Mixed examination practice 18

Short questions

1. Find $\frac{dy}{dx}$ for each of the following:

(a) $y = x^2 \arcsin x$

(b) $xe^y = 4y^2$ *[7 marks]*

2. Differentiate $f(x) = \arccos(1 - x^2)$. *[4 marks]*

3. Find the exact value of the gradient of the curve with equation

$y = \frac{1}{4 - x^2}$ when $x = \frac{1}{2}$. *[5 marks]*

4. Find the equation of the normal to the curve with equation

$4x^2 + xy^2 - 3y^3 = 56$ at the point (–5, 2). *[7 marks]*

5. Given that $y = \arctan(x^2)$ find $\frac{d^2y}{dx^2}$. *[5 marks]*

6. Find the gradient of the curve with equation $4\sin x \cos y + \sec^2 y = 5$

at the point $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. *[6 marks]*

7. The graph of $y = xe^{-kx}$ has a stationary point when $x = \frac{2}{5}$.
Find the value of k. *[4 marks]*

8. A curve has equation $f(x) = \frac{a}{b + e^{-cx}}$, $a \neq 0, b, c > 0$.

(a) Show that $f''(x) = \frac{ac^2 e^{-cx}(e^{-cx} - b)}{(b + e^{-cx})^3}$.

(b) Find the coordinates of the point on the curve where $f''(x) = 0$.

(c) Show that this is a point of inflexion. *[8 marks]*

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9. Find the coordinates of stationary points on the curve with equation

$(y - 2)^2 e^x = 4x$. *[7 marks]*

Long questions

1. A curve has equation $y = \frac{x^2}{1-2x}$.

(a) Write down the equation of the vertical asymptote of the curve.

(b) Use differentiation to find the coordinates of stationary points on the curve.

(c) Determine the nature of the stationary points.

(d) Sketch the graph of $y = \frac{x^2}{1-2x}$. *[15 marks]*

2. The function f is defined by $f(x) = \frac{x^2}{2^x}$, for $x > 0$.

(a) (i) Show that $f'(x) = \frac{2x - x^2 \ln 2}{2^x}$.

(ii) Obtain an expression for $f''(x)$, simplifying your answer as far as possible.

(b) (i) Find the exact value of x satisfying the equation $f'(x) = 0$.

(ii) Show that this value gives a maximum value for $f(x)$.

(c) Find the x-coordinates of the two points of inflexion on the graph of f. *[12 marks]*

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3. Let $f(x) = \arccos(\sqrt{1-9x^2})$ for $0 < x < \frac{1}{3}$.

(a) Show that $f'(x) = \frac{3}{\sqrt{1-9x^2}}$.

(b) Show that $f''(x) > 0$ for all $x \in]0, \frac{1}{3}[$.

(c) Let $g(x) = \arccos(kx)$. If $g'(x) = -pf'(x)$ for $0 < x < \frac{1}{3}$, find the values of p and k. *[12 marks]*

4. A curve is given by the implicit equation $x^2 - xy + y^2 = 12$.

(a) Find the coordinates of the stationary points on the curve.

(b) Show that at the stationary points, $(x-2y)\frac{d^2y}{dx^2} = 2$.

(c) Hence determine the nature of the stationary points. *[16 marks]*

5. If $f(x) = \sec x$, $0 \le x \le \pi$ the inverse function is $f^{-1}(x) = \text{arcsec}\, x$.

(a) Write down the domain of $\text{arcsec}\, x$.

(b) Sketch the graph of $y = \text{arcsec}\, x$.

(c) Show that the derivative of $\sec x$ is $\sec x \tan x$.

(d) Find the derivative of $\text{arcsec}\, x$ with respect to x, justifying carefully the sign of your answer. *[12 marks]*

(b) (i) $\frac{2}{4+x^2}$ (ii) $\frac{10}{25+4x^2}$

(c) (i) $\arcsin x+\frac{x}{\sqrt{1-x^2}}$

(ii) $2x\arccos x-\frac{x^2}{\sqrt{1-x^2}}$

(d) (i) $\frac{2x}{1+(x^2+1)^2}$

(ii) $\frac{-2x}{\sqrt{1-(1-x^2)^2}}$

2. $-\frac{3}{\sqrt{35}}$

4. $\frac{dy}{dx}=-\frac{1+\tan^2\left(\frac{1}{x}\right)}{x^2}$

5. (a) $\arcsin x+\frac{x}{\sqrt{1-x^2}}$

(b) $x\arcsin x+\sqrt{1-x^2}+c$

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Short questions

1. (a) $2x\arcsin x+\frac{x^2}{\sqrt{1-x^2}}$ (b) $\frac{e^y}{8y-xe^y}$

2. $\frac{2x}{\sqrt{1-(1-x^2)^2}}$

3. $\frac{16}{225}$

4. $y=\frac{14}{9}x+\frac{88}{9}$

5. $\frac{2(1-3x^4)}{(1+x^4)^2}$

6. $-\frac{1}{7}$

7. $\frac{5}{2}$

8. (b) $-\ln\frac{b}{c},\frac{a}{2b}$

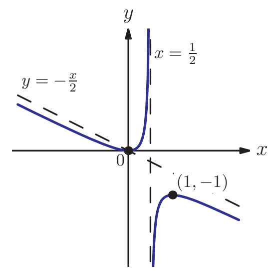
9. $\left(1,\frac{2\sqrt{e}-2}{\sqrt{e}}\right),\left(1,\frac{2\sqrt{e}+2}{\sqrt{e}}\right)$

Long questions

1. (a) $x=\frac{1}{2}$ (b) $(0,0),(1,-1)$

(c) $(0,0)$ local min $(1,-1)$ local max

(d)



2. (a) (ii) $\frac{(\ln 2)^2x^2-4x\ln 2+2}{2^x}$

(b) (i) $\frac{2}{\ln 2}$ (c) $\frac{2\pm\sqrt{2}}{\ln 2}$

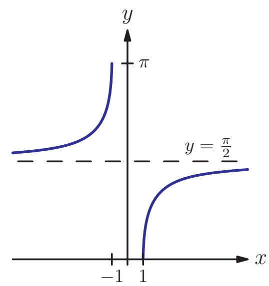
3. (c) $k=3, p=1$

4. (a) $(2,4),(-2,-4)$

(c) $(2,4)$ local max; $(-2,-4)$ local min

5. (a) $x\geq 1, x\leq -1$

(b)



(d) $\frac{1}{x\sqrt{x^2-1}}$

Chapter 19

Exercise 19A

1. (a) (i) $(x+3)^5+c$ (ii) $\frac{1}{6}(x-2)^6+c$

(b) (i) $\frac{1}{32}(4x-5)^8+c$ (ii) $2\left(\frac{1}{8}x+1\right)^4+c$

(c) (i) $-\frac{8}{7}\left(3-\frac{1}{2}x\right)^7+c$ (ii) $-\frac{1}{9}(4-x)^9+c$

(d) (i) $\frac{1}{3}(2x-1)^{\frac{3}{2}}+c$ (ii) $-\frac{4}{5}(2-5x)^{\frac{7}{4}}+c$

(e) (i) $4\left(2+\frac{x}{3}\right)^{\frac{3}{4}}+c$ (ii) $2(4-3x)^{-1}+c$