Name:

1. (4 points)

(a) Differentiate $f(x) = 3x^2 + x - 1$ from the first principles.

$$f'(x) = \lim_{\Delta x \to 0} \frac{3(x + \Delta x)^2 + (x + \Delta x) - 1 - 3x^2 - x + 1}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} \frac{6x\Delta x + (\Delta x)^2 + \Delta x}{\Delta x} =$$
$$= \lim_{\Delta x \to 0} (6x + \Delta x + 1) = 6x + 1$$

(b) Hence find the gradient of tangent line to the graph of f when $x = 2$.

$$f'(2) = 6 \times 2 + 1 = 13$$

(c) Find the coordinates of the point on the graph of f, at which the gradient is -1.

$$f'(x) = -1 \quad \Rightarrow \quad x = -\frac{1}{3}$$

$$f\left(-\frac{1}{3}\right) = -1$$

The point has coordinates $\left(-\frac{1}{3}, -1\right)$.

2. (6 points) Find the second derivative of each of the following functions:

(a) $f(x) = xe^{2x}$

$$f'(x) = e^{2x} + 2xe^{2x}$$

$$f''(x) = 2e^{2x} + 2e^{2x} + 4xe^{2x} = e^{2x}(4 + 4x)$$

(b) $g(x) = \sin(x^2 + 1)$

$$g'(x) = 2x\cos(x^2 + 1)$$

$$g''(x) = 2\cos(x^2 + 1) - 4x^2\sin(x^2 + 1)$$

(c) $h(x) = \sqrt{x} + \ln(\sin x)$

$$h'(x) = \frac{1}{2\sqrt{x}} + \cot x$$

$$h''(x) = -\frac{1}{4x\sqrt{x}} - \csc^2 x$$

3. (4 points) Solve the following equation:

$$2\cos^3 x = -3\sin x \cos x$$

for $0 \leqslant x \leqslant 2\pi$.

We move all terms to one side and factor out $\cos x$:

$$\cos x(2\cos^2 x + 3\sin x) = 0$$

So $\cos x = 0$ or $2\cos^2 x + 3\sin x = 0$.

The first equation gives $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$.

Now to solve the second equation we use the double angle identity:

$$1 - 2\sin^2 x + 3\sin x = 0$$

So we have:

$$0 = 2\sin^2 x - 3\sin x - 1$$

Factoring gives:

