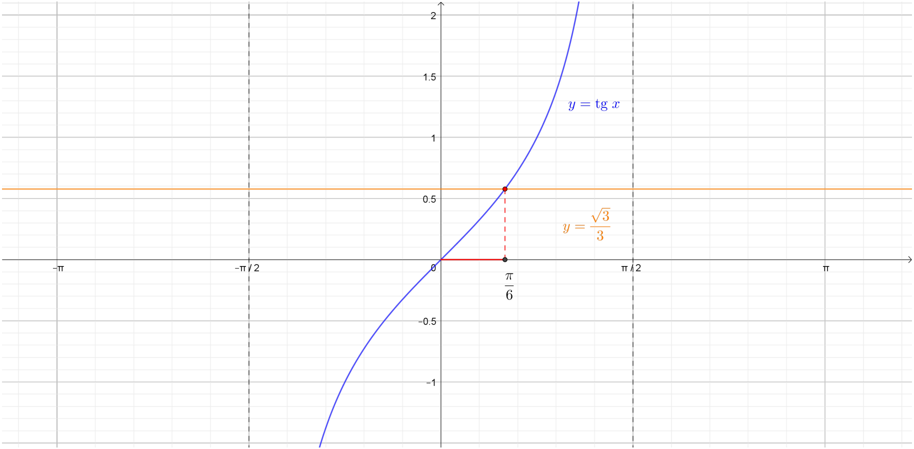
Basic trigonometric equations - example 3

Solve:

$$\tan x = \frac{\sqrt{3}}{3}$$

We draw one period of tangent function (remember that the period of $\tan x$ is π, it's best to draw it from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$) and the line $y = \frac{\sqrt{3}}{3}$.

Basic trigonometric equations - example 3



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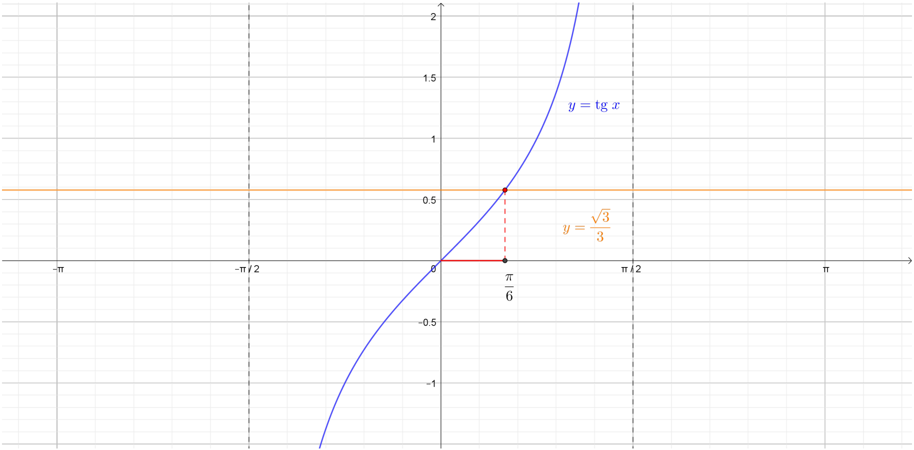
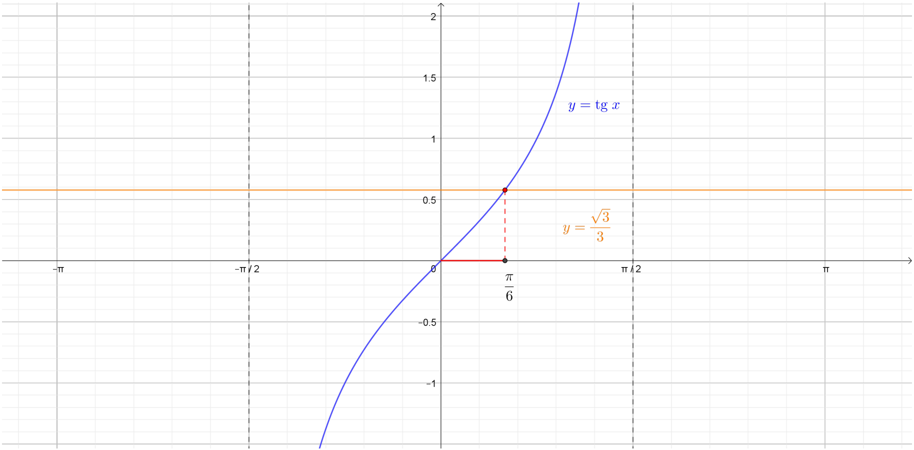


There's one solution (red point).

Basic trigonometric equations - example 3



There's one solution (red point). We know it from the table of standard angles, $x = \frac{\pi}{6}$.

Basic trigonometric equations - example 3

We get that the solutions to the equations

$$\tan x = \frac{\sqrt{3}}{3}$$

are:

Basic trigonometric equations - example 3

We get that the solutions to the equations

$$\tan x = \frac{\sqrt{3}}{3}$$

are:

$$x = \frac{\pi}{6} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 4

Solve:

$$\cot x = 1$$

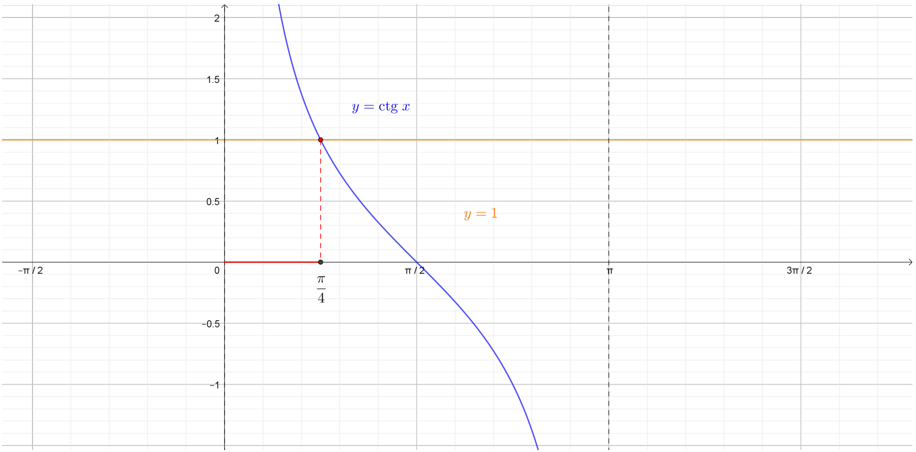
Basic trigonometric equations - example 4

Solve:

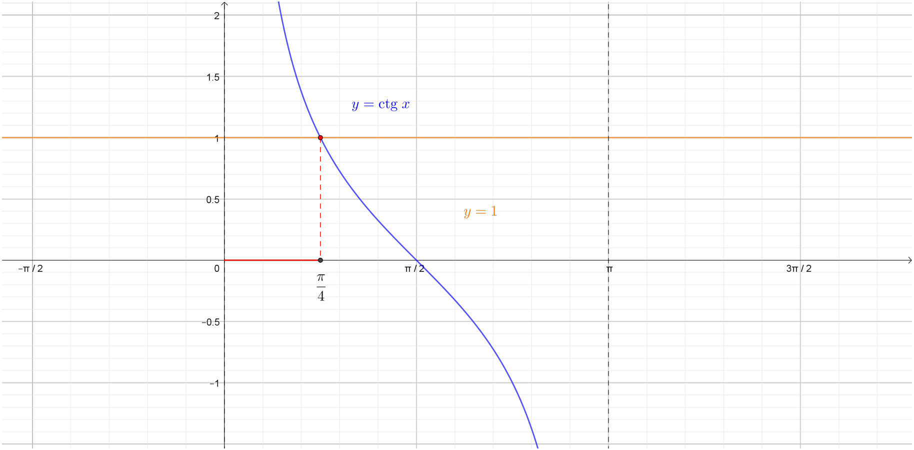
$$\cot x = 1$$

We draw one period of cotangent function (the period is π, we'll draw it between 0 and π) and the line $y = 1$.

Basic trigonometric equations - example 4

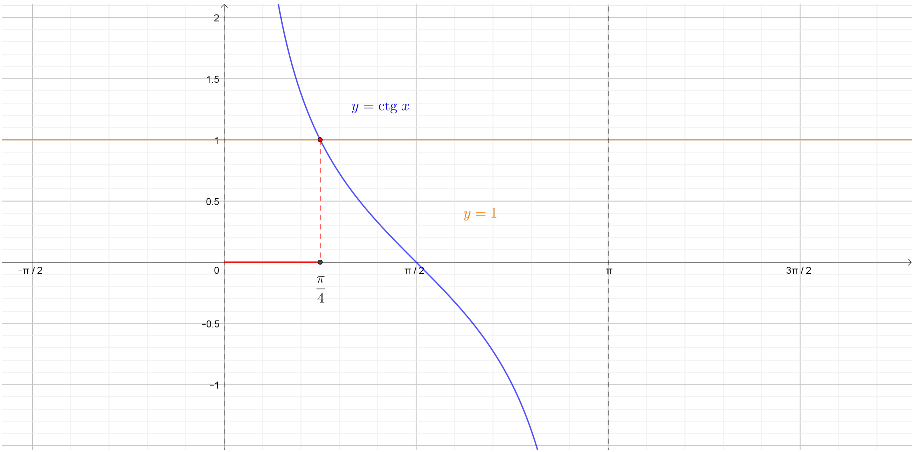


Basic trigonometric equations - example 4



We can see one solution (red point).

Basic trigonometric equations - example 4



We can see one solution (red point). It is $x = \frac{\pi}{4}$.

