Self-assessment answers: 16 Basic differentiation and its applications

1. (a) $\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$ or $\frac{1}{2\sqrt{x}}+\frac{1}{2x\sqrt{x}}$ or $\frac{x+1}{2x\sqrt{x}}$

(b) $\sec^2 x - 2\sin x$

(c) $2x - \mathrm{e}^x$

(d) $\frac{3}{x}$ *[8 marks]*

2. $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{1}{x}$

When $x = 3$: $y = 6 - \ln 3$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2-\frac{1}{3}=\frac{5}{3} \Rightarrow m=-\frac{3}{5}$

Equation of normal: $y-(6-\ln 3)=-\frac{3}{5}(x-3)$

(or $y=-\frac{3}{5}x+\frac{39}{5}-\ln 3$) *[6 marks]*

3. $\frac{\mathrm{d}y}{\mathrm{d}x}=3\mathrm{e}^x-1=0$ when $\mathrm{e}^x=\frac{1}{3} \Leftrightarrow x=\ln\left(\frac{1}{3}\right)$

$y=3\left(\frac{1}{3}\right)-\ln\left(\frac{1}{3}\right)=1-\ln\left(\frac{1}{3}\right)=1+\ln 3$

Stationary point is $(-\ln 3, 1+\ln 3)$ *[6 marks]*

4. (a) (i) $(x + h)^2 - x^2 = x^2 + 2xh + h^2 - x^2 = 2xh + h^2$

(ii) Let $f(x) = x^2$. Then,

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = 2x + h$$

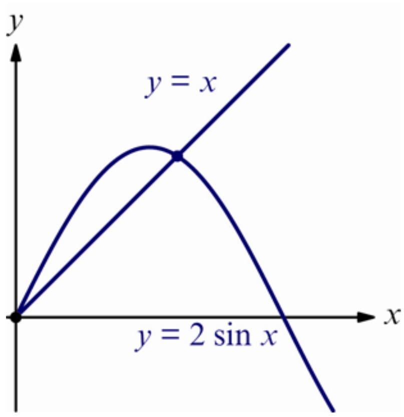
$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 2x$$

$\therefore\ f'(x) = 2x$

(b) (i) Stationary point when $f'(x) = 0$:

$2x - 4\sin x = 0$

$\Leftrightarrow\quad x = 2\sin x$



As $y = 2\sin x$ has gradient 2 at the origin and $y = x$ has gradient 1, the graphs intersect once, hence there is only one stationary point of $f(x)$. It has $x > \frac{\pi}{2}$ because $2\sin x > x$ when $x = \frac{\pi}{2}$.

(ii) $f''(x) = 2 - 4\cos x$

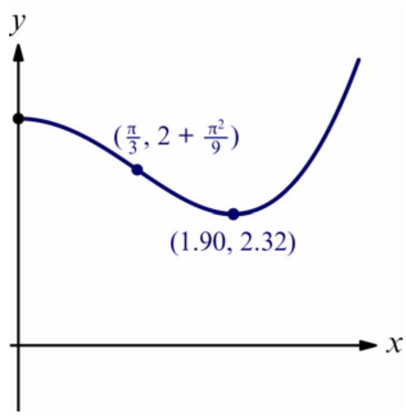
The stationary point has $x > \frac{\pi}{2}$, so $\cos x < 0$ and $f''(x) > 0$.

Hence the stationary point is a minimum.

(iii) $f''(x) = 0 \quad \Leftrightarrow \quad 2 - 4\cos x = 0 \quad \Leftrightarrow \quad \cos x = \frac{1}{2}$

$0 < x < \pi \ \therefore\ x = \frac{\pi}{3},\ y = 2 + \frac{\pi^2}{9}$

(iv)



[19 marks]

