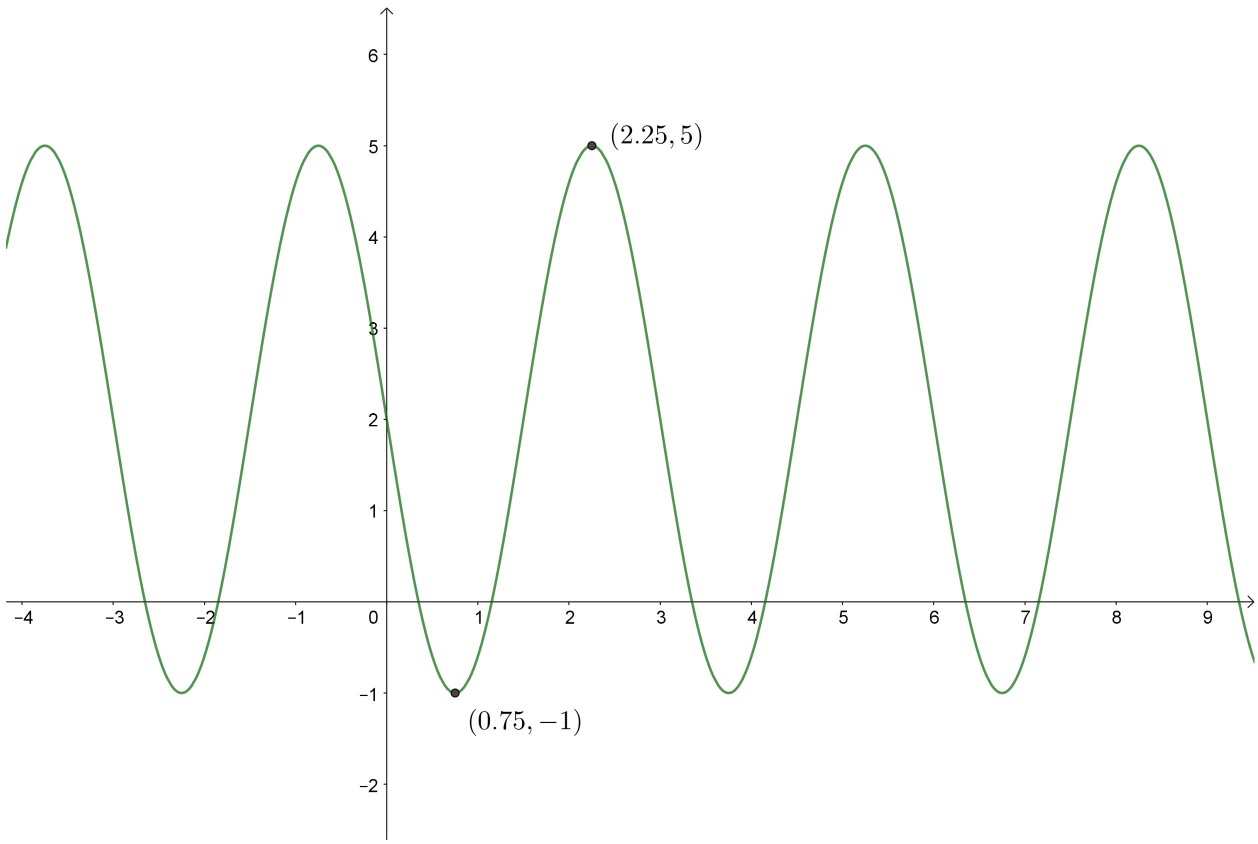
Name:

1. (4 points) The following diagram shows the graph of a function $f(x) = a\sin(bx) + c$, where $a, b, c, \in \mathbb{R}$.