Basic trigonometric equations - example 4

Therefore the solutions to

$$\cot x = 1$$

are:

Basic trigonometric equations - example 4

Therefore the solutions to

$$\cot x = 1$$

are:

$$x = \frac{\pi}{4} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - exercises

Solve the following equations:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

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Basic trigonometric equations - exercises

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  Solution:

Solve the following equations:

- Equation:

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Solution:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

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- Equation:

$$\cos x = 0$$

Basic trigonometric equations - exercises

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- Equation:

$$\cos x = 0$$

Solution:

$$x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

Basic trigonometric equations - exercises

- Equation:

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Basic trigonometric equations - exercises

- Equation:

$$\tan x = \sqrt{3}$$

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$$x = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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$$\cot x = 0$$

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Basic trigonometric equations - example 5

Solve the equation:

$$\sin x = -1$$

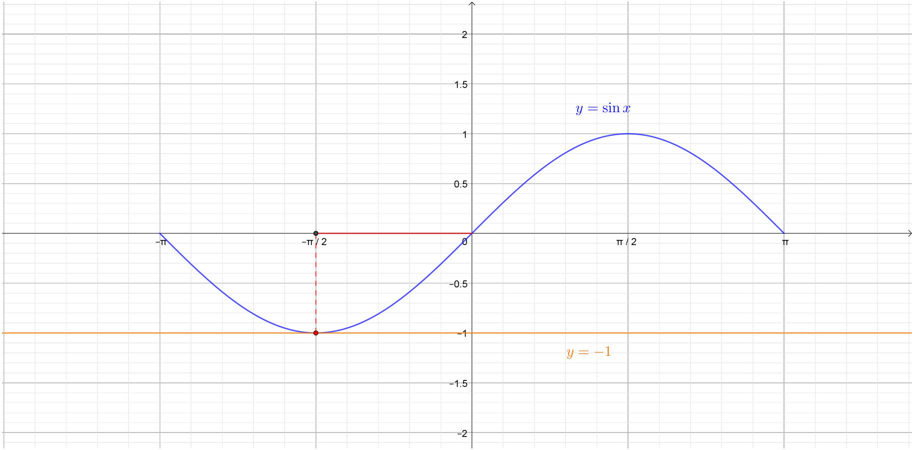
Basic trigonometric equations - example 5

Solve the equation:

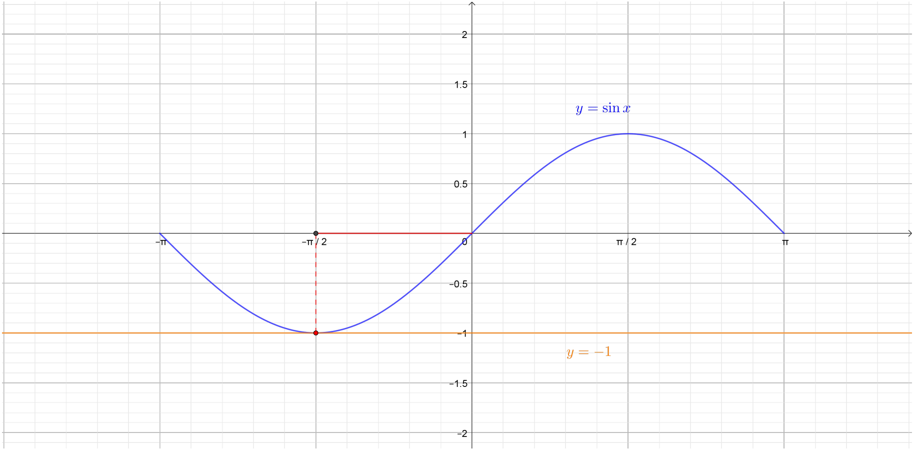
$$\sin x = -1$$

We draw one period of sine function and the line $y = -1$.

Basic trigonometric equations - example 5



Basic trigonometric equations - example 5



We can see one solution, it's of course $x = -\frac{\pi}{2}$.

Basic trigonometric equations - example 5

The solutions to

$$\sin x = -1$$

are:

Basic trigonometric equations - example 5

The solutions to

$$\sin x = -1$$

are:

$$x = -\frac{\pi}{2} + 2k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 6

Solve:

$$\cos x = -\frac{1}{2}$$

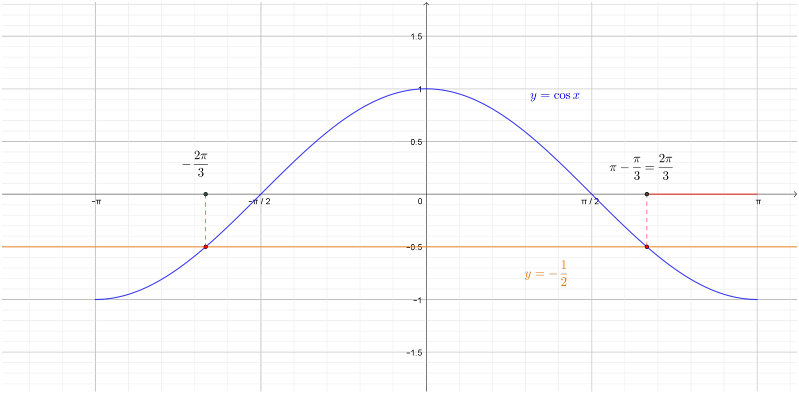
Basic trigonometric equations - example 6

Solve:

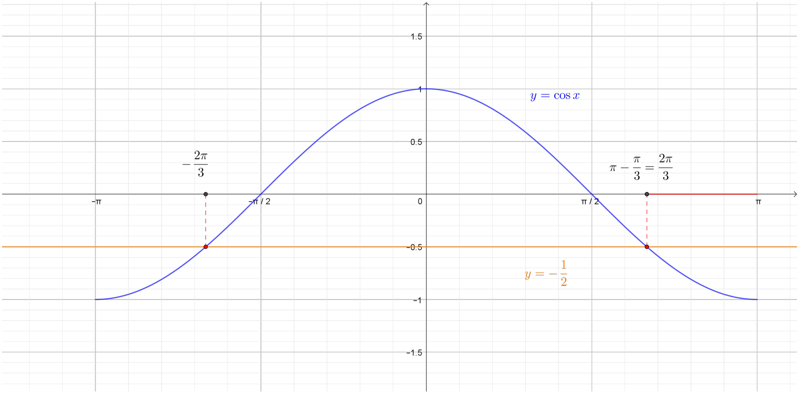
$$\cos x = -\frac{1}{2}$$

We draw one period of cosine function and the line $y = -\frac{1}{2}$.

Basic trigonometric equations - example 6



Basic trigonometric equations - example 6



We can see two solutions. If we were to solve $\cos x = \frac{1}{2}$, then we would know that $x = \frac{\pi}{3}$ is one of the solutions, here we can use the symmetry to get $x = \frac{2\pi}{3}$ as a solution and then we also get $x = -\frac{2\pi}{3}$.

Basic trigonometric equations - example 6

We get that the solutions to

$$\cos x = -\frac{1}{2}$$

are:

Basic trigonometric equations - example 6

We get that the solutions to

$$\cos x = -\frac{1}{2}$$

are:

$$x = -\frac{2\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 7

Solve:

$$\tan x = -1$$

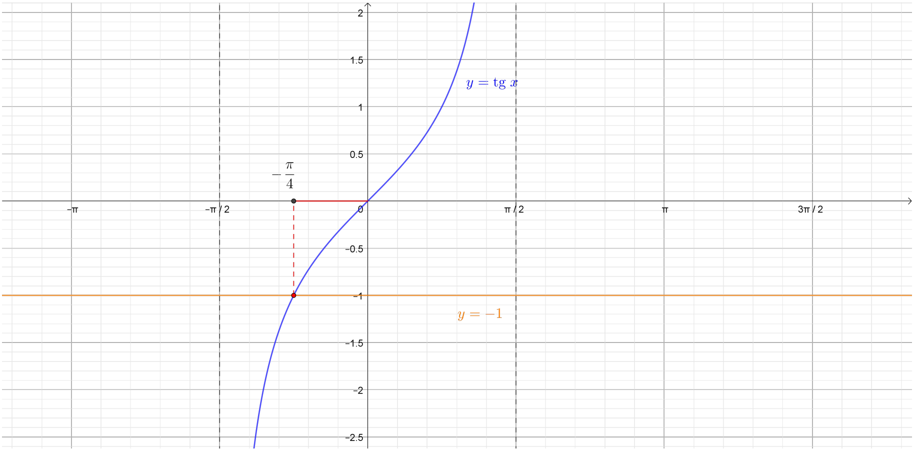
Basic trigonometric equations - example 7

Solve:

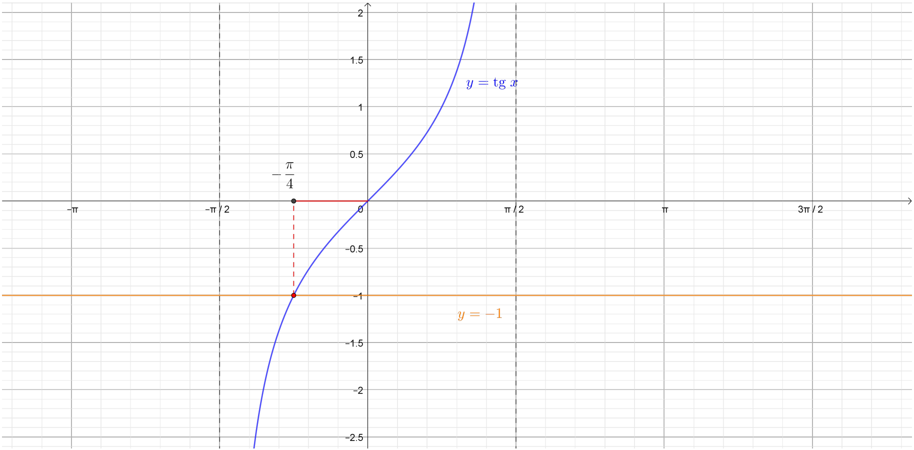
$$\tan x = -1$$

As always we draw one period of tangent function and the line $y = -1$.

