Self-assessment: 18 Further differentiation methods

1. Differentiate the following expressions with respect to x:

(a) $(2x + 1)^5$

(b) $\cos^3(2x)$

(c) $\operatorname{arctan}(x^2)$

(d) $\dfrac{2^x}{x}$ *[9 marks]*

2. Do not use a calculator to answer this question.

Find the x-coordinate of the stationary point on the graph of $y = \dfrac{e^{-2x}}{x^2}$.

(accessible to students on the path to grade 5 or 6) [5 marks]

3. Find the gradient of the curve with equation $\ln(y^2) + 3x^2 = 12$ at the point (2, 1).

(accessible to students on the path to grade 5 or 6) [5 marks]

4. Given that $f(x) = x \sin(ax)$ with $a > 0$,

(a) Find $f'(x)$ and $f''(x)$.

(accessible to students on the path to grade 5 or 6)

(b) (i) Show that the stationary points of $f(x)$ satisfy the equation $\tan(ax) = -ax$.

(ii) Use a graph to show that the above equation has only one solution for $x \in \left(-\dfrac{\pi}{2a}, \dfrac{\pi}{2a}\right)$.

(iii) Hence find the coordinates of the stationary point on the graph of $y = f(x)$ and determine its nature.

(accessible to students on the path to grade 7)

(c) Find the value of a for which $f(x)$ satisfies the equation $f''(x) + 4f(x) = 2a\cos(ax)$.

(accessible to students on the path to grade 5 or 6)

[14 marks]