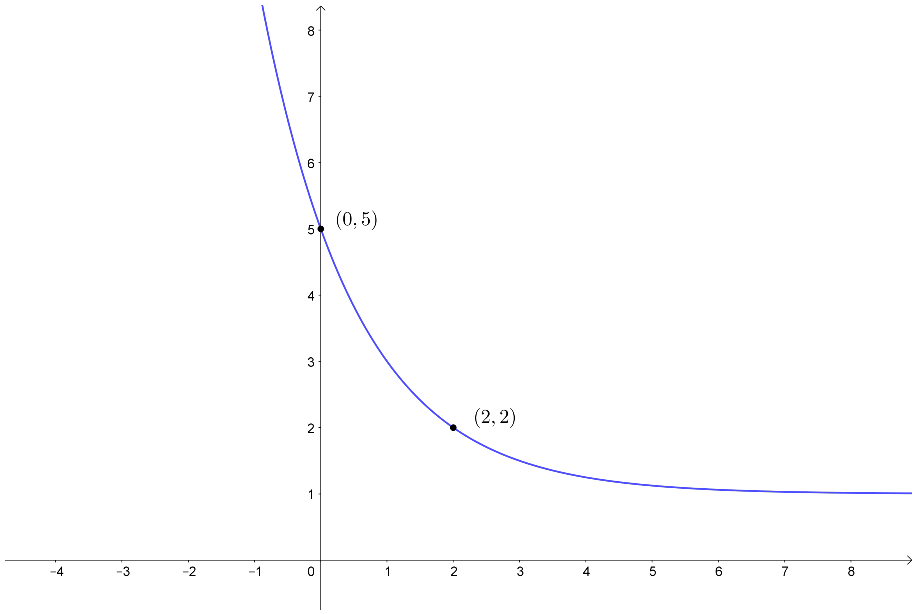


Find the values of a, b and c.

The maximum and minimum values are 5 and -1 respectively, so the principle axis is $y = 2$ (so $c = 2$) and the amplitude is 3. However the graph is reflected in x-axis so $a = -3$.

Half of the period is 1.5, so the period is 3. This makes $b = \frac{2\pi}{3}$.

2. (4 points) The following diagram shows the graph of the function $f(x) = A \times 2^{-x} + B$, where $A, B \in \mathbb{R}$.



(a) Find the values of A and B.

We set up two equations using the points:

$$\begin{cases} A + B = 5 \\ \frac{1}{4}A + B = 2 \end{cases}$$

Solving the above gives $A = 4$ and $B = 1$.

(b) Write down the equation of the horizontal asymptote of the graph of $y = f(x)$.

$$y = 1$$

(c) Solve the inequality

$$f(x) > \frac{17}{16}$$

We can solve the equation first:

$$4 \times 2^{-x} + 1 = \frac{17}{16}$$

which gives $x = 6$. So the solution to the inequality is $x < 6$.

Or we can just solve the inequality directly:

$$4 \times 2^{-x} + 1 > \frac{17}{16}$$

$$2^{-x} > \frac{1}{64}$$

$$2^{-x} > 2^{-6}$$

$$-x > -6$$

$$x < 6$$

3. (4 points) Polynomial $P(x) = 4x^3 + 5x^2 + ax + b$ is divisible by $(x + 2)$, and when divided by $(x - 1)$ there is a remainder of 6. Find the values of a and b.

We have the following information:

$$\begin{cases} P(-2) = 0 \\ P(1) = 6 \end{cases}$$

These give the following equations:

$$\begin{cases} -2a + b = 12 \\ a + b = -3 \end{cases}$$

Solving gives $a = -5$ and $b = 2$.

4. (4 points) Let $p = \log_a x$ and $q = \log_a y$. Show that:

(a) $\log_{xy} a = \dfrac{1}{p+q}$

(b) $\log_{\frac{x}{y}} a = \dfrac{1}{p-q}$

(a)

$$LHS = \log_{xy} a = \frac{\log_a a}{\log_a xy} = \frac{1}{\log_a x + \log_a y} = \frac{1}{p+q} = RHS$$

$\square$

(b)

$$LHS = \log_{\frac{x}{y}} a = \frac{\log_a a}{\log_a \frac{x}{y}} = \frac{1}{\log_a x - \log_a y} = \frac{1}{p-q} = RHS$$

$\square$

5. (4 points) Solve the simultaneous equations:

$$\begin{cases} \log_3 x + 4\log_9 y = 2 \\ 2\log_4 x + \log_2 y = 1 \end{cases}$$

Change the base to 3 in the first equation and 2 in the second:

$$\begin{cases} \log_3 x + 4\frac{\log_3 y}{\log_3 9} = 2 \\ 2\frac{\log_2 x}{\log_2 4} + \log_2 y = 1 \end{cases}$$

Simplify:

$$\begin{cases} \log_3 x + 2\log_3 y = 2 \\ \log_2 x + \log_2 y = 1 \end{cases}$$

Combine the logs:

$$\begin{cases} \log_3(xy^2) = 2 \\ \log_2(xy) = 1 \end{cases}$$

Use the definition of logs:

$$\begin{cases} xy^2 = 9 \\ xy = 2 \end{cases}$$

Dividing the first equation by the second gives $y = \frac{9}{2}$, which then gives $x = \frac{4}{9}$.