Basic trigonometric equations - example 7



Basic trigonometric equations - example 7



There's one solution. If we were to solve $\tan x = 1$, the solution would be $x = \frac{\pi}{4}$, so here we of course have $x = -\frac{\pi}{4}$

Basic trigonometric equations - example 7

So the solutions to

$$\tan x = -1$$

are:

Basic trigonometric equations - example 7

So the solutions to

$$\tan x = -1$$

are:

$$x = -\frac{\pi}{4} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - example 8

Solve:

$$\cot x = -\sqrt{3}$$

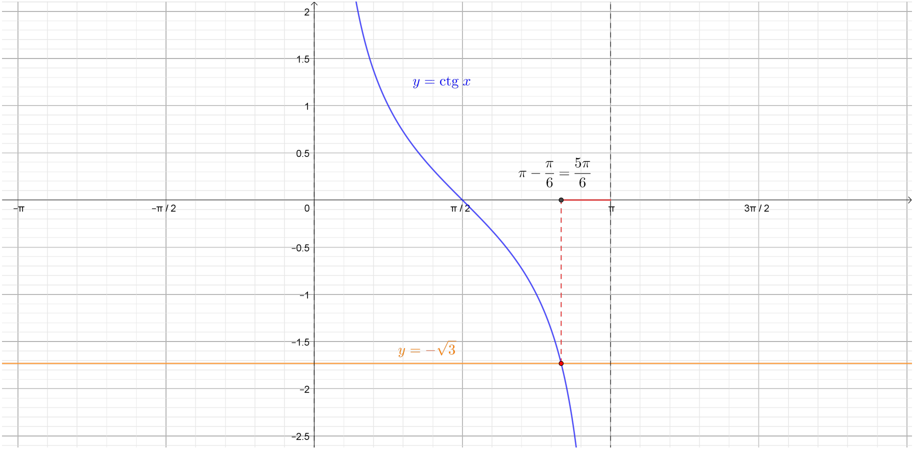
Basic trigonometric equations - example 8

Solve:

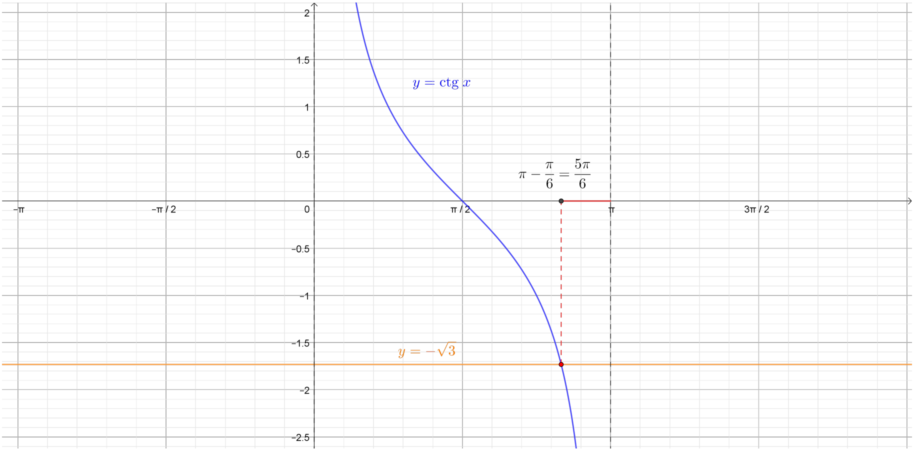
$$\cot x = -\sqrt{3}$$

We draw one period of cotangent function and the line $y = -\sqrt{3}$.

Basic trigonometric equations - example 8



Basic trigonometric equations - example 8



There's one solution. Solving $\cot x = \sqrt{3}$ would give us $x = \frac{\pi}{6}$, so here we have $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Basic trigonometric equations - example 8

So the solutions to

$$\cot x = -\sqrt{3}$$

are:

Basic trigonometric equations - example 8

So the solutions to

$$\cot x = -\sqrt{3}$$

are:

$$x = \frac{5\pi}{6} + k\pi$$

where $k \in \mathbb{Z}$.

Basic trigonometric equations - exercises

Solve the following equations:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

- Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

- Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

  Solutions:

Basic trigonometric equations - exercises

Solve the following equations:

- Equation:

$$\sin x = -\frac{\sqrt{3}}{2}$$

  Solutions:

$$x = -\frac{\pi}{3} + 2k\pi \qquad \text{or} \qquad x = -\frac{2\pi}{3} + 2k\pi \qquad \text{wheree } k \in \mathbb{Z}$$

- Equation:

$$\cos x = -\frac{\sqrt{2}}{2}$$

  Solutions:

$$x = -\frac{3\pi}{4} + 2k\pi \qquad \text{or} \qquad x = \frac{3\pi}{4} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

  Solutions:

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

  Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = -1$$

Basic trigonometric equations - exercises

- Equation:

$$\tan x = -\frac{\sqrt{3}}{3}$$

  Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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$$\cot x = -1$$

  Solutions:

Basic trigonometric equations - exercises

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$$\tan x = -\frac{\sqrt{3}}{3}$$

  Solutions:

$$x = -\frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

- Equation:

$$\cot x = -1$$

  Solutions:

$$x = \frac{3\pi}{4} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Basic trigonometric equations - example 9

In the examples above we found **all** solutions to a given equation. However in almost all IB trig equation questions you'll be required to find solutions that are in a specific interval.

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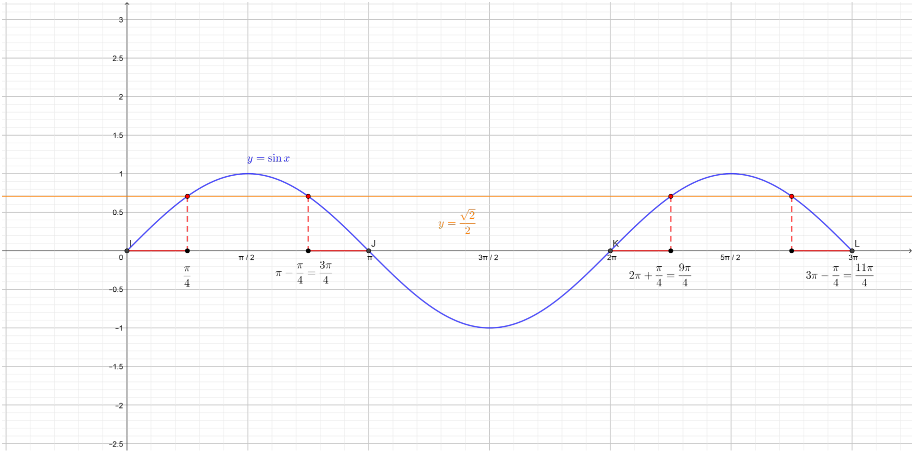
Solve

$$\sin x = \frac{\sqrt{2}}{2}$$

for $0 \leq x \leq 3\pi$.

This is even simpler. We draw $y = \frac{\sqrt{2}}{2}$ and $y = \sin x$, but only for $0 \leq x \leq 3\pi$.

Basic trigonometric equations - example 9



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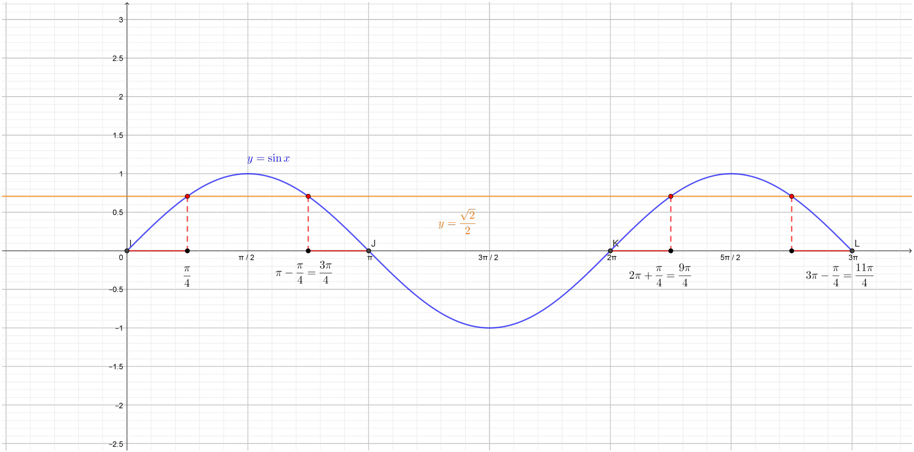
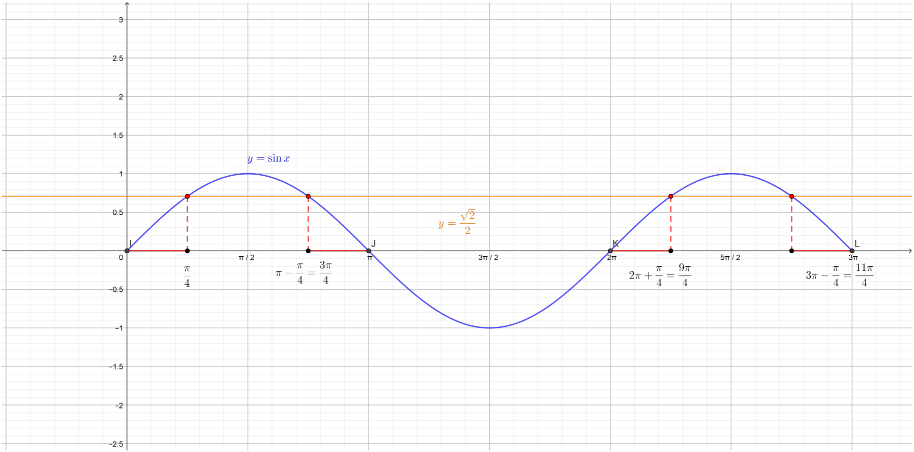


We have four solutions.