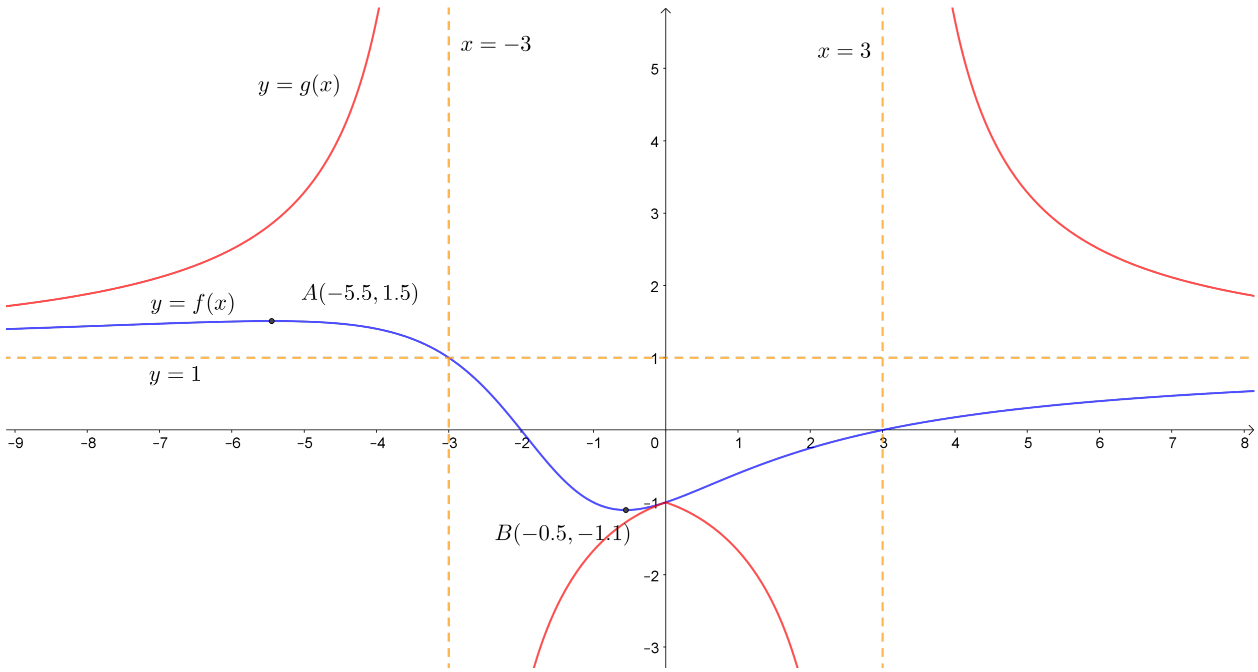
$$0 = (2\sin x + 1)(\sin x - 2)$$

So we have $\sin x = -\frac{1}{2}$ or $\sin x = 2$. The first equation gives $x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$. The second equation has of course no solutions. So in the end we have:

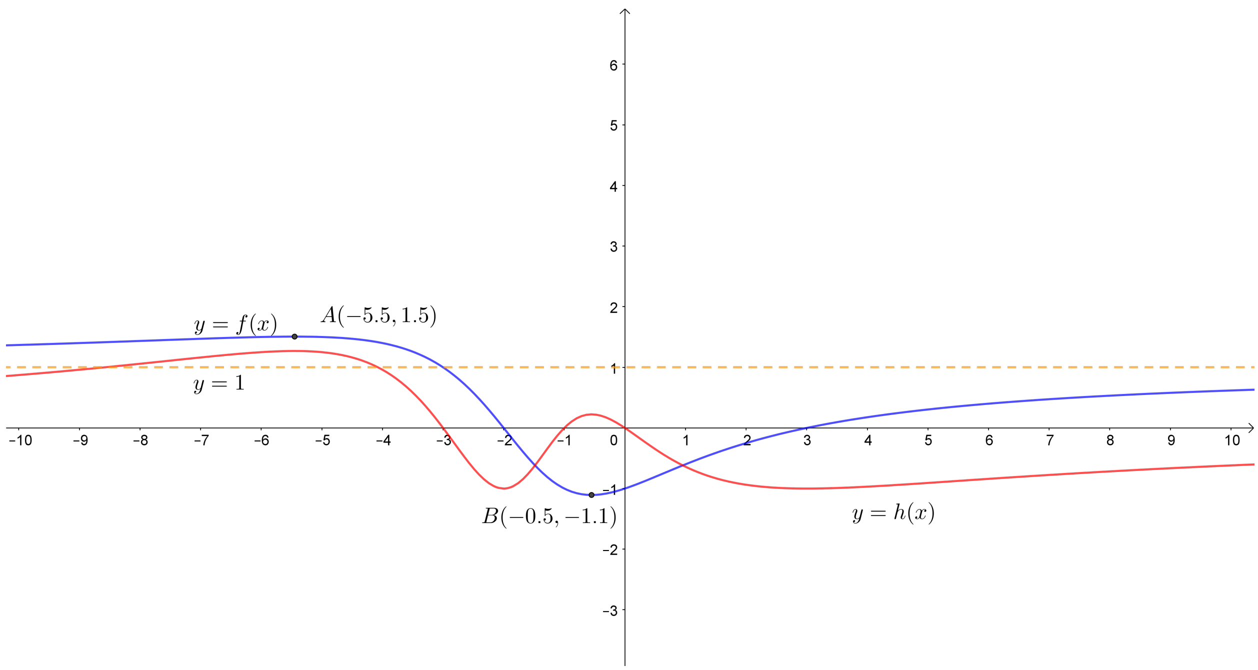
$$x \in \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$$

4. (6 points) The graph of $f(x)$ is shown below. Use the diagrams to sketch the graph of

(a) $g(x) = \dfrac{1}{f(|x|)}$



(b) $h(x) = [f(x)]^2 - 1$



y = f(x)
A(−5.5, 1.5)
y = 1
B(−0.5, −1.1)
y = h(x)
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
10
6
5
4
3
2
1
-1
-2
-3