Name:

1. (4 points) Consider the following infinite series:

$$\ln x + (\ln x)^2 + (\ln x)^3 + \ldots$$

(a) Find the values of x for which the series converges.

(b) Find the value of x for which the sum of the series is 1.

2. (5 points) Let $\alpha \in \left(\pi, \dfrac{3\pi}{2}\right)$ with $\tan \alpha = \dfrac{2}{3}$. Calculate:

(i) $\sin \alpha$

(ii) $\sin 2\alpha$

(iii) $\sin 3\alpha$

3. (5 points) Find the following limits:

(a) $\displaystyle\lim_{x\to\infty} \frac{2x^2 + 2x + 1}{1 - x^2}$

(b) $\displaystyle\lim_{x\to 3} \frac{x^2 - 4x + 3}{x^2 - 5x + 6}$

(c) $\displaystyle\lim_{x\to 0} \frac{\sqrt{4 + x} - 2}{x}$

(d) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x^2 + 3x} - 3x}{x + 1}$

(e) $\displaystyle\lim_{x\to 0} \frac{\sin^2(2x)}{x^2}$

4. (6 points) Consider the function

$$f(x) = \ln x + \ln(x - 1) - \ln(x^2 - 1)$$

(a) Find the domain of $f(x)$.

(b) Write $f(x)$ in the form $\ln\left(\dfrac{x}{x + a}\right)$, where a is a constant to be found.

(c) Find the inverse of $f(x)$.