Basic trigonometric equations - example 9



We have four solutions. We should know one from the table and find the rest using symmetries and periodicity of the graph.

Basic trigonometric equations - example 9

The solutions to

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for $0 \leq x \leq 3\pi$ are

Basic trigonometric equations - example 9

The solutions to

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for $0 \leq x \leq 3\pi$ are $x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$ or $x = \frac{9\pi}{4}$ or $x = \frac{11\pi}{4}$.

Basic trigonometric equations - example 10

Solve

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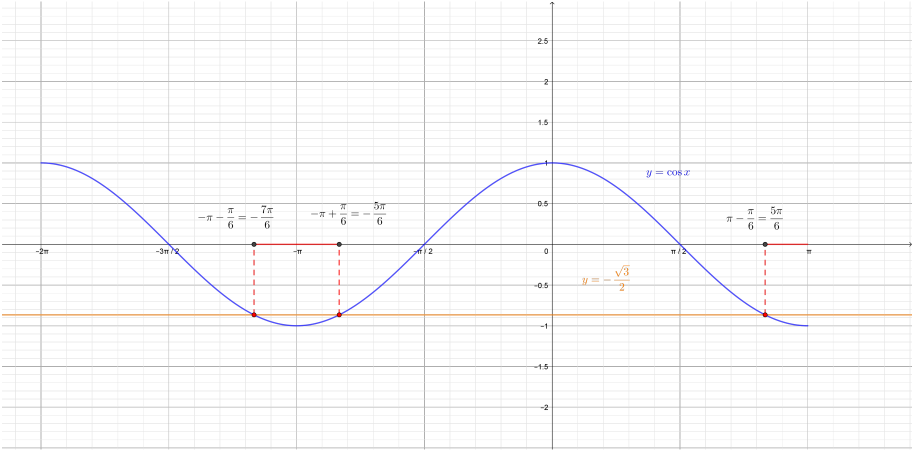
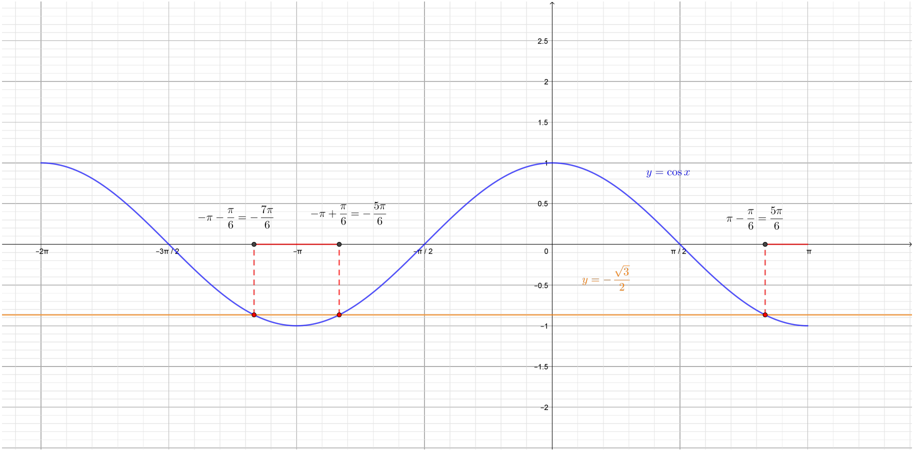
We draw $y = -\frac{\sqrt{3}}{2}$ and $y = \cos x$, but only for $-2\pi \leq x \leq \pi$.

Basic trigonometric equations - example 10

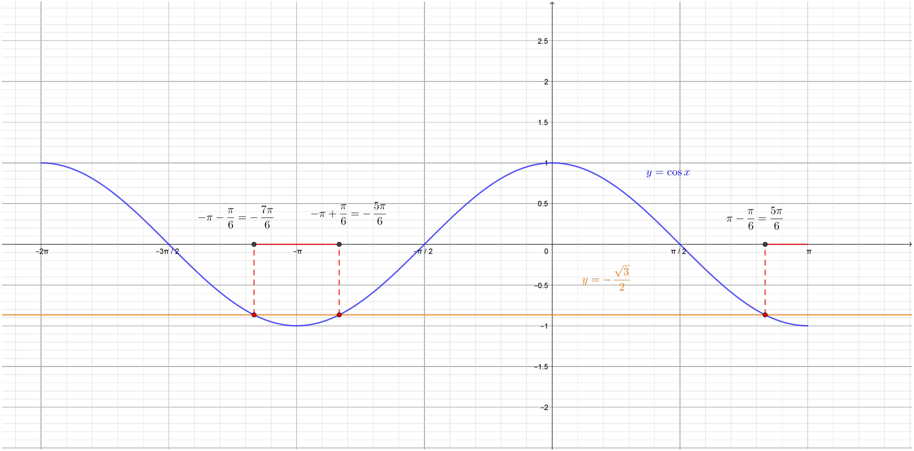


Basic trigonometric equations - example 10



We have 3 solutions.

Basic trigonometric equations - example 10



We have 3 solutions. If we were solving $\cos x = \frac{\sqrt{3}}{2}$, then we would have $x = \frac{\pi}{6}$ as a solution, based on that and symmetries we can find the actual solutions.

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The solutions to

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Basic trigonometric equations - exercises

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On the following slides I'll skip the step with drawing graphs, but you should still do it. It is a very useful habit. What it means is that when you get to a basic trig equation you should solve it as above - quick sketch and read of the solutions.

We move on to equations where some algebraic manipulation is required.

Variations of basic equations - example 1

Solve:

$$2\sin(3x) + 4 = 3$$

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Now we divide by 3, to get x:

$$x = -\frac{\pi}{18} + \frac{2k\pi}{3} \qquad \text{or} \qquad x = -\frac{5\pi}{18} + \frac{2k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

and this is our solution.

Note that when solving

$$\sin(3x) = -\frac{1}{2}$$

We don't need to draw $\sin(3x)$ (sine squeezed by a factor of $\frac{1}{3}$). It's better to draw $\sin\alpha$ (so the usual graph of sine), solve for α and then put $3x$ instead of α.

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We will get back to this in a few slides.

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rearrange to find x:

$$x = \frac{2\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and that's our solution.

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rearrange to find x:

$$x = -\frac{\pi}{15} + \frac{k\pi}{5} \qquad \text{or} \qquad x = \frac{\pi}{15} + \frac{k\pi}{5} \qquad \text{where } k \in \mathbb{Z}$$

and that's it.

Variations of basic equations - example 4

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we now solve two basic equations (instead of x we have $\frac{x}{2}$), we get:

$$\frac{x}{2} = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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we now solve two basic equations (instead of x we have $\frac{x}{2}$), we get:

$$\frac{x}{2} = -\frac{\pi}{3} + k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{3} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Multiply by 2 to get x:

$$x = -\frac{2\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

and that's our solution.

Variations of basic equations - example 5

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we solve two basic equations (instead of x we have $3x$), we get:

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divide by 3 to get x:

$$x = \frac{\pi}{6} + \frac{k\pi}{3} \qquad \text{or} \qquad x = \frac{2k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

and we have the solution.

Variations of basic equations - example 6

Solve:

$$|2\sin(7x) + 1| = 2$$

Variations of basic equations - example 6

Solve:

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divide by 7 to get x:

$$x=\frac{\pi}{42}+\frac{2k\pi}{7} \qquad \text{or} \qquad x=\frac{5\pi}{42}+\frac{2k\pi}{7} \qquad \text{where } k\in\mathbb{Z}$$

and that's our solution.

Variations of basic equations - exercises

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Solution:

$$x = \frac{\pi}{16} + \frac{k\pi}{4} \qquad \text{or} \qquad x = \frac{\pi}{8} + \frac{k\pi}{4} \qquad \text{where } k \in \mathbb{Z}$$

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so:

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Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$.

Variations of basic equations - example 7

$$x = \frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{12} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Now we need to choose values of k, so that our solutions will satisfy $0 \leq x \leq \pi$. After brief deliberation we get: $x = \frac{\pi}{12}$ or $x = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{\pi}{2} = \frac{5\pi}{12}$ or $x = -\frac{\pi}{12} + \frac{2\pi}{2} = \frac{11\pi}{12}$.

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Let's go back to the beginning:

$$2\cos 4x - 1 = 0$$

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but $0 \leq \alpha \leq 4\pi$. Remember to change the interval!

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We draw $\cos\alpha$ for $0 \leq \alpha \leq 4\pi$ and we find the solutions:

$$\alpha = \frac{\pi}{3} \quad \text{or} \quad \alpha = \frac{5\pi}{3} \quad \text{or} \quad \alpha = \frac{7\pi}{3} \quad \text{or} \quad \alpha = \frac{11\pi}{3}$$

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We go back to x, we have $\alpha = 4x$, so $x = \frac{\alpha}{4}$ and we get:

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Solve:

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$$\alpha \in \left\{-\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}\right\}$$

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We go back to x. Since $\alpha = \frac{x}{2}$, so we have $x = 2\alpha$, this gives the following solutions:

$$x \in \left\{-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}\right\}$$

Variations of basic equations - exercises

- Solve:

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$$x \in \left\{-\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}\right\}$$

Now we move on to equations which can be easily factored resulting in two or more basic equations.

Factoring - example 1

Solve:

$$2\sin^2 x + \sin x - 1 = 0$$

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We solve these basic equations to get:

$$x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi \quad \text{or} \quad x = -\frac{\pi}{2} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$

Factoring - example 2

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$$2\cos^2 x - 3\cos x - 2 = 0$$

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$$x = -\frac{2\pi}{3} + 2k\pi \qquad \text{or} \qquad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Factoring - example 3

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$$2\sin x\cos x - 2\sin x + \cos x - 1 = 0$$

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Now we solve and get:

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Factoring - example 4

Solve:

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Think about this solution. Make sure you understand where it came from.

