Self-assessment: 16 Basic differentiation and its applications

1. Differentiate the following:

   (a) $\sqrt{x} - \dfrac{1}{\sqrt{x}}$

   (b) $\tan x + 2 \cos x$

   (c) $x^2 - e^x$

   (d) $3 \ln x - 1$ 	*[8 marks]*

2. Do not use a calculator to answer this question.

   Find the equation of the normal to the curve $y = 2x - \ln x$ at the point where $x = 3$.

   (accessible to students on the path to grade 3 or 4) [6 marks]

3. Find the exact coordinates of the stationary point on the graph of $y = 3e^x - x$.

   (accessible to students on the path to grade 3 or 4) [6 marks]

4.   (a) (i) Expand and simplify $(x + h)^2 - x^2$.

      (ii) Hence prove from first principles that the derivative of x^2 is $2x$.

   (accessible to students on the path to grade 5 or 6)

   (b) The function f is defined by $f(x) = x^2 + 4 \cos x$ for $0 < x < \pi$.

      (i) By considering the graphs of $y = x$ and $y = 2 \sin x$, show that $f(x)$ has only one

         stationary point, and explain why this stationary point is between $\dfrac{\pi}{2}$ and π.

      (ii) Find $f''(x)$ and hence prove that the stationary point is a minimum.

      (iii) Find the coordinates of the point of inflection on the graph of $y = f(x)$.

      (iv) Sketch the graph of $y = f(x)$, clearly labelling the stationary point and the point of
         inflection.

   (accessible to students on the path to grade 7)

   [19 marks]