Self-assessment answers: 18 Further differentiation methods

1. (a) $5(2x+1)^4 \times 2 = 10(2x+1)^4$

(b) $3\cos^2(2x) \times (-\sin(2x)) \times 2 = -6\cos^2(2x)\sin(2x)$

(c) $\dfrac{2x}{x^4+1}$

(d) $\dfrac{2^x x\ln 2 - 2^x}{x^2}$ *[9 marks]*

2. $\dfrac{dy}{dx} = \dfrac{-2x^2e^{-2x} - 2xe^{-2x}}{x^4}$

$\dfrac{-2e^{-2x}(x+1)}{x^3}$

$= 0$ when $x = -1$ (as $e^{-2x} \neq 0$) *[5 marks]*

3. $\dfrac{2y}{y^2}\dfrac{dy}{dx} + 6x = 0$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} + 3x = 0$

At (2, 1) : $\dfrac{dy}{dx} + 6 = 0$

$\Rightarrow \dfrac{dy}{dx} = -6$ *[5 marks]*

4. (a) $f'(x) = \sin(ax) + ax\cos(ax)$

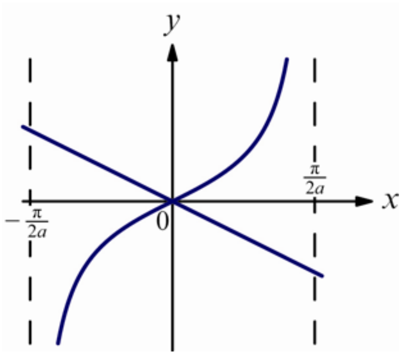
$f''(x) = a\cos(ax) + a\cos(ax) - a^2x\sin(ax)$

$= 2a\cos(ax) - a^2x\sin(ax)$

(b) (i) $f'(x) = 0 \Rightarrow \sin(ax) = -ax\cos(ax)$

$\Rightarrow \tan(ax) = -ax$

(ii)



The only intersection is $x = 0$.

(iii) $f(0) = 0$

$f''(0) = 2a > 0$

So (0, 0) is a minimum point.

(c) $f''(x) + 4f(x) = 2a\cos(ax)$

$\Leftrightarrow \quad 2a\cos(ax) - a^2x\sin(ax) + 4x\sin x = 2a\cos(ax)$

$\Leftrightarrow \quad 2a\cos(ax) + (4 - a^2)x\sin x = 2a\cos(ax)$

This is satisfied when $4 - a^2 = 0$.

$a > 0 \therefore a = 2$ *[14 marks]*