Factoring - exercises

- Solve:

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- Solve:

$$\sin^3 x - 4\sin^2 x - \sin x + 4 = 0$$

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We increase the difficulty slightly.

We increase the difficulty slightly. We add the Pythagorean identity to our arsenal.

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

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Given an angle α, such that $\cos\alpha = \frac{1}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, calculate $\sin\alpha$ and $\cot\alpha$.

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Simple problem

Given an angle α, such that $\cos\alpha = \frac{1}{3}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, calculate $\sin\alpha$ and $\cot\alpha$.

We have $\sin x = -\frac{2\sqrt{2}}{3}$ and $\cot x = -\frac{\sqrt{2}}{4}$.

Pythagorean identity - introduction

The Pythagorean identity is probably the most famous trigonometric identity. For any angle x we have:

$$\sin^2 x + \cos^2 x = 1$$

We can use it to solve simple problems like:

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We have $\sin x = -\frac{2\sqrt{2}}{3}$ and $\cot x = -\frac{\sqrt{2}}{4}$. Refer to chapter 8E in Core HL if you forgot about these types of problems.

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Solving trig equations using Pythagorean identity boils down to simplifying the equation so that it can be solved using previous methods.

Pythagorean identity - introduction

Note that there are two very simple consequences of Pythagorean identity, namely:

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

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They of course can be derived by dividing the Pythagorean identity by $\cos^2 x$ and $\sin^2 x$ respectively.

Pythagorean identity - example 1

Solve:

$$5 \sin x - 2 \cos^2 x = 1$$

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We now solve and get:

$$x = \frac{\pi}{6} + 2k\pi \qquad \text{or} \qquad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Pythagorean identity - example 2

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Factorize:

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we now solve and get that:

$$x = -\frac{2\pi}{9} + \frac{2k\pi}{3} \quad \text{or} \quad x = \frac{2\pi}{9} + \frac{2k\pi}{3} \quad \text{or} \quad x = \frac{2k\pi}{3} \qquad \text{where } k \in \mathbb{Z}$$

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$$x = -\frac{\pi}{12} + k\pi \qquad \text{or} \qquad x = -\frac{5\pi}{12} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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It may however happen that it's not so simple and then the goal would be to make sure that we have the same angle everywhere.

Double angle formulae - introduction

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In case of *cosine* we in fact have 3 formulae and we use the one which suits us.

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In case of *cosine* we in fact have 3 formulae and we use the one which suits us.

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

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The angle on the left hand side has to be twice the angle on the right hand side.

Double angle - example 1

Solve:

$$\sin 2x + \sin x = 0$$

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We use double angle formula for sine ($\sin 2x = 2\sin x \cos x$) and get:

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We use double angle formula for sine ($\sin 2x = 2\sin x \cos x$) and get:

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We factor out $\sin x$ and we get:

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$$x = k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

Double angle - example 2

Solve

$$\cos(6x) - 3\cos(3x) + 1 = 0$$

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Factor out $\cos(3x)$:

$$\cos(3x)(2\cos(3x) - 3) = 0$$

solve to get:

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$$3x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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solve to get:

$$3x = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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Solve:

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We use double angle formula ($\cos 4x = 1 - 2\sin^2 2x$), we get:

$$-2\sin^2 2x + 4\sin 2x + 6 = 0$$

Divide by -2 and factorize:

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Divide by -2 and factorize:

$$(\sin 2x + 1)(\sin 2x - 3) = 0$$

solve and get (the second equation has no solutions):

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Divide by -2 and factorize:

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$$2x = \frac{3\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

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$$\cos 4x + 2\sin 2x\cos 2x + 1 = 0$$

  Solution:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{or} \qquad x = -\frac{\pi}{8} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - introduction

We have the following formulae for sine and cosine of sum and difference of angles:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

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They can be used to calculate for example $\sin(\frac{7\pi}{12})$ or $\cos 15^\circ$:

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They can be used to calculate for example $\sin(\frac{7\pi}{12})$ or $\cos 15^\circ$:

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) =$$
$$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

Formula for *sine/cosine* of sum/difference of angles - introduction

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ =$$
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We get the same result. This, of course, is no accident, we have $\frac{7\pi}{12} = 105^\circ$, so
$\sin 105^\circ = \sin(180 - 75^\circ) = \sin 75^\circ = \cos(90^\circ - 75^\circ) = \cos 15^\circ$.

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When solving equations we will in most cases use the formulae in the opposite direction.

Formula for *sine/cosine* of sum/difference of angles - example 1

Solve

$$\sin x + \sqrt{3}\cos x = \sqrt{2}$$

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We have a sum so it's appropriate to change it to cosine of a difference or sine of a sum. We will do the later. We want to change 1 into cosine and $\sqrt{3}$ into a sine. By drawing an appropriate triangle we can see that the hypotenuse is 2 (so we need to divide both sides by 2) and the required angle is $\alpha = \frac{\pi}{3}$:

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

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$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

So we get:

$$\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Now we can apply the formula for the sine of the sum of angles to get:

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

Note that we could have tried to use the formula for cosine of a difference of the angles. In which case we would need to change 1 into sine and $\sqrt{3}$ into cosine. The hypotenuse is still 2, but the angle is $\alpha = \frac{\pi}{6}$, so we would get:

Formula for *sine/cosine* of sum/difference of angles - example 1

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$$\sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \frac{\sqrt{2}}{2}$$

Applying the formula for the cosine of a difference we get:

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

Going back, we have:

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Formula for *sine/cosine* of sum/difference of angles - example 1

Going back, we have:

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This is simple now, we have:

$$x+\frac{\pi}{3}=\frac{\pi}{4}+2k\pi \qquad \text{lub} \qquad x+\frac{\pi}{3}=\frac{3\pi}{4}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

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so:

$$x=-\frac{\pi}{12}+2k\pi \qquad \text{lub} \qquad x=\frac{5\pi}{12}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

If we used the formula for cosine of a difference we would end up with:

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

If we used the formula for cosine of a difference we would end up with:

$$\cos\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

This gives:

$$x - \frac{\pi}{6} = -\frac{\pi}{4} + 2k\pi \qquad \text{lub} \qquad x - \frac{\pi}{6} = \frac{\pi}{4} + 2k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 1

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and the final answer is of course the same.

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

We can use sine of a difference here. We draw an appropriate triangle, the hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{4}$. So we divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = 1$$

so:

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

$$\sin x - \cos x = \sqrt{2}$$

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$$\frac{1}{\sqrt{2}} \sin x - \frac{1}{\sqrt{2}} \cos x = 1$$

so:

$$\cos \frac{\pi}{4} \sin x - \sin \frac{\pi}{4} \cos x = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 2

Solve:

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We can use sine of a difference here. We draw an appropriate triangle, the hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{4}$. So we divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = 1$$

so:

$$\cos\frac{\pi}{4}\sin x - \sin\frac{\pi}{4}\cos x = 1$$

Now apply the formula for sine of a difference:

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 2

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 2

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

this gives:

$$x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 2

$$\sin\left(x - \frac{\pi}{4}\right) = 1$$

this gives:

$$x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

So finally we get:

$$x = \frac{3\pi}{4} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

so:

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

so:

$$\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x = \frac{1}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

Solve:

$$\sqrt{3}\sin x + \cos x = 1$$

We can apply the formula for sine of a sum. We draw a triangle with adjacent side $\sqrt{3}$ and opposite side 1. The hypotenuse is 2 and the angle is $\alpha = \frac{\pi}{6}$. So we divide both sides by 2:

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

so:

$$\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x = \frac{1}{2}$$

Applying the formula we get:

$$\sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 3

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this gives:

$$x+\frac{\pi}{6}=\frac{\pi}{6}+2k\pi \qquad \text{lub} \qquad x+\frac{\pi}{6}=\frac{5\pi}{6}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

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so finally we get:

$$x=2k\pi \qquad \text{lub} \qquad x=\frac{2\pi}{3}+2k\pi \qquad \text{gdzie } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference.

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{6}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

so:

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{6}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

so:

$$\cos\frac{\pi}{4}\sin 3x + \sin\frac{\pi}{4}\cos 3x = -\frac{\sqrt{3}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

Solve:

$$\sin 3x + \cos 3x = -\frac{\sqrt{6}}{2}$$

The fact that instead of x we have $3x$ makes no difference. We draw a triangle with both adjacent and opposite sides being 1. The hypotenuse is $\sqrt{2}$ and the angle is $\alpha = \frac{\pi}{6}$. We divide both side by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}\sin 3x + \frac{1}{\sqrt{2}}\cos 3x = -\frac{\sqrt{12}}{4}$$

so:

$$\cos\frac{\pi}{4}\sin 3x + \sin\frac{\pi}{4}\cos 3x = -\frac{\sqrt{3}}{2}$$

Using formula for sine of a sum we get:

$$\sin\left(3x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

$$\sin\left(3x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

$$\sin\left(3x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$$

we get

$$3x+\frac{\pi}{4}=-\frac{\pi}{3}+2k\pi \qquad \text{or} \qquad 3x+\frac{\pi}{4}=-\frac{2\pi}{3}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - example 4

$$\sin\left(3x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$$

we get

$$3x+\frac{\pi}{4}=-\frac{\pi}{3}+2k\pi \qquad \text{or} \qquad 3x+\frac{\pi}{4}=-\frac{2\pi}{3}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

So in the end we get:

$$x=-\frac{7\pi}{36}+\frac{2k\pi}{3} \qquad \text{or} \qquad x=-\frac{11\pi}{36}+\frac{2k\pi}{3} \qquad \text{where } k\in\mathbb{Z}$$

Formula for *sine/cosine* of sum/difference of angles - exercise

- Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Formula for *sine/cosine* of sum/difference of angles - exercise

- Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

Formula for *sine/cosine* of sum/difference of angles - exercise

- Solve:

$$\sin 2x - \sqrt{3}\cos 2x = 1$$

Solution:

$$x = \frac{\pi}{4} + k\pi \qquad \text{or} \qquad x = \frac{7\pi}{12} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Now we move to the final set of examples, where we apply formulae for sums and differences of sines and cosines.

