1. Find the equation of the normal to the curve $x^3y^3 - xy = 0$ at the point (1, 1).

(Total 7 marks)

2. A curve C is defined implicitly by $xe^y = x^2 + y^2$. Find the equation of the tangent to C at the point (1, 0).

(Total 7 marks)

3. Show that the points (0, 0) and $(\sqrt{2\pi}, -\sqrt{2\pi})$ on the curve $e^{(x + y)} = \cos(xy)$ have a common tangent.

(Total 7 marks)

4. The function f is defined by $f(x) = e^{x^2 - 2x - 1.5}$.

(a) Find $f'(x)$.

(2)

(b) You are given that $y = \dfrac{f(x)}{x-1}$ has a local minimum at $x = a$, $a > 1$. Find the value of a.

(6)
(Total 8 marks)

5. (a) Differentiate $f(x) = \arcsin x + 2\sqrt{1-x^2}$, $x \in [-1, 1]$.

(3)

(b) Find the coordinates of the point on the graph of $y = f(x)$ in [–1, 1], where the gradient of the tangent to the curve is zero.

(3)
(Total 6 marks)

6. Let f be a function defined by $f(x) = x - \arctan x$, $x \in \mathbb{R}$.

(a) Find $f(1)$ and $f(-\sqrt{3})$.

(2)

(b) Show that $f(-x) = -f(x)$, for $x \in \mathbb{R}$.

(2)

(c) Show that $x - \frac{\pi}{2} < f(x) < x + \frac{\pi}{2}$, for $x \in \mathbb{R}$.

(2)

(d) Find expressions for $f'(x)$ and $f''(x)$. Hence describe the behaviour of the graph of f at the origin and justify your answer.

(8)

(e) Sketch a graph of f, showing clearly the asymptotes.

(3)

(f) Justify that the inverse of f is defined for all $x \in \mathbb{R}$ and sketch its graph.

(3)
(Total 20 marks)