Formulae for sum and difference of *sine/cosine* - introduction

These are not required by IB, but nevertheless useful. They're **not** included in the formula booklet so you should learn them by hard (in fact it's best to learn the ones in the formula booklet by hard as well).

Formulae for sum and difference of *sine/cosine* - introduction

These are not required by IB, but nevertheless useful. They're **not** included in the formula booklet so you should learn them by hard (in fact it's best to learn the ones in the formula booklet by hard as well).

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha=\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2\sin\frac{3x}{2}\cos\frac{x}{2} = 0$$

so we get:

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

$$2 \sin \frac{3x}{2} \cos \frac{x}{2} = 0$$

so we get:

$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 1

Solve:

$$\sin x + \sin 2x = 0$$

We of course use the formula for the sum of the sine:

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$$\frac{3x}{2} = k\pi \qquad \text{or} \qquad \frac{x}{2} = \frac{\pi}{2} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

and finally:

$$x = \frac{2k\pi}{3} \qquad \text{or} \qquad x = \pi + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

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We've solve the above equation earlier using $\sin 2x = 2\sin x\cos x$.

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We've solve the above equation earlier using $\sin 2x = 2\sin x\cos x$. Compare the answers.

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We of course use the formula for the sum of the sine:

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We've solve the above equation earlier using $\sin 2x = 2\sin x\cos x$. Compare the answers. At first glance you may think that we got different solutions, but if you study it carefully you will see that they are indeed the same.

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$.

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why?

Formulae for sum and difference of *sine/cosine* - example 2

Solve:

$$\cos x + \cos 2x + \cos 3x = 0$$

We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x\cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

$$2\cos 2x\cos(-x) + \cos 2x = 0$$

factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

Formulae for sum and difference of *sine/cosine* - example 2

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factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

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factor out $\cos 2x$ (and change $\cos(-x)$ to $\cos x$):

$$\cos 2x(2\cos x + 1) = 0$$

This gives:

$$2x = \frac{\pi}{2} + k\pi \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 2

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In the end:

Formulae for sum and difference of *sine/cosine* - example 2

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We use the formula for the sum of cosine to $\cos x + \cos 3x$. Why? Because we get $2\cos 2x \cos(-x)$ i $\cos 2x$ and we will be able to factorize the expression:

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In the end:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{2\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 3

Solve:

$$\sin 2x - \cos 3x = 0$$

Formulae for sum and difference of *sine/cosine* - example 3

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$$\sin 2x - \cos 3x = 0$$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

Formulae for sum and difference of *sine/cosine* - example 3

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$$\sin 2x - \cos 3x = 0$$

This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

$$\sin 2x - \sin(\frac{\pi}{2} - 3x) = 0$$

Formulae for sum and difference of *sine/cosine* - example 3

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This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

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Now apply the formula for difference of sines:

Formulae for sum and difference of *sine/cosine* - example 3

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This may seem problematic at first as there is no obvious formula that applies here, but we can simply change cosine into sine using the formula that changes a function into co-function:

$$\sin 2x - \sin(\frac{\pi}{2} - 3x) = 0$$

Now apply the formula for difference of sines:

$$2\sin\left(\frac{2x - (\frac{\pi}{2} - 3x)}{2}\right)\cos\left(\frac{2x + (\frac{\pi}{2} - 3x)}{2}\right) = 0$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-\left(\frac{\pi}{2}-3x\right)}{2}\right)\cos\left(\frac{2x+\left(\frac{\pi}{2}-3x\right)}{2}\right)=0$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-\left(\frac{\pi}{2}-3x\right)}{2}\right)\cos\left(\frac{2x+\left(\frac{\pi}{2}-3x\right)}{2}\right)=0$$

Simplify to get:

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-\left(\frac{\pi}{2}-3x\right)}{2}\right)\cos\left(\frac{2x+\left(\frac{\pi}{2}-3x\right)}{2}\right)=0$$

Simplify to get:

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$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

This gives:

$$\frac{5x}{2}-\frac{\pi}{4}=k\pi \qquad \text{or} \qquad -\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \qquad \text{where } k\in\mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - example 3

$$2\sin\left(\frac{2x-(\frac{\pi}{2}-3x)}{2}\right)\cos\left(\frac{2x+(\frac{\pi}{2}-3x)}{2}\right)=0$$

Simplify to get:

$$2\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)\cos\left(-\frac{x}{2}+\frac{\pi}{4}\right)=0$$

This gives:

$$\frac{5x}{2}-\frac{\pi}{4}=k\pi \qquad \text{or} \qquad -\frac{x}{2}+\frac{\pi}{4}=\frac{\pi}{2}+k\pi \qquad \text{where } k\in\mathbb{Z}$$

And finally:

$$x=\frac{\pi}{10}+\frac{2k\pi}{5} \qquad \text{or} \qquad x=-\frac{\pi}{2}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

Formulae for sum and difference of *sine/cosine* - exercises

- Equation:

$$\sin x = \sin 3x$$

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Solution:

$$x = \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced examples

On the next slides we will look at some advanced examples, where we need to make some important observations.

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Make sure you think about these example before looking at the solutions. There may be multiple ways to solve those.

Advanced problems - example 1

Solve:

$$\sin^4 x + \cos^4 x = \cos 2x$$

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$$1 - 2\sin^2 x \cos^2 x = \cos 2x$$

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Now we have two options. We can try to make all angles equal $2x$ or make them all equal x.

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Advanced problems - example 1

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We try to make all angles equal to $2x$. Recall that $\sin 2x = 2\sin x \cos x$, so $\sin^2 2x = 4\sin^2 x \cos^2 x$. It makes sense to multiply our equation by 2 to get:

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This is equivalent to:

$$(\cos 2x - 1)^2 = 0$$

Advanced problems - example 1

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$$2\sin^2 x(1 - \cos^2 x) = 0$$

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so $\sin x = 0$ or $\cos x = \pm 1$, both of these give:

$$x = k\pi \qquad \text{gdzie } k \in \mathbb{Z}$$

Advanced problems - example 2

Solve:

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We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first.

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We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

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We have 3 different angles: $x, 2x$ i $3x$. Let's get rid of $3x$ first. We can write $\sin 3x$ as $\sin(x + 2x)$ and we get:

$$\sin x \cos 2x + \cos x \sin 2x + \cos 2x = 1 + 2\sin x \cos 2x$$

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Moving all terms to one side:

$$\cos x \sin 2x - \sin x \cos 2x + \cos 2x - 1 = 0$$

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The first two term give us a formula for $\sin(2x - x)$, so $\sin x$.

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Advanced problems - example 2

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which gives us the following solutions:

$$x = k\pi \quad \text{or} \quad x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 3

Solve:

$$\sin^3 x + \cos^3 x = 1$$

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$$\sin^3 x - \sin^2 x + \cos^3 x - \cos^2 x = 0$$

We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

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We factor out $\sin^2 x$ and $\cos^2 x$:

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Now an important observation. $\sin^2 x \geq 0$, but $\sin x - 1 \leq 0$, because *sine* cannot be greater than 1.

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Now an important observation. $\sin^2 x \geq 0$, but $\sin x - 1 \leq 0$, because *sine* cannot be greater than 1. Similarly $\cos^2 x \geq 0$ and $\cos x - 1 \leq 0$.
So $\sin^2 x(\sin x - 1) \leq 0$ oraz $\cos^2 x(\cos x - 1) \leq 0$

Advanced problems - example 3

$$\sin^2 x(\sin x - 1) + \cos^2 x(\cos x - 1) = 0$$

Both terms are non-positive, but their sum is 0, so they both must be 0.

$$\sin^2 x(\sin x - 1) = 0 \qquad \text{and} \qquad \cos^2 x(\cos x - 1) = 0$$

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Solving this gives::

$$x = 2k\pi \qquad \text{or} \qquad x = \frac{\pi}{2} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 4

Solve:

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Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2\sin 2x \sin(-x) = 2\sin(-x)\cos 2x$$

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Solve:

$$\cos x - \cos 3x = \sin x - \sin 3x$$

Seems obvious that we want to apply the formula for sum of sines and cosines:

$$-2 \sin 2x \sin(-x) = 2 \sin(-x) \cos 2x$$

Sine is an odd function, so $\sin(-x) = -\sin x$, using this and moving all terms to one side:

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Factoring out $2\sin x$:

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Factoring out $2\sin x$:

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Advanced problems - example 4

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So $\sin x = 0$ or $\sin 2x = -\cos 2x$.

Advanced problems - example 4

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So $\sin x = 0$ or $\sin 2x = -\cos 2x$. The second equation can be turned into $\tan 2x = -1$ (by dividing both sides by $\cos x$).

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So $\sin x = 0$ or $\sin 2x = -\cos 2x$. The second equation can be turned into $\tan 2x = -1$ (by dividing both sides by $\cos x$). We solve and get:

$$x = k\pi \qquad \text{or} \qquad 2x = -\frac{\pi}{4} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

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So finally we have:

$$x = k\pi \qquad \text{or} \qquad x = -\frac{\pi}{8} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Advanced problems - example 5

Solve:

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Move all terms to one side:

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

Advanced problems - example 5

Solve:

$$\sin^2 x + \sin^2 2x = \sin^2 3x$$

Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

We can use difference of squares (with the hope that we can get $\sin 2x$ to factor out):

$$(\sin x - \sin 3x)(\sin x + \sin 3x) + \sin^2 2x = 0$$

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Move all terms to one side:

$$\sin^2 x - \sin^2 3x + \sin^2 2x = 0$$

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$$(\sin x - \sin 3x)(\sin x + \sin 3x) + \sin^2 2x = 0$$

We use the formula for sum and difference of sines:

$$2\sin(-x)\cos 2x \cdot 2\sin 2x \cos x + \sin^2 2x = 0$$

Advanced problems - example 5

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$$2\sin(-x)\cos 2x \cdot 2\sin 2x\cos x + \sin^2 2x = 0$$

We have that $\sin(-x) = -\sin x$ and we get:

$$-4\sin x\cos x\cos 2x\sin 2x + \sin^2 2x = 0$$

Now we have the expression $\sin x\cos x$ which should remind us of the formula $\sin 2x = 2\sin x\cos x$, we use it to get:

Advanced problems - example 5

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We have that $\sin(-x) = -\sin x$ and we get:

$$-4\sin x\cos x\cos 2x\sin 2x + \sin^2 2x = 0$$

Now we have the expression $\sin x\cos x$ which should remind us of the formula $\sin 2x = 2\sin x\cos x$, we use it to get:

$$-2\cos 2x\sin^2 2x + \sin^2 2x = 0$$

Now it's a breeze, we factor out $\sin^2 2x$:

$$\sin^2 2x(1 - 2\cos 2x) = 0$$

Advanced problems - example 5

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Advanced problems - example 5

$$\sin^2 2x(1-2\cos 2x)=0$$

We get:

$$2x=k\pi \quad \text{or} \quad 2x=-\frac{\pi}{3}+2k\pi \quad \text{or} \quad 2x=\frac{\pi}{3}+2k\pi \qquad \text{where } k\in\mathbb{Z}$$

Advanced problems - example 5

$$\sin^2 2x(1 - 2\cos 2x) = 0$$

We get:

$$2x = k\pi \quad \text{or} \quad 2x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad 2x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

So the final answer is:

$$x = \frac{k\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{6} + k\pi \quad \text{or} \quad x = \frac{\pi}{6} + k\pi \qquad \text{where } k \in \mathbb{Z}$$

Advanced problems - example 6

Solve:

$$\cot 8x \cot 10x = -1$$

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Advanced problems - example 6

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Multiply by the denominator (which we know is non-zero) and move to one side to get:

Advanced problems - example 6

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This looks like a formula for a cosine of a difference.

Advanced problems - example 6

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So:

$$x = \frac{\pi}{4} + \frac{k\pi}{2} \qquad \text{where } k \in \mathbb{Z}$$

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But beware, all of these solutions are outside of our domain, so in the end our equation has no solutions.

Advanced problems - example 6

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$$x \in \emptyset$$

The next slides include problems that appeared on a Polish Matura (advanced level).

Polish Matura - problem 1

May 2015,

Polish Matura - problem 1

May 2015, simple multiple choice question to begin with:

Zadanie 4. (0–1)

Równanie $2\sin x + 3\cos x = 6$ w przedziale $(0, 2\pi)$

A. nie ma rozwiązań rzeczywistych.

B. ma dokładnie jedno rozwiązanie rzeczywiste.

C. ma dokładnie dwa rozwiązania rzeczywiste.

D. ma więcej niż dwa rozwiązania rzeczywiste.

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This is a very important question, because it shows that it's always important to think about the equation before solving it. $\sin x$ is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course

Polish Matura - problem 1

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This is a very important question, because it shows that it's always important to think about the equation before solving it. $\sin x$ is less than or equal to 1, similarly $\cos x$, so the left hand side is certainly not greater than 5 (note that the maximum value of the left hand side is of course (find it) $\sqrt{13}$), so there'll be no solutions - answer A.

Polish Matura - problem 2

May 2017,

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$.

Polish Matura - problem 2

May 2017,

Zadanie 10. (0–4)

Rozwiąż równanie $\cos 2x + 3\cos x = -2$ w przedziale $\langle 0, 2\pi \rangle$.

We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Polish Matura - problem 2

May 2017,

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Factorize:

Polish Matura - problem 2

May 2017,

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We change $\cos 2x$ into $2\cos^2 x - 1$. Move all terms to one side to get:

$$2\cos^2 x + 3\cos x + 1 = 0$$

Factorize:

$$(2\cos x + 1)(\cos x + 1) = 0$$

Polish Matura - problem 2

May 2017,

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Factorize:

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Now we sketch the graph of cosine for $0 \leq x \leq 2\pi$.

Polish Matura - problem 2

May 2017,

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Factorize:

$$(2\cos x + 1)(\cos x + 1) = 0$$

Now we sketch the graph of cosine for $0 \leq x \leq 2\pi$. We get:

$$x = \frac{2\pi}{3} \qquad \text{or} \qquad x = \pi \qquad \text{or} \qquad x = \frac{4\pi}{3}$$

Polish Matura - problem 3

May 2018,

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)

Rozwiąż równanie $\sin 6x + \cos 3x = 2\sin 3x + 1$ w przedziale $\langle 0, \pi \rangle$.

We change $\sin 6x$ into $2\sin 3x \cos 3x$:

$$2\sin 3x \cos 3x + \cos 3x = 2\sin 3x + 1$$

Polish Matura - problem 3

May 2018,

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Factor out $\cos 3x$ and move all terms to one side:

Polish Matura - problem 3

May 2018,

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$$\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$$

Polish Matura - problem 3

May 2018,

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Factor out $\cos 3x$ and move all terms to one side:

$$\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$$

Now we can factor out $(2\sin 3x + 1)$:

Polish Matura - problem 3

May 2018,

Zadanie 11. (0–4)
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We change $\sin 6x$ into $2\sin 3x\cos 3x$:

$$2\sin 3x\cos 3x + \cos 3x = 2\sin 3x + 1$$

Factor out $\cos 3x$ and move all terms to one side:

$$\cos 3x(2\sin 3x + 1) - (2\sin 3x + 1) = 0$$

Now we can factor out $(2\sin 3x + 1)$:

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Polish Matura - problem 3

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Polish Matura - problem 3

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Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$.

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$.

Polish Matura - problem 3

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Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$. We now get the following solutions:

Polish Matura - problem 3

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$$\alpha = \frac{7\pi}{6} \quad \text{or} \quad \alpha = \frac{11\pi}{6} \quad \text{or} \quad \alpha = 0 \quad \text{or} \quad \alpha = 2\pi$$

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Since $\alpha = 3x$, then $x = \frac{\alpha}{3}$, so:

Polish Matura - problem 3

$$(2\sin 3x + 1)(\cos 3x - 1) = 0$$

Now it's fairly simple, beware though that we have $3x$ and the domain is $0 \leq x \leq \pi$. We subsitute $\alpha = 3x$ and then we have $0 \leq \alpha \leq 3\pi$. We now get the following solutions:

$$\alpha = \frac{7\pi}{6} \quad \text{or} \quad \alpha = \frac{11\pi}{6} \quad \text{or} \quad \alpha = 0 \quad \text{or} \quad \alpha = 2\pi$$

Since $\alpha = 3x$, then $x = \frac{\alpha}{3}$, so:

$$x = \frac{7\pi}{18} \quad \text{or} \quad x = \frac{11\pi}{18} \quad \text{or} \quad x = 0 \quad \text{or} \quad x = \frac{2\pi}{3}$$

Polish Matura - problem 4

May 2019,

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

A. $-\frac{\sqrt{3}}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{1}{2}$ **D.** $\frac{\sqrt{3}}{2}$

If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get:

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)

Liczba $\cos^2 105° - \sin^2 105°$ jest równa

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If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get::

$$\cos^2 105° - \sin^2 105° = \cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$$

Polish Matura - problem 4

May 2019,

Zadanie 2. (0–1)
Liczba $\cos^2 105° - \sin^2 105°$ jest równa

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If we remember the formulae, then we should immediately notice $\cos 2x = \cos^2 x - \sin^2 x$, so we get::

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Answer A.

Polish Matura - problem 5

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Polish Matura - problem 5

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Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

Rozwiąż równanie $(\cos x)\left[\sin\left(x-\frac{\pi}{3}\right)+\sin\left(x+\frac{\pi}{3}\right)\right]=\frac{1}{2}\sin x$.

Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

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$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

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Polish Matura - problem 5

May 2019,

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Now it becomes very simple. Of course we have $\cos\frac{\pi}{3} = \frac{1}{2}$.

Polish Matura - problem 5

May 2019,

Zadanie 14. (0–4)

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Now it becomes very simple. Of course we have $\cos\frac{\pi}{3} = \frac{1}{2}$. We move all terms to one side and we get:

Polish Matura - problem 5

May 2019,

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Looks complicated, but the first step is obvious - we add the sines using appropriate formula. We get:

$$\cos x \cdot 2\sin x \cos\frac{\pi}{3} = \frac{1}{2}\sin x$$

Now it becomes very simple. Of course we have $\cos\frac{\pi}{3}=\frac{1}{2}$. We move all terms to one side and we get:

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2} \sin x = 0$$

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

We factor out $\sin x$:

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

We factor out $\sin x$:

$$\sin x(\cos x - \frac{1}{2}) = 0$$

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2} \sin x = 0$$

We factor out $\sin x$:

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This gives:

Polish Matura - problem 5

$$\sin x \cos x - \frac{1}{2}\sin x = 0$$

We factor out $\sin x$:

$$\sin x(\cos x - \frac{1}{2}) = 0$$

This gives:

$$x = k\pi \quad \text{or} \quad x = -\frac{\pi}{3} + 2k\pi \quad \text{or} \quad x = \frac{\pi}{3} + 2k\pi \qquad \text{where } k \in \mathbb{Z}$$

Now we move on to IB exam questions.

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual.

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

$$\tan x + \frac{2\tan x}{1 - \tan^2 x} = 0$$

IB exam - problem 1

[Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \leq x < 360^\circ$.

Note that the interval is in degrees - this is unusual. It's quite obvious that we will use double angle formula for tangent:

$$\tan x + \frac{2\tan x}{1 - \tan^2 x} = 0$$

Now it makes sense to multiply both sides by $1 - \tan^2 x$ to get:

IB exam - problem 1

[Maximum mark: 6]

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Now it makes sense to multiply both sides by $1-\tan^2 x$ to get:

$$\tan x - \tan^3 x + 2\tan x = 0$$

IB exam - problem 1

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So $\tan x = 0$ or $\tan x = \pm\sqrt{3}$. We should draw the graph of $\tan x$ for $0^\circ \leq x < 360^\circ$, so that we don't miss any solutions. In the end we get:

$$x \in \{0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ\}$$

IB exam - problem 2

[Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

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We will start by using double angle formulae for sine and cosine. For cosine it makes sense to use $\cos 2x = 2\cos^2 x - 1$, because this will allow us to cancel 1 on both sides.

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$$2\sin x \cos x - 2\cos^2 x + 1 = 1 + \sin x - \cos x$$

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$$2\sin x\cos x - 2\cos^2 x + 1 = 1 + \sin x - \cos x$$

Moving all terms to one side we get:

$$2\sin x\cos x - 2\cos^2 x - \sin x + \cos x = 0$$

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$$x \in \left\{-\frac{3\pi}{4}, -\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{3}\right\}$$

IB exam - problem 3

[Maximum mark: 8]

Consider the equation $\dfrac{\sqrt{3}-1}{\sin x}+\dfrac{\sqrt{3}+1}{\cos x}=4\sqrt{2},\ 0<x<\dfrac{\pi}{2}$. Given that $\sin\left(\dfrac{\pi}{12}\right)=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos\left(\dfrac{\pi}{12}\right)=\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(a) verify that $x=\dfrac{\pi}{12}$ is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0<x<\dfrac{\pi}{2}$. [5]

IB exam - problem 3

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(a) verify that $x=\dfrac{\pi}{12}$ is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0<x<\dfrac{\pi}{2}$. [5]

The first part is easy, since we're given all the information.

IB exam - problem 3 (a)

We will start with the left hand side:

$$
\begin{aligned}
LHS &= \frac{\sqrt{3}-1}{\sin\frac{\pi}{12}} + \frac{\sqrt{3}+1}{\cos\frac{\pi}{12}} = \\
&= \frac{4(\sqrt{3}-1)}{\sqrt{6}-\sqrt{2}} + \frac{4(\sqrt{3}+1)}{\sqrt{6}+\sqrt{2}} = \\
&= \frac{4(\sqrt{3}-1)}{\sqrt{2}(\sqrt{3}-1)} + \frac{4(\sqrt{3}+1)}{\sqrt{2}(\sqrt{3}+1)} = \\
&= \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \\
&= \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2} = RHS
\end{aligned}
$$

so $x = \dfrac{\pi}{12}$ is a solution.

IB exam - problem 3 (b)

We can multiply both sides by $\sin x \cos x$ to get rid of denominators. We get

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$$(\sqrt{3}-1)\cos x + (\sqrt{3}+1)\sin x = 4\sqrt{2}\sin x \cos x$$

Now we can try to use the formula for sine of sums on the left hand side. The opposite side is $\sqrt{3}-1$, the adjacent side is $\sqrt{3}+1$. The hypotenuse is $2\sqrt{2}$.

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$$\frac{\sqrt{3}-1}{2\sqrt{2}}\cos x + \frac{\sqrt{3}+1}{2\sqrt{2}}\sin x = 2\sin x \cos x$$

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$$\frac{\sqrt{3}-1}{2\sqrt{2}}\cos x + \frac{\sqrt{3}+1}{2\sqrt{2}}\sin x = 2\sin x \cos x$$

This is happens to be perfect for the right hand side as well.

IB exam - problem 3 (b)

We get:

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$$\sin\frac{\pi}{12}\cos x+\cos\frac{\pi}{12}\sin x=\sin 2x$$

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Apply the formula for the sine of sum:

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Apply the formula for the sine of sum:

$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

IB exam - problem 3 (b)

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Now there are two ways to proceed.

IB exam - problem 3 (b)

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Now there are two ways to proceed. We can move all terms to one side and use the formula for difference of sines:

$$\sin\left(\frac{\pi}{12} + x\right) - \sin 2x = 0$$

IB exam - problem 3 (b)

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Now there are two ways to proceed. We can move all terms to one side and use the formula for difference of sines:

$$\sin\left(\frac{\pi}{12} + x\right) - \sin 2x = 0$$

So:

$$2\sin\left(\frac{\frac{\pi}{12} - x}{2}\right)\cos\left(\frac{\frac{\pi}{12} + 3x}{2}\right) = 0$$

IB exam - problem 3 (b)

$$2\sin\left(\frac{\frac{\pi}{12}-x}{2}\right)\cos\left(\frac{\frac{\pi}{12}+3x}{2}\right)=0$$

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$$\frac{\frac{\pi}{12}-x}{2}=0$$

which gives $x=\dfrac{\pi}{12}$ an answer we already had.

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which gives $x=\dfrac{\pi}{12}$ an answer we already had. Solving the second part (cosine) gives:

$$\frac{\frac{\pi}{12}+3x}{2}=\frac{\pi}{2}$$

So $x=\dfrac{11\pi}{36}$ and this is our second solution.

IB exam - problem 3 (b)

Let's go back to:

$$\sin\left(\frac{\pi}{12}+x\right)=\sin 2x$$

and let's discuss another approach.

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$$\sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

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We have sine function on both sides. Of course if the arguments are the same then the values will also be the same, so we can have:

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$$\frac{\pi}{12}+x=2x$$

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and this gives $x = \dfrac{\pi}{12}$.

But two sines are also equal if one argument is π minus the other ($\sin\alpha = \sin(\pi - \alpha)$), so we could also have:

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But two sines are also equal if one argument is π minus the other ($\sin\alpha=\sin(\pi-\alpha)$), so we could also have:

$$\frac{\pi}{12}+x=\pi-2x$$

and this gives the solution $x=\dfrac{11\pi}{36}$.

IB exam - problem 3 (b)

Note that in general if we have:

$$\sin\alpha = \sin\beta$$

Then:

$$\alpha = \beta \quad \text{or} \quad \alpha = \pi - \beta \quad \text{or} \quad \alpha = 2\pi + \beta \quad \text{or} \quad \alpha = 3\pi - \beta \quad \text{or} \quad \ldots$$

IB exam - problem 3 (b)

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If we have:

$$\cos \alpha = \cos \beta$$

Then

$$\alpha = \beta \quad \text{or} \quad \alpha = 2\pi - \beta \quad \text{or} \quad \alpha = 2\pi + \beta \quad \text{or} \quad \alpha = 3\pi - \beta \quad \text{or} \quad \ldots$$

IB exam - problem 4

[Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ) = \cos(x+30^\circ)$, $0^\circ \le x \le 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \dfrac{1}{\sqrt{2}}$. [3]

IB exam - problem 4

[Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ) = \cos(x+30^\circ)$, $0^\circ \le x \le 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \dfrac{1}{\sqrt{2}}$. [3]

Note that this is part of a longer question which involved topics we haven't covered yet.

IB exam - problem 4

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(a) Solve $2\sin(x+60^\circ)=\cos(x+30^\circ)$, $0^\circ \leq x \leq 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \frac{1}{\sqrt{2}}$. [3]

Note that this is part of a longer question which involved topics we haven't covered yet.
For part (a) it makes sense to apply the formula for the sine and cosine of sums.

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

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Now this becomes:

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3}\cos x = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3}\cos x = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$

This gives:

$$3\sin x = -\sqrt{3}\cos x$$

IB exam - problem 4 (a)

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This gives:

$$3\sin x = -\sqrt{3}\cos x$$

which is equivalent to:

$$\tan x = -\frac{\sqrt{3}}{3}$$

IB exam - problem 4 (a)

We get:

$$2(\sin x \cos 60^\circ + \sin 60^\circ \cos x) = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

Now this becomes:

$$\sin x + \sqrt{3} \cos x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$$

This gives:

$$3 \sin x = -\sqrt{3} \cos x$$

which is equivalent to:

$$\tan x = -\frac{\sqrt{3}}{3}$$

In the given interval we have only one solution, namely $x = 150^\circ$.

IB exam - problem 4 (b)

For part (b) we can calculate both sine and cosine separately using angles 60° and 45°, so that we have:

IB exam - problem 4 (b)

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$$
\begin{aligned}
LHS &= \sin 105^\circ + \cos 105^\circ = \\
&= \sin(60^\circ + 45^\circ) + \cos(60^\circ + 45^\circ) = \\
&= \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ + \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ = \\
&= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = RHS
\end{aligned}
$$

$\square$

IB exam - problem 4 (b)

Alternatively we can change $\cos 105^\circ$ into $-\sin 15^\circ$ and apply the formula for difference of sines:

IB exam - problem 4 (b)

Alternatively we can change $\cos 105^\circ$ into $-\sin 15^\circ$ and apply the formula for difference of sines:

$$\begin{aligned} LHS &= \sin 105^\circ + \cos 105^\circ = \\ &= \sin 105^\circ - \sin 15^\circ = \\ &= 2\sin 45^\circ \cos 60^\circ = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = RHS \end{aligned}$$

$\square$

That's it. That's all the basics. Note that all IB questions will require you to solve trigonometric equations in a specific interval, but it's still useful to be aware of the general solution. Make sure you understand all examples discussed in the presentation. If you have any questions you can email me at t.j.lechowski@gmail.com.